APPLIED ARITHMETIC

THE THREE ESSENTIALS

BOOK 3

LENNES AND JENKINS

THE COVER DESIGN

The boy and girl are about to push open the dark gates of Ignorance, which, when closed, keep them from entering the threshold of Education, where abide Happiness, Content and Comfort.

Based on Applied Arithmetic, they have a solid foundation on which to stand while forcing back the gates.

The theme of the border design includes the primitive counting method of the Indians, pictured in the knotted leather thongs. The artist has employed in the corners the devices of the ancient abacus and the modern type of mechanical calculator.

LENNES AND JENKINS

LIPPINCOTT'S SCHOOL TEXT SERIES
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APPLIED ARITHMETIC
THE THREE ESSENTIALS

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BOOK III

PHILADELPHIA, LONDON, CHICAGO
J. B. LIPPINCOTT COMPANY

Electrotyped and Printed by J. B. Lippincott Company
The Washington Square Press, Philadelphia, U. S. A.

PREFACE

THIS book is the third of a three-book series, and is intended to cover the work in arithmetic of the seventh and eighth grades. The principles which have guided the authors may be grouped under three main heads:

1. *Selection and Organization of Subject Matter.* Recent discussion and practice, as revealed in the literature and in published curricula, seem to show substantial agreement as to what topics should be included in the earlier parts of a course in arithmetic, while there is yet considerable difference of opinion as to topics to be included in the later parts. All topics whose inclusion or exclusion is now being debated are placed among the supplementary topics at the end of Books Two and Three. The main body of each book furnishes a minimum course which may be studied without break in continuity. At convenient points in the text supplementary topics may be taken up. It is believed that in this respect these books will serve each of many different needs just as effectively as if they were prepared to meet such need exclusively. The business of the maker of text books is to furnish the teachers and supervising officers effective instruments for carrying out their purposes, rather than to seek to impose rigidly his own personal predilections.

It has been the purpose of the authors to arrange the subject matter in such a manner that the greatest simplicity of treatment may be attained. In the third book this is exemplified by the consistent use of the principle of product and factors in the solution of problems, as shown on pages 90, 103, 116. If the teacher thinks best, a symbol such as x may now be used to represent the unknown number. The development is so organized that this will not disturb the continuity in the slightest. In any event, the solution of these problems is effected by means of the simple ideas developed in Book One in connection with such simple combina-

tions as $3 \times 4 = 12$ and $2 \times 3 \times 4 = 24$. See page 262 for a summary of the use of the equation or the principle of product and factors.

2. *Derivation and Application.* The subject matter of arithmetic may be divided into two parts:

(*a*) The four fundamental operations on integers and common and decimal fractions.

(*b*) The application of these operations to the solution of problems.

In this book the fundamental operations are reviewed at the beginning of each year. It is believed that such reviews are necessary after any extended period of vacation, and that the text should provide the necessary material at appropriate points.

In the applications of arithmetic the only new element is the situation which gives rise to the problem. In percentage there is nothing new except the fact that *hundredths* or *per cents* are used systematically. In discount, the only new element is the fact that a reduction in price stated as a certain number of hundredths or per cents of the original price is often made. Similar remarks apply to commission, interest, profit and loss, taxes, insurance, and other applications of percentage. A serious effort has been made to make this clear to the child, and to put him in the way of obtaining solid information about the only new matter with which he is confronted as each topic is taken up. On this point see pages 84, 102, 122, 126, 131, 134, etc.

3. *Motivation.* The subject matter of arithmetic can be motivated most effectively only when the freest possible use is made of the child's many spontaneous interests. The authors believe they have not neglected any opportunity to interest the child in the subject matter itself and its manifold applications. They have recognized, however, that it is possible to so connect the learning of arithmetic with other activities which in themselves are of compelling interest to the child, that the combination will be a source of joy and life when the arithmetic elements alone would lead to sadness and forced labor. For this reason material has

been provided which may be made the basis for systematic use of group competition. See pages 32, 72, 120, 121.

The most effective applications of arithmetic can be made only when considerable local material is brought in. In the upper grades such material is also an important element in motivation. The child is becoming more and more interested in the activities of the grown-up. He is curious to know how things are really done, and if properly directed will gladly gather information from his own environment. For this reason many suggestions for bringing in local material are made throughout the book.

Some other points may be mentioned. It is found on investigation that the fractions which are in common use are very simple. Denominators other than 2, 3, 4, 8, 12, 16, 32, occur so seldom as to be almost negligible. In fractions to be added, subtracted, or divided they may be said to not occur at all. A problem like $3/5 + 2/7$ is as rare in practical life as the buffalo on our Western prairies. For this reason much practice has been given on the manipulation of very simple fractions, while the more complicated fractions have been given less space. See pages 50, 51, 55, 56, 172, 173, 174, 175.

As in Books One and Two the standard methods of subtraction have been given equal prominence. The addition method has been carried through consistently in fractions and in denominate numbers. See pages 14, 42, 56.

A simple form for family accounts is given on pages 10 to 13. The table of food values on pages 334, 335 will enable the teacher to make many interesting problems on costs of foods. The farm accounts given on pages 38 to 40 are such as are used by progressive farmers. On pages 104 to 106 are given the different standard methods for computing interest. Any one of these methods may be selected without the slightest break in continuity of development. In this respect as in many others these books leave the teacher and the supervising officers free to select their own method. The important subject of stocks and bonds is developed with simplicity and thoroughness. See pages 186 to 188.

The comparative study on page 189 of the applications of percentage should be helpful.

No effort has been spared to make the books attractive in appearance and convenient in the arrangement of subject matter on the page.

The reason for producing this series lies, not in any one of the features mentioned here, but in the belief that, by careful and systematic use of all that is best in present knowledge and usage, it should be possible to produce a series of texts that would more adequately meet the requirements of the modern school than is done by any texts now in existence.

These books have been built leisurely. The first draft was made nearly ten years ago. During the intervening time the work of selecting what has proved most certainly valuable, both in what is old and what is new, and in organizing and relating the various parts, has been in constant progress. It is difficult, if not impossible to make proper acknowledgment to all who helped in this work. The most prominent among these, however, have been Dr. Theodore Lindquist, Head of the Department of Mathematics in the State Normal School at Emporia, Kansas, Mr. H. C. Pearson, Principal of the Horace Mann School in New York City, and several of the Horace Mann teachers.

THE AUTHORS.

DECEMBER, 1919.

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APPLIED ARITHMETIC

THE THREE ESSENTIALS

BOOK III

CHAPTER I

REVIEW OF THE FUNDAMENTAL OPERATIONS

1. **The Subject Matter of Arithmetic.** Arithmetic may be divided into two parts. One part deals with the operations of addition, subtraction, multiplication, and division as applied to whole numbers, to common fractions, and to decimals. The other part deals with the use of these operations in solving problems which arise in the practical affairs of life.

The first part of this book contains a review of the fundamental operations, while the main body of it is devoted to business forms and industrial practices, and to a further study of mensuration.

In connection with the review, considerable drill in the fundamental operations will be given, which should develop greater speed and accuracy in performing them. In practical affairs all computations must be absolutely accurate, and this should be the standard in the schools.

In studying business forms it should be borne in mind that many of these are changing from time to time. Indeed at one and the same time different forms will be found in use in different houses engaged in the same business, even though they are located in the same city. Hence it is not sufficient to become acquainted with any one set of forms. It is necessary to learn to understand the *purpose of these forms*. It is only by *understanding* such purposes and how they are achieved that one will be able to learn quickly the details of any business in which he may be employed.

2. **The Decimal Number System.** The number system which is now used practically everywhere is called the *decimal number system.* This *number system* should not be confused with our *system of notation.* When we speak of the decimal number system we refer simply to the fact that we count to ten, then say eleven (ten and one), twelve (ten and two), etc., up to twenty (two tens), then twenty-one (twenty and one), etc., up to thirty (three tens), and so on. We call ten tens *one hundred*, and ten hundreds *one thousand.*

3. **The Base 10.** We say that 10 is the *base* of our number system. The use of 10 as a base is supposed to have had its origin in the fact that we have ten fingers. Primitive people, like children, often count on their fingers.

4. **The Arabic Notation. Place Value.** Our system of *notation* for numbers, called the *Arabic notation,* is based on the idea of *place value.* That is, the number represented by a figure depends upon its position. Thus, in 728, the 8 represents ones, the 2 represents tens, and the 7 represents hundreds.

5. **Digits and their Order. Periods.** The figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 are often called *digits.* If a digit represents ones, it is said to be of ones' order; if it represents tens, it is said to be of tens' order; and so on.

The digits in a number are grouped in *periods,* as shown here:

Trillions period	Billions period	Millions period	Thousands period	Ones period	Decimals period
896,	220,	193,	485,	728	.294

ORAL EXERCISES

1. What does each digit represent in 1,234,567,890? Name each period.

2. What does each digit represent in 12,345,678.9? In 1,234,567.89? In 123,456.789? In 12,345.6789? In 0.47? In 1.0203?

6. Roman Notation. In the Roman notation seven letters are used to represent numbers. These are:

Letters	I	V	X	L	C	D	M
Values	1	5	10	50	100	500	1000

The other numbers are represented by combinations of these according to the following principles:

1. *If a letter precedes one of greater value its value is subtracted. The values of all other letters are added.*

Thus IV = 5 − 1 = 4, and XXIX = 10 + 10 − 1 + 10 = 29, while VI = 5 + 1 = 6, and XXXI = 10 + 10 + 10 + 1 = 31.

2. *A bar placed above a letter indicates that its value is to be multiplied by* 1000.

Thus, $\bar{\mathrm{V}}$ = 5000; $\bar{\mathrm{C}}$ = 100,000; and $\bar{\mathrm{M}}$ = 1,000,000.

These principles are further illustrated by the following:

XL = 40, CD = 400, $\bar{\mathrm{X}}$DLX = 10,560; $\bar{\mathrm{D}}\bar{\mathrm{L}}\bar{\mathrm{X}}$ = 560,000.

We thus see that in the Roman notation the principle of place value is entirely absent except that the location of a letter shows whether its value is to be added or subtracted.

WRITTEN EXERCISES

Write the following in Roman numerals:

1. 987
2. 298
3. 6460
4. 8290
5. 10400
6. 74940
7. 1914
8. 1916

Write the following in Arabic numerals:

9. CCCLXXXVI
10. LXXXIX
11. DCCLXXV
12. MDCCCLXXX
13. MCMXVI
14. MXDII
15. MDCCLXXV
16. MDCCCLXV
17. MMDCLVII

7. The Unit. One of anything is called a *unit*. This *one* may be a single object or it may be a collection of objects taken together for the purpose of counting.

Thus, we may say that there are 24 marbles in a bag, in which case the unit is one marble. But we may also say that the bag contains two dozen marbles, in which case one dozen is the unit.

In making measurements, the standard of measure used is called the *unit of measure.*

Thus, if we say a room is 36 feet long, the foot is the unit of measure; and if we say the room is 12 yards long, the yard is the unit of measure. In general, we express magnitudes in terms of certain units of measure.

8. Abstract and Concrete Numbers. A number which is stated without giving the unit is called an *abstract number.*

A number stated in terms of a certain unit is called a *concrete number.*

Thus 24, 16, and 34 are abstract numbers, while 24 feet, 16 miles, and 34 boys are concrete numbers.

9. Like Numbers. Numbers expressed in terms of the same unit are called *like numbers.* Abstract numbers are also called like numbers.

10. Reduction to Proper Units. In certain cases it is necessary to reduce numbers to the same unit before performing the required operations. Thus, if we are to find the area of a rectangle whose length is given as 12 yards, and width as 18 feet, we must reduce both dimensions to the same unit before multiplying, choosing feet or yards according as we want the result in square feet or in square yards. Again, if we are required to find how long 100 bushels of grain will last a horse consuming 12 quarts a day, we must reduce to quarts or to bushels.

In such cases the exercise of common sense is all that is needed. Do not forget that rules can never replace good common sense.

11. **Choice of Units.** In making a measurement, the unit to be used depends upon the kind of thing to be measured, and its magnitude. The width of a page of this book would be measured in inches, the width of a door or window probably in feet, and the length of the schoolroom in feet or in yards. The length of a field is usually measured in rods, and the distance between cities in miles. Sometimes different units are used interchangeably for the same quantity to be measured.

ORAL EXERCISES

1. To find the volume of a box 3 yards long, $3\frac{2}{3}$ feet wide, and 18 inches deep, what unit would you use? Reduce the dimensions to this unit.

2. To find the volume of a box $2\frac{1}{2}$ yards long, $2\frac{3}{4}$ feet wide, and 15 inches deep, what unit would you use? Reduce the dimensions to this unit.

3. Name substances whose weight you would give in ounces, in pounds, in tons.

4. Name substances whose measure you would give in pints, in quarts, in gallons, in barrels.

5. Name substances whose measure you would give in cubic inches, in cubic feet, in cubic yards.

6. Name areas which you would give in square inches, in square yards, in square rods, in acres, in square miles.

7. Name substances whose measure you would give in dry quarts, in pecks, in bushels.

8. What is the unit used in stating how much lumber is produced in a state?

9. What is the unit used in stating how much cotton is produced in the United States?

12. Addition. *The process of finding a number which is equal to two or more numbers when taken together is called addition.*

The numbers to be added are called *addends.*

The result obtained in addition is called the *sum.*

Thus, in $8+9=17$, 8 and 9 are addends, and 17 is the sum.

Only like numbers can be added.

13. Drill in Addition. Checking. All operations in arithmetic should be checked so the computer may *know* that the results are correct. The best check in addition is to add carefully, and then to add the columns again in the opposite direction.

279
135
389
803
22

Computers usually add a column downward first. The result is put down in pencil, and the numbers to be carried are written in small figures below. Thus, the sum of the first column is 23. The 3 is written in ones' place, in the sum, and the 2 right below the 3.

WRITTEN EXERCISES

Add the following rapidly, adding each column in both directions:

1.	2.	3.	4.	5.
498	123	765	136	438
273	456	432	391	647
439	789	109	396	684
529	234	786	237	621
234	576	543	846	324
356	980	210	979	376
579	345	533	274	294
543	678	757	354	678
438	901	646	901	342
826	456	868	198	680
547	789	979	234	567
745	466	104	342	198
594	319	982	147	845
276	872	293	298	397
210	421	704	467	782

14. Grouping. Greater speed in addition can be attained by grouping two or more numbers. Numbers whose sum is 10 should always be grouped.

In this example the braces indicate how the numbers may be grouped. We see at once that the sum of the first group is 18; of the second, 9; and of the third, 16. We simply say: 18, 27, 43. Carrying the 4 to the second column we say: 4, 14, 27, 47, 53.

	36	
10	79	18
	83	
13	55	9
	94	
20	37	
	84	16
6	65	
	533	

WRITTEN EXERCISES

In this manner add the following:

(1)	(2)	(3)	(4)	(5)	(6)
45389	3691	4	67	859	359
67123	5432	5	84	176	827
17519	5112	2	39	176	827
3280	2781	9	61	478	377
5014	2137	1	78	478	888
3062	910	7	40	356	999
91395	2311	4	50	1843	333
32104	1667	2	91	946	744
5103	99	5	19	8510	660
73706	333	6	64	7350	470
3217	4632	2	79	187	1908
53518	5161	8	94	6291	1764
7316	3313	2	37	4879	1281
5728	88	9	86	9846	9000
2459	4200	3	79	1046	3748
5192	5193	7	34	9846	394
6334	7164	8	56	1046	275
793	2891	4	78	8845	394
6750	390	5	90	372	275
8000	4615	6	84	542	117
5930	7769	7	43	8192	897

15. Horizontal Addition. For the purpose of practical accounting, horizontal addition is important. (See next page.)

Example. Add 47+34+29+134+390+247+64=945.

As in column addition, add numbers of the same order. Thus, the sum of the numbers in ones' place is 35. Write down 5 and carry 3. The chief difficulty in this kind of addition is to make sure that numbers of the same order are added. However, a little practice will make this fairly easy.

WRITTEN EXERCISES

Add the following horizontally:

1. 41+27+81+36+92+36+87+53=
2. 14+37+61+93+38+22+63+45=
3. 261+93+567+473+374+369+56+83=
4. 50+39+42+97+843+576+493+57=
5. $51.64+16.74+12.60+39.18+47.60+.89+43+25=
6. $1.30+4.60+2.73+3.72+3.67+.91+.84+.34=
7. $0.46+.28+1.50+2.46+1.91+.71+.4.76+.57=
8. $.310+1.30+2.40+42.75+.85+.61+3.80+9.60=
9. 191+13+49+78+23+74+39+465=
10. 476+49+78+705+47+98+73+564=
11. 287+87+39+816+89+461+27+651=
12. 64+64+213+927+164+153+56+379=
13. 39+42+149+338+351+493+55+793=
14. 27+91+250+429+681+493+55+793=
15. 47+384+361+510+394+671+66+397=
16. 17+560+472+608+176+385+77+937=
17. 563+138+583+134+583+724+88+739=

WRITTEN EXERCISES

The following examples show how horizontal addition sometimes saves copying.

In the following reports give the required totals:

1. Report of attendance at a large school.

	Mon.	Tues.	Wed.	Thurs.	Fri.	Totals
1st Grade..........	263	275	272	266	259	
2d Grade...........	247	253	245	250	249	
3d Grade...........	225	220	230	224	229	
4th Grade..........	204	204	208	207	211	
5th Grade..........	191	187	184	195	190	
6th Grade..........	169	174	175	173	171	
7th Grade..........	122	119	124	129	125	
8th Grade..........	102	95	97	99	101	
Totals..........						

2. Report of a circulating library for one week.

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Totals
Fiction......	4365	2960	3435	4194	3892	5438	
History......	1284	1106	1045	1016	1128	1386	
Biography...	964	734	1016	886	843	1167	
Science......	397	408	342	526	385	624	
Poetry......	651	591	476	491	527	854	
Religion.....	260	302	287	293	317	458	
Totals......							

3. Report on the mail handled in one week in a city post office.

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Totals
Ordinary letters.	37694	35094	42934	39745	45678	51942	
Registered letters	2143	1842	1202	1064	1769	2431	
Post-cards......	8670	11943	10645	9937	8674	11794	
Parcels.........	1897	1735	1632	2198	1367	934	
Newspapers.....	59670	67893	61376	71456	76261	69247	
Totals........							

January	Rent	Light	Telephone	Clothing (husband)	Clothing (wife)	Clothing (older child)	Clothing (younger child)	Laundry
1	$30.00	....	$1.50	....	....	....	....	....
2	....	$3.75	....	$.50	$.40	....	$.60	....
3	....	....	....	....	....	....	....	$.40
4	....	....	....	....	....	....	....	....
5	....	....	....	....	3.50	$1.35	....	....
6	....	....	....	....	....	....	....	....
7	....	....	....	....	....	....	....	....
8	....	....	....	2.60	....	....	....	....
9	....	....	....	....	....	....	....	....
10	....	....	....	....	4.20	....	.75	.65
11	....	....	....	....	....	....	....	....
12	....	....	....	....	....	....	....	....
13	....	....	....	....	....	....	....	....
14	....	....	....	1.50	....	....	....	....
15	....	....	....	....	....	....	....	....
16	....	....	....	....	....	....	1.00	....
17	....	....	....	....	....	....	....	.85
18	....	....	....	....	....	....	....	....
19	....	....	....	....	8.60	....	....	....
20	....	....	....	....	....	....	....	....
21	....	....	....	....	....	....	....	....
22	....	....	....	5.00	....	....	....	....
23	....	....	....	....	....	....	....	....
24	....	....	....	....	....	....	....	.90
25	....	....	....	....	....	....	....	....
26	....	....	....	....	....	....	....	....
27	....	....	....	....	....	....	....	....
28	....	....	....	....	3.10	....	....	....
29	....	....	....	3.20	....	....	....	....
30	....	....	....	....	....	....	....	....
31	....	....	....	....	....	....	....	.95
Totals								

Help	Meat	Milk	Groceries	Transportation	Books and papers	Doctor, and medicine	Miscellaneous	Total	Income	Saved
....	$.41	....	$.49	....	....	....	$.35	....	....	....
....	.35	....	.63	....	$.30	....	2.80	....	....	....
$1.50	.85	$1.00	.87	$1.20	....	....	....	....	....	....
....	....	....	....	....	....	....	.50	....	....	....
....	.45	....	.90	....	....	....	....	....	....	....
....	.42	....	.44	.10	....	....	....	....	....	....
....	.36	....	.73	....	....	....	1.60	....	....	....
....	.94	....	.29	....	....	$4.50	....	....	....	....
....	....	....	.34	....	....	.95	....	....	....	....
1.50	.47	1.00	.90	.20	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	.36	....	.45	.10	.04	....	....	....	....	....
....	.39	....	.27	....	....	....	.90	....	....	....
....	.73	....	.38	....	....	....	.47	....	....	....
....	....	....	.74	.20	.02	.10	.25	....	....	....
1.50	.45	....	.97	....	....	....	....	....	....	....
....	....	....	....	.20	....	....	....	....	....	....
....	.27	1.00	.90	.25	....	....	.20	....	....	....
....	.46	....	.47	....	....	....	.20	....	....	....
....	.64	....	....	....	....	....	....	....	....	....
....	.37	....	.61	.20	.04	....	....	....	....	....
1.50	.29	....	.34	....	.03	....	.75	....	....	....
....	.87	....	.81	....	....	....	.30	....	....	....
....	....	....	....	.10	.05	....	.50	....	....	....
....	.53	....	.38	.15	....	....	....	....	....	....
....	.12	1.00	.67	.25	....	....	....	....	....	....
....	.42	....	.76	.10	.10	....	3.60	....	....	....
....	.19	....	.28	....	....	....	....	....	....	....
1.50	.34	....	.43	....	.50	....	....	....	....	....
....	.63	....	.84	.20	.03	.85	.25	....	....	....
....	....	....	....	....	....	....	....	....	....	....
									$148.50	

16. **Expenditures of a Family.** A family consisting of a husband, wife, and two small children decided to keep exact accounts of all money spent for various purposes. At the end of the year they were thus able to see just how their income had been spent, and to study the question as to whether it might have been spent more wisely.

Find the expenditure for each purpose for the month of January as given on pages 10 and 11, and also the total expenditures for this month, and the amount saved. Also find the total daily expenditures. Add horizontally.

On pages 12 and 13 are the expenditures of this family for each month of the year. Find the yearly expenditure for each item, the total income, the total yearly expenditure and the total amount saved. Also find the amount saved each month.

1914	Rent	Light	Telephone	Clothing for husband	Clothing for wife	Clothing for older child	Clothing for younger child	Laundry
January......	$30	$3.75	$1.50	$12.80	$19.80	$1.35	$2.35	$3.75
February.....	30	3.60	1.80	3.60	4.20	3.30	5.70	3.40
March.......	30	3.40	1.40	5.89	3.65	2.60	3.40	2.20
April.........	30	3.32	1.60	18.00	32.45	.75	1.60	2.70
May.........	30	3.16	1.55	.75	4.85	15.60	2.80	2.95
June.........	30	2.15	1.25	25.00	3.65	3.70	4.00	3.10
July..........	30	1.18	.89	4.00	11.70	.50	.75	3.
August.......	30	.56	2.60	12.60	8.90	2.30	1.65	3.65
September....	30	1.80	.55	1.75	26.60	3.65	8.40	3.40
October......	30	2.19	.89	25.00	10.00	1.14	.85	2.50
November....	30	2.46	1.90	4.60	1.50	3.75	2.15	2.10
December....	30	3.86	1.60	5.60	4.50	2.60	1.70	2.60
Totals.....								

After a family have kept such accounts for a number of years, they will be able to see how their tastes have changed, and to consider whether they are spending their income more or less wisely as the years go by.

ORAL EXERCISES

1. Considering the income of this family, which expenditures do you think are too low? Which do you think are too high?

2. Why should a family save some money? Find all the reasons you can for saving money, and also all reasons you can against saving.

3. Why should a family keep account with expenditure? Do people in general keep such accounts? If not, why not?

Help	Meat	Milk	Groceries	Transportation and vacations	Books and papers	Doctors, dentist and medicine	Miscellaneous, including insurance, etc.	Totals	Income	Saved
7.50	11.31	4.00	14.89	3.25	1.11	6.40	12.67	....	148.50	....
6.00	10.60	4.00	16.39	1.65	.60	....	28.06	....	138.50	....
6.00	10.80	4.00	13.41	1.60	3.42	....	7.12	....	145.75	....
6.00	9.65	3.50	12.81	2.30	.57	1.80	6.80	....	163.25	....
7.50	9.90	4.00	11.46	1.45	9.24	....	31.40	....	173.60	....
6.00	8.60	4.00	14.84	2.80	3.41	.45	6.60	....	134.45	....
4.50	8.40	3.50	13.65	2.60	3.61	....	8.90	....	180.20	....
4.50	7.35	3.50	13.80	15.30	.50	3.60	70.60	....	175.40	....
4.50	7.50	3.50	11.40	2.60	3.62	....	5.30	....	139.75	....
4.50	7.70	3.50	12.65	1.40	3.45	....	5.40	....	137.60	....
6.00	8.40	4.00	19.90	10.70	2.26	4.60	3.67	....	148.20	....
6.00	9.20	4.00	14.60	1.90	1.39	....	89.65	....	151.50	....

17. Subtraction. The process of subtraction is defined in two ways:

(a) Subtraction is the process of finding how much must be added to one number to make the sum another given number.

(b) Subtraction is the process of taking one number away from another.

Thus, "What must be added to 7 to make 15?" is a problem in subtraction, as is also "Take 7 from 15."

In this example which number is the *minuend?* Which is the *subtrahend?* What is the *remainder* or *difference?*

Example. From 124071 subtract 94783.

$$\begin{array}{r} 124071 \\ \underline{94783} \\ 29288 \end{array}$$

Explain this process by the method you have learned.

WRITTEN EXERCISES

Heights of Mountains. The height of Mount Everest is 29,002 feet; of Mount Aconcagua, 22,860 feet; of Mount Blanc, 15,781 feet; of Mount McKinley, 20,300 feet; and of Pikes Peak, 14,104 feet.

1. What is the difference between the heights of:
   (a) Mount Everest and Mount Blanc?
   (b) Mount Blanc and Mount McKinley?
   (c) Mount McKinley and Pikes Peak?
   (d) Mount Aconcagua and Mount Everest?

Lengths of Rivers. The Mississippi is 2960 miles long, the Nile, 3370 miles; the Amazon, 4000 miles; the Volga, 2400 miles; and the Yangtsze-Kiang, 3200 miles.

2. Find the difference in the lengths of:
   (a) The Nile and the Mississippi.
   (b) The Nile and the Amazon.
   (c) The Amazon and the Volga.
   (d) The Volga and the Yangtsze-Kiang.

Example. From 1747 subtract the sum of 196, 426, 81, 35, 283.

1747
196
426
81
35
283
726

The minuend is written at the top, and the numbers whose sum is to be subtracted from it are written in a column below. Each column is added upward, and below the line a number is written, which, when added, is just sufficient to make the last figure of the sum equal to the figure at the top of the column. Thus we say 8, 9, 15, 21, *and 6* make 27. The 6 is written in ones' place below the line, and the 2 is carried. The sum of the second column including the 2 carried is 32, to which 2 must be added. The 2 is written below the line, and the 3 is carried. The sum of the last column including the 3 carried is 10, to which 7 must be added to make 17. The result is 726. Thus a number has been found which, when added to the sum of the given numbers, equals 1747.

This method of subtracting the sum of given numbers is frequently useful. In a check book the records of three or four checks are usually put in a column, and their sum is subtracted from the balance brought forward.

WRITTEN EXERCISES

Find the balance left in the bank as indicated by the following statements:

1. Brought forward:	$364.95	2. Brought forward:	$763.74
Checks	146.20	Checks	208.33
	42.70		141.90
	8.90		30.20
	54.60		6.48
Balance:		Balance:	
3. Brought forward:	$1290.33	4. Brought forward:	$234.18
Checks	34.60	Checks	35.60
	291.80		37.40
	460.00		51.40
	37.50		72.46
Balance:		Balance:	

18. Addition and Subtraction of Decimal Fractions. The idea of place value in our number system is present also in decimal fractions. That is, in a number like 77.77 each numeral 7 represents 10 times as much as is represented by the same numeral in the next place to the right. For this reason decimal fractions are added and subtracted exactly in the same manner as whole numbers.

Example. Add 3.042, and .065, 891.2, 340.

3.042 .065 891.200 340.000	In adding long columns, it is customary to annex zeros so that all decimals will be of the same order. Sometimes a zero is written to the left of the decimal point to call special attention to it.

WRITTEN EXERCISES

Copy and add the following:

1. 4.892, 604.5, 8918.7, 4.37, .087, 0.45, 0.091.
2. 65.435, 8.794, 0.647, 0.045, 8.391, 76.91, 8.375.
3. 3.291, 6.840, .49, 0.671, 4.395, 7.210, .491.
4. .548, .098, 47.91, 864.4, 390., 67.432, 8.891.
5. 7.543, 8.76, .987, 10.31, 12.37, 494.2, 198.8.
6. 47.34, 0.579, 88.52, 943.7, 67.291, 82.346, 21.91.
7. 43.478, .24, 961.78, 51.761, 19.276, 6.432, 1.912.

Copy and subtract the following:

1. 9.402 − 4.06
2. 18.2 − 7.387
3. 24.6 − 18.394
4. 50.30 − 40.672
5. 6.02 − 2.049
6. 3.728 − .089
7. 4.931 − 2.786
8. 2.708 − 1.891
9. 1.273 − .824
10. 30.08 − 27.461

19. **Multiplication** is a short process of taking one number as many times as there are *ones* in another number.

Add:		
47	Thus, instead of adding	Multiply 47
47	47+47+47+47, we may	4
47	multiply 47 by 4.	188
47		
188		

The number multiplied is called *multiplicand*.

The number by which we multiply is called the *multiplier*.

The result obtained by multiplying is called the *product*.

The sign $\times$ is the sign of multiplication.

The multiplicand and multiplier are also called *factors* of the product.

The word "factor" is here used so as to include fractions, while in treating of factoring the word is so used as to apply only to integers.

20. **Abstract and Concrete Numbers in Multiplication.** A multiplier is always an abstract number. That is, we never multiply by $8, by 4 days, or by 6 cows. If a concrete number is multiplied by an abstract number, the product is always of the same denomination as the multiplicand.

In practice it is best, however, to decide independently the denomination in terms of which the result is to be stated, and then to carry out the work with abstract numbers. Any other scheme is sure to cause trouble, and is abandoned later both by those who use arithmetic in practical life, and also by those who study more advanced mathematics.

21. **The Order of Factors.** One of the fundamental facts about multiplication is that the order of the factors does not affect the product.

That is, $3\times4=4\times3$.

It is usually best to use the smaller number as the multiplier.

22. Multiplying by a Product. Another fundamental fact about multiplication is, that if a multiplier is the product of two factors, then instead of multiplying by the multiplier itself, we may multiply successively by these two factors.

Thus: $6\times7=3\times14=42$. That is, instead of multiplying 7 by 6, we may multiply 7 by 2 and the product by 3. Again, $80\times28=10\times224=2240$. Here we multiply by 8 first and the product by 10.

ORAL EXERCISES

Give the products of the following orally:

1. 10×56	2. 10×380	3. 100×51	4. 100×74
5. 10×47	6. 10×50	7. 100×291	8. 100×61
9. 20×65	10. 40×31	11. 800×21	12. 500×43
13. 50×17	14. 70×94	15. 300×56	16. 700×139

$$\begin{array}{r} 492 \\ \underline{327} \\ 3444 \\ 984 \\ \underline{1476} \\ 160884 \end{array}$$

To multiply by 327, we first multiply by 7 *ones*, then by 2 *tens*, and finally by 3 *hundreds*. The sum of these partial products is the complete product. Explain how we multiply by 20 and by 300. Why are the zeros omitted in writing the partial products?

In multiplying a number ending in one or more zeros, the work is sometimes written down as in the following:

To multiply 351 by 4600 we first multiply by 46 as usual and then annex two zeros, which is equivalent to multiplying by 100.

$$\begin{array}{r} 351 \\ \underline{4600} \\ 2106 \\ \underline{1404} \\ 1614600 \end{array}$$

WRITTEN EXERCISES

See how many minutes you need to multiply the following:

1. $\begin{array}{r} 859 \\ \underline{38} \end{array}$	3. $\begin{array}{r} 643 \\ \underline{391} \end{array}$	5. $\begin{array}{r} 823 \\ \underline{754} \end{array}$	7. $\begin{array}{r} 4221 \\ \underline{697} \end{array}$	9. $\begin{array}{r} 1604 \\ \underline{206} \end{array}$
2. $\begin{array}{r} 488 \\ \underline{921} \end{array}$	4. $\begin{array}{r} 4230 \\ \underline{890} \end{array}$	6. $\begin{array}{r} 629 \\ \underline{183} \end{array}$	8. $\begin{array}{r} 9140 \\ \underline{706} \end{array}$	10. $\begin{array}{r} 3894 \\ \underline{786} \end{array}$

23. **Multiplying Decimals.** By expressing decimals in the form of common fractions or mixed numbers, and multiplying them, the following rule may be obtained:

Multiply decimals as if they were whole numbers, and then point off as many decimal places in the product as the combined number of decimal places in the multiplier and multiplicand.

Thus, $.4 \times .12 = .048$; $.03 \times .24 = .0072$.

ORAL EXERCISES

Give the products of the following:

1. $.3 \times .8$	8. $.5 \times .6$	15. $.8 \times .4$	22. $.9 \times 7$
2. $.9 \times .7$	9. $.05 \times .6$	16. $.8 \times .04$	23. $.9 \times .9$
3. $.04 \times 8$	10. $.05 \times .06$	17. $.08 \times .4$	24. 90×7
4. $.3 \times 6$	11. 80×40	18. $.08 \times .04$	25. $90 \times .7$
5. $5. \times 60$	12. 8×40	19. $80 \times .04$	26. $90 \times .07$
6. $.5 \times 60$	13. 8×4	20. $8 \times .004$	27. $9 \times .07$
7. $.5 \times 6$	14. $.8 \times 40$	21. 9×7	28. $.9 \times .07$

WRITTEN EXERCISES

Multiply the following:

1. 4.297 .071	4. .671 .038	7. .192 .084	10. .0391 .0087
2. 8.54 1.46	5. 3.84 6.89	8. 529. 76.4	11. .092 .364
3. .59 .07	6. 20.79 .241	9. 10.39 4.83	12. .045 83.19

13. By how much does 1.4142×1.4142 differ from 2?

14. By how much does 1.7321×1.7321 differ from 3?

See in how many minutes you can copy and multiply the following:

1. 6432	2. 4307	3. 5004	4. 4827
636	637	798	438
5. 1357	6. 5327	7. 3425	8. 9786
793	446	137	354
9. 312.7	10. 2040	11. 5321	12. 3748
50.6	304	521	237
13. 52.02	14. 14.16	15. 15.87	16. 3224
43.5	5.17	8.36	863
17. 1718	18. 1895	19. 9142	20. 6374
819	947	975	739

21. A man gets $5.50 every day he works. How much does he earn in one year if he works 312 days? How much would he earn if he worked only 289 days?

$$\begin{array}{r} 312 \\ 5.50 \\ \hline 15600 \\ 1560 \\ \hline 1716.00 \end{array} \text{ (dollars)}$$

Solution: We find the product of the two numbers as if they were abstract. Why is 5.50, instead of 312, used as the multiplier? How do we know that the product is dollars? (See § 20.)

22. The distance from New York to Liverpool is 3540 statute miles. How many miles does a steamer travel in one year if it makes 15 round trips between New York and Liverpool?

23. If this steamer consumes on an average 4800 tons of coal each time it crosses the Atlantic, how many dollars' worth of coal does it consume in one year if the coal costs on an average $2.15 a ton?

24. At the rate of 43 bushels to the acre, how many bushels of corn will a farmer get from a field containing 114 acres? At 57¢ a bushel, what is the value of this corn?

24. **Short Cuts in Multiplication.** From $25=\frac{100}{4}$ it follows that to multiply by 25 we may multiply by 100 and divide by 4.

$$\text{Thus, } 25\times 49086=\frac{4908600}{4}=1227150.$$

In practice the zeros are annexed mentally, and the result is written down directly, thus: $25\times 49086=1227150$

Similarly: To multiply by 50 annex 2 zeros and divide by 2. To multiply 125 annex 3 zeros and divide by 8 $(125=\frac{1000}{8})$.

WRITTEN EXERCISES

In this manner write down the products of the following:

1. 5×793082	4. 125×7640	7. 125×9418
2. 25×34940	5. 250×3950	8. 25×1849
3. 50×6430	6. 50×8764	9. 250×7470

Example 1. Multiply 940840 by 102.

$$\begin{array}{r} 94084000 \\ \underline{1881680} \\ 95965680 \end{array}$$

Solution: Annex two zeros to the multiplicand, then multiply 940840 by 2 and add. This saves writing the multiplicand once.

Example 2. Multiply 3946 by 98.

$$\begin{array}{r} 394600 \\ \underline{7892} \\ 386708 \end{array}$$

Solution: $98=100-2$. Hence multiply by 100 and subtract 2×3946.

WRITTEN EXERCISES

1. Give short cuts for multiplying by 99, by 103, by 998, by 1004.

Multiply the following, using short cuts when you can:

2. 9×3461	5. 103×431	8. 998×7642
3. 99×5731	6. 104×5421	9. 997×7855
4. 98×386	7. 999×6530	10. 996×9864

PROBLEMS

1. The area of the State of Illinois is 56,650 square miles. Is this greater or less than the area of a rectangle 340 miles long, and 170 miles wide? How much?

2. Make a problem like the preceding about your own state. Find the dimensions of a rectangle as nearly the shape and size of your state as you can.

3. Light travels 186,300 miles a second, and goes from the sun to the earth in 498 seconds. Find the distance from the sun to the earth.

4. For a certain grade of freight, the rate between New York and Chicago is $3.60 a ton. At this rate how much is earned by a train of 38 cars, each containing 35 tons of freight?

5. A steel bar 1 foot long expands .00000639 feet for every degree of rise in temperature. How many feet will 240 miles of rails expand if the temperature rises from 20° below zero to 95° above zero?

The required result is $.00000639 \times 240 \times 5280 \times 115$. Explain this fully.

6. A New York firm buys a bill of goods in England amounting to £6895 (pounds sterling). How much in American money do they pay in settling this bill if one pound is worth $4.8724?

7. A Boston firm bought goods in Paris amounting to 57,860 francs. How much was this in American money if one franc was worth $.195?

8. A Chicago firm bought goods in Japan amounting to 19,860 yen. How much is this in American money if one yen is worth $49\frac{4}{5}$ cents ($0.498)?

9. A farmer sold 37 head of beef cattle at $.095 a pound live weight. How much did he get for his cattle if they averaged 1248 pounds?

PROBLEMS

1. A field containing 46.4 acres yields an average crop of 64.5 bushels of corn per acre. At 58 cents a bushel, what is the value of this crop?

2. If a dairy cow needs 45 lbs. of silage per day, how much will 14 cows eat in 186 days?

3. A schoolroom should receive 75 cubic yards of air per hour for each pupil. How many cubic feet is this? In a schoolroom containing 37 persons, how many cubic feet of air should be supplied during a morning session of 3 hours?

4. In your schoolroom, how many cubic feet of air per hour should be brought in to keep it ventilated?

5. A man bought 27 cattle, weighing on an average 1045 pounds, paying 9½ ($.095) cents a pound. After feeding them 4 months, they weighed on an average of 1329 pounds, and were sold for 10¾ cents ($.1075) a pound. How much more did the man receive for the cattle than he paid for them?

6. Two trains leave Chicago at the same time, one going west at the rate of 36 miles an hour, and one going east at the rate of 42 miles an hour. How far apart will they be in 8 hours?

7. If steel rails weigh 36 pounds to the foot, how many tons of rail are needed for a 90-mile track (two rails, each 90 miles long)?

8. At 48 cents a square foot, what is the value of a lot 32 feet by 128 feet?

9. One year the State of Minnesota had 5,500,000 acres sown in wheat. At 16.5 bushels to the acre, what was the total yield? At 87 cents a bushel, what was the value of this crop?

10. If the outfit for a soldier cost $102.75, what was the cost of equipping the 46,843 soldiers assembled in one camp?

25. Division. Find the missing numbers in $3 \times ? = 36$ in $12 \times ? = 48$, and in $27 \times ? = 108$.

These examples involve the process of division, and illustrate the following definition:

Division is the process of finding one of two numbers when their product and the other number are given.

The given product is called the *dividend*, the given number is called the *divisor*, and the number to be found is called the *quotient*.

Division may also be regarded as finding how many times the divisor is contained in the dividend. That is, how many times may the divisor be subtracted from the dividend before the dividend is exhausted? From this point of view, division is a repeated subtraction, just as multiplication may be regarded as a repeated addition.

26. Estimating Quotients. The most difficult step in division is to estimate the quotient figures.

Example 1. Estimate the quotient in $812\overline{)4943}$.

Solution: Taking quotient of hundreds, $49 \div 8 = 6$ with a remainder. On multiplying 812 by 6 this is found the correct quotient.

Example 2. Estimate the quotient in $693\overline{)5835}$.

Taking quotient of hundreds, $58 \div 7 = 8$ with a remainder. Since 693 is nearly 700, we use 7 instead of 6 as the divisor.

Example 3. Estimate the quotient in $884\overline{)3528}$.

$35 \div 9 = 3$ with a remainder. But we find on multiplying that 4 is the correct figure in the quotient.

Notice that in $35 \div 9$ the remainder is very large.

Example 4. Estimate the quotient in $542\overline{)4242}$.

Taking quotient of hundreds $42 \div 5 = 8$. On multiplying we find, however, that the quotient figure is only 7. Notice that the divisor is much larger than 500.

ORAL AND WRITTEN EXERCISES

Estimate the quotients in the following. After having made all the estimates, test them by multiplying. The estimates should be made rapidly and accurately.

1. 57)289	5. 948)8639	9. 269)2137
2. 84)367	6. 473)3478	10. 598)4786
3. 174)893	7. 347)1946	11. 385)2789
4. 487)3987	8. 538)4692	12. 426)2970

Examples. Divide 679305 ÷ 7 by short division, and 345204 ÷ 453 by long division.

Compare these two examples with care. The processes used are exactly the same. In the first example the divisor is so small that we multiply and subtract without writing down the numbers; while in the second we find it necessary to write down the products and the remainders. In general, *mental* and *written* arithmetic differ just in this, that in mental arithmetic the results are carried in the memory, while in written arithmetic much of the work is written down, so that there is less to remember.

WRITTEN EXERCISES

1. 56892	(a) 2
2. 92462	(b) 3
3. 65978	(c) 4
4. 37809	(d) 5
5. 53086	(e) 6
6. 35806	(f) 7
7. 60850	(g) 8
8. 98790	(h) 9
9. 25924	(i) 11
10. 3976	(j) 12

From the figures on the left make 100 examples in short division by dividing each number in the first column by each number in the second. In the same manner make 100 examples in long division from the two columns on the right.

1. 498640	(a) 768
2. 92879	(b) 436
3. 12040	(c) 346
4. 3826	(d) 214
5. 53086	(e) 412
6. 45720	(f) 574
7. 75402	(g) 475
8. 39278	(h) 376
9. 5924	(i) 224
10. 16790	(j) 982

27. Division of Decimals. The only additional difficulty in dividing decimals lies in placing the decimal point in the quotient.

Example 1. Divide 324.78 by 21.

$$\begin{array}{r} 7 \\ 15.4\cancel{6} \\ 21\overline{)324.78} \\ \underline{21} \\ 114 \\ \underline{105} \\ 97 \\ \underline{84} \\ 138 \\ \underline{126} \\ 12 \end{array}$$

Solution: First divide 32 (tens) by 21, obtaining a quotient of 1 (ten), which we place directly above the 2 in the dividend. The division proceeds the same as in division of integers. The decimal point in the quotient is placed directly above the decimal point in the dividend. The quotient to the nearest hundredth is 15.47. Why?

Example 2. Divide 43.4 by 2.14.

$$\begin{array}{r} 20.28 \\ 214.\overline{)4340.} \\ \underline{428} \\ 600 \\ \underline{428} \\ 1720 \\ 1712 \end{array}$$

Solution: First multiply both dividend and divisor by 100 by moving the decimal point two places to the right This makes the divisor a whole number, and the example becomes like Example 1.

The result given is the nearest approximation in a two-place decimal.

Example 3. Divide 4.67 by 2300.

$23\cancel{0}\cancel{0}\overline{).0467}$ *Solution:* First simplify by dividing both dividend and divisor by 100. How is 4.67 divided by 100?

WRITTEN EXERCISES

Find quotients to four decimal places:

1. 7.91 ÷ 0.17	5. 5.604 ÷ .057	9. 41.27 ÷ 6.43
2. 84.95 ÷ 2.64	6. 1.48 ÷ 470	10. .00492 ÷ 8.4
3. .391 ÷ .072	7. 52.86 ÷ 5.3	11. 847.4 ÷ 640
4. 9.51 ÷ 310	8. 6.18 ÷ 3.7	12. 7.98 ÷ 1.39

28. Errors in Computation. All computers make occasional errors. Since it is essential that we should know the results to be accurate, it is necessary to check them.

Problems in multiplication may be checked by dividing the product by one of the factors, or by interchanging multiplier and multiplicand, and repeating the multiplication.

Problems in division may be checked by multiplying the divisor by the quotient and adding the remainder.

The most practical check, however, is to work with care, and then to go over the work again.

The most common error in multiplication is to omit one zero in the product, or to misplace the decimal point. The decimal point is also liable to displacement in division.

ORAL EXERCISES

State how many figures there are to the right of the decimal point in each of the following products:

1. 138.91×213.7	3. 62.91×34.5	5. 54.03×72.26
2. 502.2×30.97	4. 73.12×148.31	6. 22.04×790.3

State how many figures there are to the left of the decimal point in each of the following quotients:

7. $84.79 \div 37.6$	9. $27.83 \div 48.9$	11. $1.942 \div 258$
8. $186 \div 132.4$	10. $.0827 \div .0065$	12. $0.374 \div 0.241$

In many cases the exercise of a little common sense will help to avoid gross errors. In each of the following put a decimal point to make the statement reasonable:

13. A man weighed 17825 pounds.

14. A head of beef cattle sold for $8475.

15. A passenger train ran 514 miles an hour.

16. A bullet was fired with a velocity of 2150 feet a second.

SHORT CUTS IN DIVISION

29. Dividing by 10, 1000, etc. We have already learned to divide a number by 10, 100, or 1000, by striking out one, two or three zeros, or by moving the decimal point one, two, or three places to the left.

That is, $450 \div 10 = 45$, $24000 \div 100 = 240$, $342 \div 10 = 34.2$, and $645 \div 100 = 6.45$.

ORAL EXERCISES

Read the quotients of the following:

1. $1620 \div 10$
2. $3529 \div 100$
3. $5031 \div 100$
4. $7000 \div 100$
5. $5243 \div 1000$
6. $14270 \div 100$
7. $93140 \div 1000$
8. $50000 \div 100$
9. $3100 \div 1000$

30. Dividing by 20, 60, 700, etc. To divide a number by 20, we may first divide it by 10 and then by 2. Similarly, to divide a number by 60, we may first divide it by 10 and then by 6

Again, $240 \div 20 = 24 \div 2 = 12$, and $6900 \div 300 = 69 \div 3 = 23$.

Example. Divide 45900 by 700.

Solution: $7\cancel{0}\cancel{0})\underline{459\cancel{0}\cancel{0}}$
$\quad 65 - 400$

We first divide by 100, and then by 7, as in the preceding examples, obtaining a quotient 7, and a remainder 4. The real remainder is 400, however, since the dividend has been divided by 100.

WRITTEN EXERCISES

Find the quotient and remainder, if any, in each of the following:

1. $5280 \div 40$
2. $3720 \div 30$
3. $5500 \div 500$
4. $2900 \div 600$
5. $53000 \div 700$
6. $3360 \div 60$
7. $7550 \div 50$
8. $88400 \div 80$
9. $13200 \div 900$
10. $5000 \div 600$
11. $16400 \div 400$
12. $5100 \div 300$
13. $39600 \div 1200$
14. $9400 \div 900$
15. $36800 \div 800$

31. Dividing by 5, 25, 50, 125, 56, etc. From $5=\frac{10}{2}$, $25=\frac{100}{4}$, $50=\frac{100}{2}$, $125=\frac{1000}{8}$ we find short cuts for dividing by 5, 25, 50, and 125. That is, to divide by 5, divide by 10 and multiply by 2.

Thus, $2876 \div 5 = 2 \times 287.6 = 575.2$.

When we know the method, we write at once $2876 \div 5 = 575.2$ by simply multiplying by 2 and pointing off one place.

Similarly, to divide by 25, divide by 100 and multiply by 4, and to divide by 125 divide by 1000 and multiply by 8.

Example. Divide 394767 by 56.

This can be reduced to two short divisions as follows:

```
7)394767
 8)56395-2
    7049-3
```

First divide by 7, and then the quotient by 8. The last remainder must be multiplied by 7 and added to the first remainder. Hence, the remainder is $7 \times 3 + 2 = 23$.

This method is applicable when the divisor is the product of one-figure numbers.

WRITTEN EXERCISES

Write the quotients in the following:

1. $15600 \div 25$	4. $59000 \div 125$	7. $92000 \div 125$
2. $3910 \div 5$	5. $32100 \div 25$	8. $12400 \div 25$
3. $7400 \div 25$	6. $9200 \div 5$	9. $21000 \div 125$

Miscellaneous examples in division involving short cuts:

10. $54247 \div 49$	16. $67200 \div 700$	22. $2380 \div 60$
11. $65480 \div 42$	17. $320 \div 5$	23. $8400 \div 300$
12. $56700 \div 72$	18. $5800 \div 25$	24. $6900 \div 700$
13. $4400 \div 64$	19. $17000 \div 125$	25. $43000 \div 500$
14. $39500 \div 81$	20. $1600 \div 50$	26. $89200 \div 32$
15. $468000 \div 600$	21. $2380 \div 80$	27. $60000 \div 800$

32. Numbers Not Known Exactly. Certain numbers can never be known exactly. Thus, the population of the United States is never known within 1000, and possibly not within 100,000. Many escape enumeration; there are large numbers of births and deaths during the period when the enumeration is made. For this reason it is sufficiently accurate to give the population of the United States for the 1910 census as 91,972,000, or possibly 92,000,000, though the number actually reported was 91,972,266.

The census returns of 1880, 1890, 1900, and 1910 gave the following numbers of doctors, lawyers, clergymen, and teachers in the United States:

	1910	1900	1890	1880
Population.	91,972,266	75,994,575	62,947,714	50,155,783
Doctors.	157,966	132,002	104,805	85,671
Lawyers.	122,149	114,460	89,630	64,137
Clergymen.	133,988	111,638	68,803	64,896
Teachers.	619,285	446,133	347,344	227,710

Example. Find to the nearest integer, the number of individuals for each doctor in the United States in 1900.

Solution: Divide 75,994,575 by 132,002, or what will amount to the same thing, divide 76,000,000 by 132,000. The required result is 576.

WRITTEN EXERCISES

1. Find to the nearest integer, the number of individuals for each doctor, each lawyer, each clergyman, and each teacher for each of the years given above.

Did the number engaged in these professions increase more or less rapidly than the population?

2. In 1900 there were in the United States 208,903 boot and shoemakers and repairers. If the population was 75,995,000, for how many persons did each shoemaker make shoes?

WRITTEN EXERCISES

1. The meter has been given in inches as follows by the men named:

1817 Hossler	39.36994 inches
1818 Kater	39.36990 inches
1835 Baily	39.36973 inches
1866 Clarke	39.36970 inches
1885 Comstock	39.36984 inches

Find the average of these to five decimal places.

2. If 39.36982 inches is taken as one meter, find, correct to three decimal places, the number of millimeters in one inch (1 meter = 1000 millimeters).

3. The pressure exerted upon a flat surface by wind blowing against it is given by the following table.

Miles per hr.	Force in pounds per square foot	
5	0.123	Gentle wind
10	0.492	Brisk gale
15	1.107	
20	1.968	Very brisk
25	3.075	
30	4.428	
35	6.027	High wind
40	7.872	Very high storm
60	17.712	Great storm
80	31.488	

Find the pressure in tons on a wall 55 feet long and 22 feet high, for each of these velocities.

4. In 1900 there were in the United States 229,649 tailors, 346,844 dressmakers, 113,193 butchers, 24,120 clockmakers and repairers, 30,278 bookbinders, 22,733 hat- and cap-makers, 10,220 broom- and brush-makers, 12,271 glovemakers, and 26,941 photographers. Find how many persons there were in the United States for every individual in each of these trades.

		A	B	C	D	E
	1.	468,786	76,421	3,764	386	76
	2.	296,762	12,839	8,292	294	43
I.	3.	529,474	29,839	3,960	862	67
	4.	987,609	98,877	7,098	493	34
	5.	376,401	35,467	6,930	867	81
	6.	578,493	49,270	9,040	217	38
	7.	278,627	26,840	5,187	172	18
II.	8.	937,418	87,490	6,840	712	38
	9.	197,535	74,560	1,983	493	49
	10.	246,806	20,953	6,204	934	27
	11.	456,782	54,624	4,908	349	94
	12.	321,090	72,918	2,602	924	72
III.	13.	286,426	47,020	8,426	721	53
	14.	135,797	94,764	2,473	271	65
	15.	417,399	53,808	3,524	641	56
	16.	176,326	76,413	7,264	524	35
	17.	420,494	35,894	4,193	425	95
IV.	18.	765,433	68,374	3,714	245	43
	19.	890,122	75,192	7,242	542	27
	20.	647,511	67,543	2,230	452	59
	21.	132,679	58,890	3,782	839	34
	22.	920,442	12,480	9,146	398	72
V.	23.	345,674	13,174	7,824	893	91
	24.	198,027	47,238	9,753	339	13
	25.	345,676	65,180	1,326	948	47

The material on pages 32, 72, 73, 120, 121 may be used from time to time in drills in the fundamentals of Arithmetic.

The teacher will suggest games of competition, and possibly you can help her by suggesting some that you would like to try.

See how long it takes you to copy neatly and add the following 4 examples.

1.	2.	3.	4.
47875	54200	24520	74810
9284	7649	41927	23389
3256	74821	26240	18914
8494	19452	13129	17262
4013	76547	27849	14520
27800	23579	45678	82870
53092	12460	32100	39563
35107	35798	54321	62245
12700	80246	67840	8451
76543	24135	94821	62749
12141	62545	64281	71984
51617	2376	73648	91543

See how long it takes you to multiply the following examples.

5. 8914 × 679	6. 4589 × 682	7. 14974 × 928	8. 47940 × 1863
9. 8592 × 543	10. 6982 × 478	11. 13410 × 2187	12. 28956 × 9842
13. 18942 × 394	14. 67852 × 1678	15. 5234 × 297	16. 6543 × 782
17. 4934 × 876	18. 1678 × 347	19. 8920 × 234	20. 2874 × 153

See how long it takes you to divide the following examples. Find each quotient to the nearest integer.

21. $867\overline{)493400}$ 22. $347\overline{)867100}$ 23. $653\overline{)287400}$

24. $394\overline{)249800}$ 25. $760\overline{)8678500}$ 26. $287\overline{)345600}$

27. $679\overline{)419800}$ 28. $286\overline{)985400}$ 29. $829\overline{)1497400}$

MISCELLANEOUS PROBLEMS

33. **Cost of Living in City and Country.** A family living in a large city wish to find out which is more expensive—to borrow money and buy a house in a suburb, or to rent an apartment in the city.

Following are data which bear on the question:

1. *Expenses in the City.* Rent $70 a month; car-fare for man, 10¢ a day, 6 days a week for 50 weeks. Car-fare for two children, 10¢ a day each, 5 days a week for 40 weeks. Additional car-fare averages 80¢ a week for the year. Extra expense for the family going to the country 10 weeks in the summer, $30.00 a week. Tips to janitors and hall boys, $2.50 a month for 12 months.

Expenses in the Suburb. Interest on an investment of $10,500 at 6%. Car-fare for man, $9.20 a month for 12 months (the children walk to school now). Additional car-fare averages $2.80 a week for 48 weeks. Taxes, $145. Repairs on house, $125 a year. Coal, $93.00 a year. Janitor, $9.50 a month. The family takes only a four weeks' trip to the country, involving an extra expense of $30 a week. All other expenses are approximately the same for the city and the suburb.

Which of the two propositions is the cheaper, and how much?

Solve this problem, using the following data:

2. *Expenses in the City.* Rent, $75 a month. Car-fare for man, 10¢ a day, 6 days a week for 46 weeks. Car-fare for 3 children, 10¢ a day, 5 days a week, 38 weeks. Other car-fare averages 95 cents a week for 40 weeks. 12 weeks in the country at an extra expense of $35 a week. Tips in apartment, $2.00 a month.

Expenses in the Country. Interest on an investment of $12,600 at 5½%. Car-fare for man, $7.40 a month for 10 months. Additional car-fare averages $2.25 a week. Taxes, $135. Repairs, $160. Coal, $105. Janitor, $8.75 a month. The family does not go to the country at all in the summer.

34. **Cost of Vacation Trips.** A family of three persons living in Boston decide to spend a year's vacation in Europe.

Expenses at Home. They rented a house at $80 a month, and had the following other expenses: A servant, $5.00 a week; groceries, $35.00 a month; meat, $15.00 a month; milk, $4.50 a month; heat and light averaging $21.00 a month; amusements, $16.00 a month. They went to the country 12 weeks in the summer at an extra expense of $20.00 a week.

Expenses Abroad. Three tickets, Boston to Naples, $95.00 each. Incidental expenses on board, $12.50 each. Three months in Italy cost $60.00 a month per person. Four months in France averaged $80.00 a month a person. Four months in Germany and England, $85.00 a month per person. Three tickets from Liverpool to Boston, $90.00 each, and incidental expense on board, $10.00 each.

They bought a year's supply of clothing, making an estimated saving of $80.00 each.

What was the total cost of their year abroad? What was the net cost after deducting the home expenses which they saved, including rental on the house?

A New Yorker's Trip to England. A man living in New York, where rooms and meals cost him $20 a week, decided to spend his 10 weeks' vacation in England. How much more will this cost him than to remain in New York, if he pays $180.00 for a round trip ticket; $10.00 for incidentals each way on board; and averages $20.00 a week for 8 weeks in England? Other expenses are about the same in New York and England. He buys necessary clothing at an estimated saving of $50.00.

Children's Vacation in the Country. John and Mary, aged 11 and 13, went from their home in Chicago to visit their uncle, who lives on a farm in northern Wisconsin. On their trip they had the following expenses: One ticket each way $7.85; other expenses on the train, $9.40 for John, and $10.75 for Mary. They bought presents for their cousins costing $9.30; other expenses, $6.90 for John, and $8.50 for Mary. Find the total expenses for both for this trip.

MISCELLANEOUS PROBLEMS

1. One ton (2000 lbs.) of hard coal contains 1825 pounds of carbon. How many tons of carbon are there in a carload of 47 tons of hard coal?

2. A ton of soft coal contains 1520 pounds of carbon. How many tons of carbon are there in a carload of 38 tons of soft coal?

3. The circumference of the driving wheels of a locomotive is 22 feet. How far will the locomotive go when these wheels make 5280 revolutions?

4. How many revolutions must the driving wheels of Example 3 make to cover a distance of 5280 miles?

5. In a bin 10 feet long and 7 feet wide there are 10 tons of coal. What is the average depth of the coal in the bin if one ton occupies 35 cubic feet?

6. One cubic foot of a certain kind of coal weighs 57 pounds. A bin 14 feet long and 12 feet wide is filled to an average depth of 5 feet. How many tons of coal are there in it?

7. A coal car 8 feet wide, 36 feet long, is filled with coal to a depth of 4 feet. How many tons are there in the car if one cubic foot weighs 58 pounds?

8. The product of two numbers is 864,528. One of the numbers is 248; find the other number.

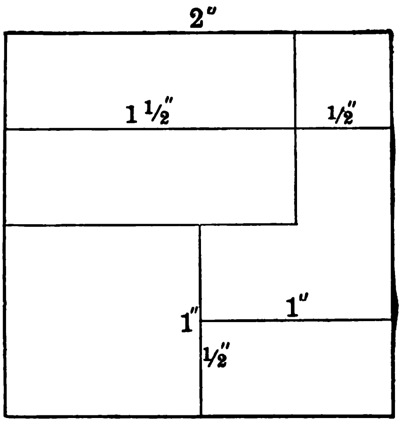
9. An ounce of pure gold is worth $20.67. What is the value of a gold nugget weighing 284.6 ounces?

10. At the rate of $20.67 an ounce for pure gold, how many ounces are there in $1000 worth of gold?

11. A carload of 24 cattle weighed 28,152 pounds. What was the average weight per head?

12. At 11¾ cents ($.1175) a pound, what was the value of the cattle in Example 11?

In most practical problems there is a series of small problems such that the result of each will be needed for those that follow. In such problems, keep the answers until you have finished all of them.



This diagram represents a farm drawn to the scale of one inch to 80 rods. The lines represent fences.

WRITTEN EXERCISES

1. How many rods of fencing are there on this farm, including the fence around it.

2. How many posts are required for all the fences on this farm if the posts are set 10 feet apart? Notice that there is one post at each corner.

3. At 11¢ a post, how much will these posts cost?

4. At 3¢ each, what is the cost of digging holes for the posts?

5. A roll of fence wire contains 80 rods of wire. How many such rolls are required to build these fences, if 6 wires are used?

6. How much does this wire cost at 2¢ a pound, if it runs 1 pound to the rod?

7. What is the total cost of material for the fence?

8. When the holes are dug, two men can build 25 rods of fence in one day. At $1.75 a day each, how much does their work amount to?

9. If you live on a farm, find the actual cost of fence posts, fence wire, and the cost of putting up a fence. Then compute the total cost of fencing a farm you know.

35. A Farmer's Account with an Oat Field. A farmer tried to find out whether or not it pays to raise oats on a certain field. Following were his expenses:

Man work 134 hours (plowing) at 22c	$.....
Horse work 258 hours (plowing) at 13c	.....
68 bu. seed oats at 65c	.....
Fertilizer	10.00
Insurance	3.00
Twine	2.50
Coal for Threshing	.96
Meals for Threshers	3.00
Threshing 668 bus. at 2c	.....
Use of land (23 acres at $3.00 per acre)	.....
Man work 270 hours at 25c dragging, seeding, hauling, etc.	.....
Horse work 411 hours at 15c, dragging, seeding, hauling, etc.	.....
Use of equipment	9.80
Interest	7.54

His receipts from the field were:

668 bu. of oats at 50c	$.....
12 tons of straw at $6.00	.....
Pasture	5.00

These data are arranged in the form of an account as follows:

Account with Oat Field

DR.		CR.	
134 hours man work at 22c	$.....	668 bu. of oats at 50c	.....
258 hours horse work at 13c	.....	12 tons of straw at $6.00	.....
68 bu. seed oats at 65c	.....	Pasture	$5.00
Fertilizer	10.00		
Insurance	3.00	Total	$.....
Twine	2.50	Total expenses	$.....
Coal for threshing	.96		
Meals for threshers	3.00	Gain	$.....
Threshing 668 bu. at 2c	.....		
Use of land, 23 acres at $3.00	.....	Gain per acre	$.....
270 hours man work at 25c	.....		
411 hours horse work at 15c	.....		
Use of equipment	9.80		
Interest	7.54		
Total	$.....		

Fill in all missing numbers in this account. What was the total gain? What was the gain per acre?

36. **Account with an Orchard.** Arrange the following in the form of an account, and show whether there is a loss or gain, and how much.

Expenditures:

Spraying material	$7.00
Ground crops, 1 bu. buckwheat, $1.25	......
4 lb. rape, 45c	......
1 lb. cowhorn turnips, 50c	......
241 apple barrels at 42c	......
Freight on apples	41.15
Use of land (3 acres at $10)	......
Man work 583 hours at 23½c	......
Horse work 326 hours at 15c	......
326 equipment hours at 4c	......
Interest	8.21
Taxes	12.70
Miscellaneous expenditure	18.40

Receipts:

Fruit used in house, 5 bu. peaches	$8.00
2 bu. peas	2.25
2 bu. cherries, $2.50; 18 bu. apples, $15.00	......
Apples sold, 140 bu. at 85c; 125 bu. at 25c; 210 bu. at 80c	......

37. **Account with a Wheat Field.** Arrange as above.

Expenditures:

Seed wheat, 85 bushels at $1.10	$....
Insurance	12.00
Twine and cutting	35.00
Threshing 856 bu. at 4c	......
Other expenses of threshing	9.60
Use of land, 45 acres at $3.50	......
Man work, 285 hours at 23c	......
Horse work, 380 hours at 14c	......
190 hours use of equipment at 4c	......
Interest	14.50
Taxes	21.60
Miscellaneous expenses	51.60

Receipts:

820 bu. of wheat at 95c	......
35 bu. of wheat at 85c	......
38 tons of straw at $5.00	......

If you live on a farm, make a real account with a field or an orchard. Make a list of all the data you need, and get them as they become available. Consult your teacher about this.

38. A Daily Milk Report in Pounds.

No. of cow	Morning	Evening	Total	No of cow	Morning	Evening	Total
1	13.4	11.7	....	8	15.9	15.3	....
2	12.8	10.4	....	9	21.4	19.2	....
3	....	....	....	10	10.4	12.3	....
4	5.4	5.	....	11	16.	14.2	....
5	15.3	11.3	....	12	....	....	....
6	8.3	7.9	....	13	17.3	14.5	....
7	11.8	10.1	....	14	16.8	13.2	....
Total				Total			

How many pounds of milk is this for the day?

How much is this milk worth at 18 cents a gallon, if one gallon of milk weighs 8.6 pounds?

MISCELLANEOUS PROBLEMS

1. A man worked 268 days. How many weeks and days did he work, counting 6 working days to the week?

2. A boy attended school 184 days in one year. How many school weeks and days would this be, counting 5 school days to the week?

3. A carpenter worked in all 1237 hours on a building. How many working days and hours would this be if he worked 8 hours a day?

4. A mason worked 387 hours building a wall. How many days and hours did he work, counting 9 hours to the day?

5. How many square yards and square feet in a pavement 40 feet wide and 70 feet long?

6. At $8 a head, how many sheep can a farmer buy for $437? How much would he have left over?

1. The attendance at a grammar school for one week was:

Monday	1485
Tuesday	1512
Wednesday	1508
Thursday	1498
Friday	1502

What is the average daily attendance for the week?

2. The area of the State of Massachusetts is 8315 square miles. The population of this state for each census since 1800 was as follows:

1800	422,845	1860	1,231,066
1810	472,040	1870	1,457,351
1820	523,287	1880	1,783,085
1830	610,408	1890	2,238,947
1840	737,699	1900	2,805,346
1850	994,514	1910	3,366,416

Find the average number of people per square mile at each census.

3. If sound travels 1090 feet per second, how many miles will sound go in 1 minute?

4. Through water sound travels 4730 feet per second. How many miles will sound go through water in one minute?

5. How long will it take sound to go a distance of 6 miles through air?

6. At 45¢ a cubic yard, find the cost of digging a ditch for a water main if the ditch is 8 feet deep, 3 feet wide, and $2\frac{3}{4}$ miles long.

7. A coal bin is 12 feet long, 9 feet wide, and 8 feet deep. How many tons of coal will this bin hold if one ton occupies 35 cu. ft. of space?

8. A brick wall is 45 feet long, 22 feet high, and 1 foot thick. How many bricks in this wall, if 22 bricks make one cubic foot?

39. Denominate Numbers. A denominate number is a number expressed in terms of some standard unit of measure.

Thus, 12 yds., 8 lbs., 3 bus., etc., are denominate numbers.

The following examples illustrate the operations on denominate numbers.

Example 1. Add:

12 yds. 2 ft. 8 in.	*Solution:* First add 9 inches and 8 inches, and reduce result to 1 foot 5 inches. Complete the addition and explain each step.
6 yds. 1 ft. 9 in.	
19 yds. 1 ft. 5 in.	

Example 2. Subtract:

9 yds. 1 ft. 5 in.	*Solution:* First add one yard and one foot to both subtrahend and minuend, or change 1 yard to feet, and 1 foot to inches, then subtract and explain each step.
3 yds. 2 ft. 8 in.	
5 yds. 1 ft. 9 in.	

Example 3. Multiply:

12 yds. 2 ft. 10 in.	First multiply the number of lowest denomination, in this case 10 inches, by 7, and reduce the product (70 in.) to feet and inches. We have 5 feet to carry and 10 inches over. Complete the multiplication and explain each step.
7	
90 yds. 1 ft. 10 in.	

Example 4. Divide:

$$\begin{array}{r} \text{3 gal. 2 qts. } 1\tfrac{3}{4}\text{ pts.} \\ 4\overline{)\text{14 gal. 3 qts. 1 pt.}} \end{array}$$

Solution: First divide the number of the highest denomination. Reduce the remainder, 2 gal., to the next lowest denomination, and add to the number of that denomination in the dividend, obtaining 8 qts.+3 qts.=11 qts.

Then, 11 qts.÷4=2 qts., with remainder 3 qts.

Complete the division and explain each step.

Example 5. Divide:

$$\begin{array}{r} \text{2 pks. } 7\tfrac{3}{4}\text{ qts.} \\ 8\overline{)\text{5 bu. 3 pks. 6 qts.}} \end{array}$$

The 5 bushels must be reduced to pecks before the division can be started.

PROBLEMS

1. If you study arithmetic 25 minutes each day, how many hours and minutes do you study arithmetic in one week, or 5 days?

2. Using the figures of the preceding problem, how many hours and minutes do you study arithmetic in a school year consisting of 38 weeks?

3. If 12 ounces of flour make a one-pound loaf of bread, how many barrels and pounds of flour are used daily by a bakery turning out 50,000 pound loaves a day? (One barrel of flour contains 196 pounds.)

4. If 3 feet of a certain kind of fence wire weigh 7½ ounces, how many pounds are required for a six-wire fence 130 rods long?

5. A train runs a mile in 1 minute and 43 seconds. At this rate, how long will it take the train to run 72 miles?

6. If a family of 5 persons consume 65 pounds of meat in a month of 30 days, what is the average amount of meat consumed by each person per day? Give result to the nearest ounce.

7. In a sewing class a piece of goods 27 yards 2 feet long was used to make 21 kitchen aprons. What was the average amount of goods used for each apron? Give result to the nearest inch.

8. A milkman delivers on an average one quart and one pint of milk to each of his customers. How many gallons, quarts, and pints does he deliver to 27 customers?

9. If a carriage wheel rolls 13 feet and 7 inches in making one revolution, how far will it roll in making 680 revolutions?

10 If one horse consumes 12 quarts of oats in one day, how long will 150 bushels of oats last one team of horses? How many bushels will be needed to feed a team 365 days?

40. Definition of a Fraction. A *fraction* is one or more of the equal parts of a unit.

Thus, $\frac{2}{3}$ represents 2 of the 3 equal parts of one.

41. Denominator, Numerator. The number stating into how many equal parts the unit has been divided is called the *denominator*.

The number stating how many of these parts are taken to form the fraction is called the *numerator*.

Thus, in $\frac{2}{3}$, 3 is the denominator; and 2, the numerator.

The word denominator means *namer*, and the word numerator means *numberer*. The former *names* the kind of parts taken; and the latter *numbers* them, or tells how many there are.

42. Terms of a Fraction. The denominator and numerator of a fraction are called its *terms*.

The unit which is divided into equal parts may be a single thing, such as one apple, or one foot, or it may be a group of objects, such as twelve persons or six apples.

ORAL EXERCISES

In the figure, regard the line AB as a unit.

1. Point out $\frac{1}{12}$ in this figure.

2. Point out $\frac{1}{6}$, $\frac{2}{12}$, $\frac{1}{4}$, and $\frac{1}{3}$ in this figure.

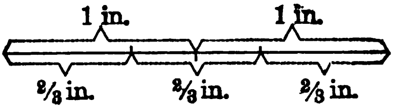
3. What fractions are represented by AE, BC, BD, AF?

4. In the figure point out $\frac{5}{12}$, $\frac{7}{12}$, $\frac{11}{12}$. Show what is meant by the denominator and what is meant by the numerator of each of these fractions.

5. In the figure point out $\frac{3}{4}$ and $\frac{6}{8}$, and show what is meant by the numerator and denominator of each.

43. A Fraction as an Indicated Division. There is another very important way of regarding a fraction, which we must make clear to ourselves. We can do this best by an example.

When a line two inches long is divided into three equal parts, each part represents $\frac{2}{3}$ of an inch. Hence $2/3$ represents the quotient when 2 is divided by 3.



EXERCISES

1. Draw a line 3 inches long, and divide it into 4 equal parts. How long is each part? What quotient is represented by each part?

2. Draw a line 4 inches long, and divide it into 5 equal parts. How long is each part? What quotient is represented by each part?

3. Draw a line 4 inches long, and divide it into 3 equal parts. How long is each part? What quotient is represented by each part?

Hence we see that *a fraction may be regarded as an indicated division, in which the numerator is to be divided by the denominator.*

The fractional form is frequently used to indicate division. Thus, to divide 16×18 by 12×12 we may write $\frac{16\times18}{12\times12}$ and then cancel.

The present form of writing fractions came into use among the Arabs about the year 1000. They wrote a fraction like $\frac{2}{3}$ in various forms, such as $2\div3$, $2/3$, and $\frac{2}{3}$. Thus, we see that the Arabians regarded a fraction as an indicated division.

A fraction whose numerator is unity is called a *unit fraction.*

The Egyptians, over 3000 years ago, used fractions; but they always reduced them to unit fractions. Thus, instead of $\frac{2}{3}$, they used $\frac{1}{2}+\frac{1}{6}$, though they did not use this form.

44. Factoring. To deal effectively with fractions we need to know something about the factors of numbers.

The *factors* of a number are the integers whose product it is.

Thus, 2 and 6 are factors of 12, as are also 3 and 4, and also 2, 2, and 3. We see that according to this definition only integers may be regarded as factors. (Compare page 17.)

Any one of the factors of a number is called an *exact divisor* of the number.

An exact divisor of a number divides it without remainder and with an integral quotient.

ORAL EXERCISES

State in which of the following pairs of numbers the first number is a factor of the second. (If you cannot tell otherwise, divide.)

1. 4, 8.	5. 3, 21.	9. 6, 46.	13. 3, 342.
2. 3, 15.	6. 7, 21.	10. 12, 132.	14. 5, 95.
3. 5, 18.	7. 4, 28.	11. 14, 168.	15. 9, 8172.
4. 6, 18.	8. 9, 54.	12. 13, 178.	16. 10, 3470.

45. Odd and Even Numbers. A number is called *even* if it has 2 as a factor.

If an integral number is not *even*, it is *odd*.

Zero is regarded as an even number.

Thus, 0, 2, 4, 6, 8, 10 are even numbers, while 1, 3, 5 are odd.

46. Prime and Composite Numbers. An integer which is divisible only by itself and 1 is called a *prime* number. Integers which are not prime numbers are called composite numbers.

Thus, 1, 2, 3, 5, 7, 11 are prime numbers, while 4, 6, 8, 9, 10 are composite numbers.

Make a list of all prime numbers less than 50. Which ones of the composite numbers have more than two factors?

47. Tests of Divisibility. In practice, factoring consists in finding exact divisors of numbers.

A number is divisible by 2 if its last digit to the right is even.

Thus, 18, 648, 536, 39,890 are divisible by 2, while 37, 381, 93, 78, 497 are not divisible by 2.

A number is divisible by 4 if its last two digits to the right represent a number which is divisible by 4.

That is, 536, 39,892, 300, 356 are divisible by 4. (Every number is a divisor of zero.)

A number is divisible by 8 if its last 3 digits to the right represent a number which is divisible by 8.

Thus, 29,864 and 12,096 are divisible by 8, while 13,978 and 8946 are not divisible by 8.

A number is divisible by 3 if the sum of the digits is divisible by 3.

Thus, 574,851 is divisible by 3 because $5+7+4+8+5+1=30$ is divisible by 3.

A number is divisible by 9 if the sum of its digits is divisible by 9.

A number is divisible by 6 if it is divisible by both 2 and 3.

A number is divisible by 5 if its last digit to the right is 5 or 0.

The best way of testing for divisibility by 7 or 11 is to divide.

ORAL EXERCISES

1. Which of the numbers 3840, 6593, 87, 4560 are divisible by 2? Which of these numbers are divisible by 4?

2. Which of the numbers 2838, 91,876, 7948, 30,474 are divisible by 8?

3. Which of the numbers 9726, 3197, 8190, 6736, are divisible by 3? Which of them are divisible by 9?

4. Which of the numbers in example 3 are divisible by 6. Give other numbers that are divisible by 6.

5. Give several numbers that are divisible by 5.

48. Prime Factors. **A factor which is a prime number is called a** *prime factor.*

Thus, 2 and 3 are prime factors of 6. 2, 3, and 5 are prime factors of 30, while 6, 10, and 15 are *composite* factors of 30.

In some products the same factor is repeated.

Thus, $4=2\times2$; $8=2\times2\times2$, and $50=2\times5\times5$. The prime factors of 8 are 2, 2, 2.

To find the prime factors of a number, divide it by one of its prime factors, then divide the quotient by one of its prime factors, and so on until the last quotient is a prime number.

Example. Find the prime factors of 5460.

Solution:

$$\begin{array}{r} 2\underline{)5460} \\ 2\underline{)2730} \\ 5\underline{)1365} \\ 3\underline{)273} \\ 7\underline{)91} \\ 13 \end{array}$$

Hence, the prime factors are 2, 2, 5, 3, 7, 13.

The work may be shortened by dividing by larger numbers which are not a prime factor, as shown in the second form. Thus:

$$\begin{array}{r} 20\underline{)5460} \\ 3\underline{)273} \\ 7\underline{)91} \\ 13 \end{array}$$

We know at once that the prime factors of 20 are 2, 2, 5; so we can write down the required prime factors from the factors obtained here.

WRITTEN EXERCISES

Find the prime factors of each of the following:

1. 108	6. 1728	11. 66000	16. 3780
2. 2240	7. 1908	12. 2280	17. 1680
3. 8451	8. 4771	13. 2520	18. 2310
4. 625	9. 9702	14. 2160	19. 8250
5. 1896	10. 4620	15. 1485	20. 2295

49. A General Rule on Fractions. The following is one of the most useful rules on fractions:

Both terms of a fraction may be multiplied or divided by the same number without changing the value of the fraction.

Thus, $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$ $\quad \frac{6}{18}=\frac{3}{9}=\frac{1}{3}$, etc.

ORAL EXERCISES

1. If the denominator of a fraction is multiplied by 2, how is the value of the fractional unit changed?

2. If the fractional unit is made only half as large as in the original fraction, how must the number of fractional units be changed to keep the value of the fraction unchanged?

3. Ask and answer other questions like these. Can you make the above rule from these questions and answers?

50. Reducing Fractions to Lowest Terms. A fraction whose numerator and denominator have no common factor is in its lowest terms.

Thus, $\frac{3}{4}$, $\frac{5}{7}$, and $\frac{3}{10}$ are in their lowest terms, while $\frac{2}{4}$, $\frac{3}{9}$ and $\frac{7}{28}$ are not in their lowest terms.

The following rule is easily understood:

A fraction may be reduced to its lowest terms by dividing both terms by their greatest common factor.

Thus, $\frac{2}{4}$ is reduced to $\frac{1}{2}$ by dividing both terms by 2, and $\frac{3}{9}$ is reduced to $\frac{1}{3}$ by dividing both terms by 3.

ORAL EXERCISES

1. Which of the following fractions are not in the lowest terms? $\frac{1}{6}$, $\frac{3}{7}$, $\frac{3}{15}$, $\frac{27}{45}$, $\frac{8}{21}$, $\frac{8}{24}$, $\frac{9}{21}$, $\frac{9}{24}$, $\frac{9}{32}$. Reduce each such to the lowest terms.

2. Reduce each of the following to its lowest terms: $\frac{4}{8}$, $\frac{3}{15}$, $\frac{7}{21}$, $\frac{8}{24}$, $\frac{5}{25}$, $\frac{9}{15}$, $\frac{6}{16}$, $\frac{12}{32}$, $\frac{15}{21}$, $\frac{16}{35}$.

Example. Reduce $\frac{132}{192}$ to lowest terms.

Solution: $\begin{matrix}4)\\4)\end{matrix}\frac{132}{192}=\frac{33}{48}=\frac{11}{16}$ The common factors 4 and 3 are removed successively.

This example illustrates the following convenient rule:

To reduce a fraction to the lowest terms, divide both terms by a common factor, then both terms of the resulting fraction by a common factor, and so on until a fraction in its lowest terms is obtained.

This rule makes it unnecessary to find the greatest common factor of the two terms of the fraction.

ORAL EXERCISES

Using the rule just given, reduce the following fractions to their lowest terms:

1. $\frac{12}{42}$	6. $\frac{4}{32}$	11. $\frac{16}{88}$	16. $\frac{4}{92}$
2. $\frac{16}{48}$	7. $\frac{16}{96}$	12. $\frac{24}{54}$	17. $\frac{8}{96}$
3. $\frac{8}{28}$	8. $\frac{18}{66}$	13. $\frac{12}{44}$	18. $\frac{12}{78}$
4. $\frac{24}{36}$	9. $\frac{12}{54}$	14. $\frac{8}{52}$	19. $\frac{16}{68}$
5. $\frac{15}{45}$	10. $\frac{20}{36}$	15. $\frac{6}{84}$	20. $\frac{18}{108}$

WRITTEN EXERCISES

Reduce each of the following to its lowest terms:

1. $\frac{64}{92}$	8. $\frac{128}{384}$	15. $\frac{315}{378}$	22. $\frac{78}{96}$
2. $\frac{45}{81}$	9. $\frac{87}{132}$	16. $\frac{246}{822}$	23. $\frac{35}{115}$
3. $\frac{56}{84}$	10. $\frac{49}{112}$	17. $\frac{125}{500}$	24. $\frac{49}{343}$
4. $\frac{38}{95}$	11. $\frac{144}{192}$	18. $\frac{240}{800}$	25. $\frac{36}{216}$
5. $\frac{64}{84}$	12. $\frac{116}{276}$	19. $\frac{126}{180}$	26. $\frac{28}{63}$
6. $\frac{48}{88}$	13. $\frac{27}{108}$	20. $\frac{210}{615}$	27. $\frac{48}{256}$
7. $\frac{145}{285}$	14. $\frac{288}{864}$	21. $\frac{39}{159}$	28. $\frac{54}{729}$

51. **Proper and Improper Fractions.** A fraction which is less than 1 is called a *proper fraction.* Other fractions are called *improper fractions.*

52. **Mixed Numbers.** A number consisting of an integer and a fraction is called a *mixed number.*

A mixed number or an integer may be reduced to an improper fraction.

An improper fraction may be reduced to an integer or a mixed number.

Example 1. $3\frac{2}{3}=\frac{9}{3}+\frac{2}{3}=\frac{11}{3}$.

Example 2. $\frac{35}{6}=\frac{30}{6}+\frac{5}{6}=5\frac{5}{6}$.

In practice we see these results at once, without the intervening steps.

53. **Numbers in Simplest Forms.** A number is in its simplest form if it is an integer or if its fractional part is a proper fraction in its lowest terms.

ORAL EXERCISES

Name rapidly improper fractions equal to the following:

1. $1\frac{1}{2}$	5. $2\frac{2}{3}$	9. $4\frac{1}{2}$	13. $5\frac{5}{6}$
2. $1\frac{2}{3}$	6. $3\frac{1}{4}$	10. $5\frac{2}{3}$	14. $5\frac{1}{6}$
3. $1\frac{3}{5}$	7. $3\frac{3}{15}$	11. $5\frac{3}{4}$	15. $6\frac{2}{3}$
4. $1\frac{5}{6}$	8. $3\frac{2}{7}$	12. $6\frac{2}{5}$	16. $7\frac{3}{4}$

Reduce the following to the simplest form.

17. $\frac{5}{2}$	21. $\frac{16}{3}$	25. $\frac{36}{6}$	29. $\frac{43}{8}$
18. $\frac{9}{4}$	22. $\frac{16}{5}$	26. $\frac{45}{7}$	30. $\frac{57}{6}$
19. $\frac{11}{5}$	23. $\frac{17}{6}$	27. $\frac{57}{9}$	31. $\frac{49}{6}$
20. $\frac{15}{4}$	24. $\frac{24}{5}$	28. $\frac{38}{5}$	32. $\frac{54}{9}$

54. **Multiples.** A number which is divisible by another number is called a *multiple* of that number.

Thus, 12, 24, 36, 48 are multiples of 12.

A factor of a number is a factor of every multiple of that number.

Thus, every factor of 12 is a factor of 24, of 36, of 48, and so on.

If a number is a multiple of 2 and also of 3, then it is a multiple of 6, which is the product of 2 and 3. Similarly, if a number is a multiple of 3 and also of 5, then it is a multiple of 15.

In general, if a number is a multiple of each of two numbers which have no common factor, then it is a multiple of the product of these numbers.

A multiple of the product of two numbers is a multiple of each of these numbers.

ORAL EXERCISES

1. Which of the following are multiples of 2: 18, 21, 36, 45, 63, 72, 81?

2. Which of the following are multiples of 3: 27, 36, 42, 45, 48, 49, 51, 63, 64, 72, 76, 81?

3. How can you tell whether a number is a multiple of 4? Which of the following numbers are multiples of 4 : 36, 84, 94, 144, 168, 282, 148?

4. How can you tell whether a number is a multiple of 15? Which of the following numbers are multiples of 15: 75, 545, 365, 465, 695, 795, 1265, 1365, 2435, 3435?

5. Which of the following numbers are multiples of 30: 750, 3860, 5920, 8340, 62900, 37140?

6. Which of the following numbers are multiples of 21: 4935, 6838, 9614, 65142?

Suggestion. Test each number for divisibility by 3 in the usual manner, and if found divisible by 3, divide by 7. If the number is not divisible by 3 we know at once it is not divisible by 21.

55. **The Least Common Multiple.** The smallest number which is a multiple of each of two or more numbers is called the *least common multiple* (L. C. M.) of these numbers.

The L. C. M. of *two* numbers may usually be found by the method shown in Example 1 below. The L. C. M. of *three or more* numbers may be found by the method used in Example 2.

Example 1. Find the L. C. M. of 8 and 18.

Solution: Write down a series of multiples of 18 (the larger number). These are 36, 54, 72, 90, etc. We then see that 72 is the smallest among these which is a multiple of 8. Hence 72 is the L. C. M. of 8 and 18.

Example 2. Find the L. C. M. of 12, 16, 18.

Solution: $12=2\times2\times3.$

$16=2\times2\times2\times2.$

$18=2\times3\times3.$

Hence a multiple of 12, 16, and 18 must contain the factors 2, 2, 3, and 2, 2, 2, 2, and 2, 3, 3.

We see at once that $2\times2\times2\times2\times3\times3$ is such a number.

Hence the required L. C. M. is $2\times2\times2\times2\times3\times3=144$.

WRITTEN EXERCISES

Find the L. C. M. of each of the following:

1. 4, 8, 12, 20.
2. 7, 14, 21, 28.
3. 9, 15, 18, 21.
4. 2, 3, 4, 5.
5. 12, 15, 16.
6. 12, 16, 18, 20.
7. 3, 5, 7, 9.
8. 5, 6, 7, 10.
9. 6, 9, 12, 15.
10. 16, 24, 36.
11. 4, 6, 8, 10.
12. 15, 18, 24.
13. 12, 30, 40.
14. 9, 15, 24, 30.
15. 16, 18, 24, 36.
16. 14, 21, 35, 49.

56. Common Denominators. Fractions which have the same denominators are called **like fractions.** It is often necessary to reduce two or more fractions to like fractions.

Thus, to add $\frac{1}{2}$ and $\frac{1}{3}$, these fractions must be reduced to 6ths. That is, $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

Similarly, $\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$.

Again, to add $\frac{1}{4}$ and $\frac{5}{6}$, these fractions must be reduced to 12ths. That is, $\frac{1}{4}+\frac{5}{6}=\frac{3}{12}+\frac{10}{12}=\frac{13}{12}=1\frac{1}{12}$.

Similarly, $\frac{5}{6}-\frac{1}{4}=\frac{10}{12}-\frac{3}{12}=\frac{7}{12}$.

It is easily seen that the smallest number which is a common denominator of two or more fractions is the L. C. M. of their denominators.

Thus, in the above examples 6 is the L. C. M. of 2 and 3, and 12 is the L. C. M. of 4 and 6.

Change $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{5}{9}$ into like fractions.

$$\frac{1}{2}=\frac{18\times1}{18\times2}=\frac{18}{36}$$
$$\frac{1}{4}=\frac{9\times1}{9\times4}=\frac{9}{36}$$
$$\frac{5}{9}=\frac{4\times5}{4\times9}=\frac{20}{36}$$

Solution: First step. Find the L. C. M. of 2, 4, and 9. This is found to be 36.
Second step: From $36\div2=18$ we find that both terms of $\frac{1}{2}$ must be multiplied by 18.

Similarly, from $36\div4=9$ we find that the terms of $\frac{1}{4}$ must be multiplied by 9.

Explain how $\frac{5}{9}$ is reduced to $\frac{20}{36}$.

In reducing fractions to common denominators, the steps in their proper order are:

First: *Find the L. C. M. of the given denominators.*

Second: *Divide the L. C. M. by the denominator of each fraction and multiply both terms by the quotient.*

It should be clearly understood that the general principle stated on page 49 is used in this process, and that the real difficulty consists in finding the proper number by which to multiply both terms of each fraction.

ORAL EXERCISES

Reduce the following fractions to common denominators:

1. $\frac{1}{2}, \frac{1}{3}$.
2. $\frac{1}{2}, \frac{2}{3}$.
3. $\frac{1}{2}, \frac{1}{4}$.
4. $\frac{1}{2}, \frac{3}{4}$.
5. $\frac{1}{3}, \frac{1}{4}$.
6. $\frac{2}{3}, \frac{1}{4}$.
7. $\frac{1}{3}, \frac{3}{4}$.
8. $\frac{2}{3}, \frac{3}{4}$.
9. $\frac{1}{3}, \frac{1}{5}$.
10. $\frac{2}{3}, \frac{1}{5}$.
11. $\frac{1}{3}, \frac{2}{5}$.
12. $\frac{1}{3}, \frac{3}{5}$.
13. $\frac{1}{3}, \frac{4}{5}$.
14. $\frac{2}{3}, \frac{2}{5}$.
15. $\frac{2}{3}, \frac{3}{5}$.
16. $\frac{2}{3}, \frac{4}{5}$.
17. $\frac{1}{4}, \frac{1}{8}$.
18. $\frac{3}{4}, \frac{3}{8}$.
19. $\frac{2}{5}, \frac{1}{4}$.
20. $\frac{1}{5}, \frac{1}{6}$.
21. $\frac{2}{5}, \frac{1}{6}$.
22. $\frac{3}{5}, \frac{5}{6}$.
23. $\frac{4}{5}, \frac{1}{7}$.
24. $\frac{5}{6}, \frac{5}{8}$.
25. $\frac{3}{8}, \frac{9}{16}$.

Add:

26. $\frac{1}{2}+\frac{1}{3}$
27. $\frac{1}{2}+\frac{3}{4}$
28. $\frac{2}{5}+\frac{1}{4}$
29. $\frac{1}{2}+\frac{2}{3}$
30. $\frac{2}{3}+\frac{3}{4}$
31. $\frac{3}{5}+\frac{1}{4}$
32. $\frac{1}{3}+\frac{1}{4}$
33. $\frac{1}{5}+\frac{1}{4}$
34. $\frac{4}{5}+\frac{1}{4}$
35. $\frac{2}{3}+\frac{1}{4}$
36. $\frac{1}{5}+\frac{3}{4}$
37. $\frac{4}{5}+\frac{3}{4}$

38–62. Find the difference between the fractions in each of Examples 1–25.

In adding mixed numbers, the whole numbers are added separately, and the fractions separately. The fractions should be added first, and the sum reduced to the simplest form. If there is a whole number in this sum it is then added to the other whole numbers.

Thus: $1\frac{1}{2}+3\frac{1}{3}+4\frac{1}{4}=9\frac{1}{12}$.

In this case the sum of the fractions is $\frac{13}{12}=1\frac{1}{12}$.

WRITTEN EXERCISES

1. $3\frac{1}{4}+4\frac{2}{3}+1\frac{1}{2}$
2. $7\frac{4}{5}+2\frac{1}{3}+4\frac{1}{2}$
3. $12\frac{7}{8}+4\frac{1}{9}+16+28\frac{1}{4}$
4. $212\frac{1}{3}+180\frac{5}{9}+3\frac{1}{2}$
5. $15\frac{3}{4}+7\frac{3}{8}+8\frac{3}{5}$
6. $9\frac{1}{12}+8\frac{1}{6}+7\frac{1}{3}+2\frac{1}{4}$
7. $4\frac{3}{8}+2\frac{3}{4}+1\frac{1}{2}+\frac{2}{3}$
8. $6\frac{5}{8}+2\frac{3}{4}+9\frac{1}{2}+7\frac{1}{3}$
9. $9\frac{3}{5}+6\frac{7}{10}+8\frac{4}{5}$
10. $10\frac{4}{9}+5\frac{2}{3}+1\frac{5}{12}+\frac{3}{4}$

57. Subtraction of Mixed Numbers. In the subtraction of mixed numbers a special device is sometimes necessary.

Example. Subtract $4\frac{1}{4}-1\frac{1}{3}$

$4\frac{1}{4}-1\frac{1}{3}=4\frac{3}{12}-1\frac{4}{12}$
$=4\frac{15}{12}-2\frac{4}{12}=2\frac{11}{12}$

First Solution: Add 1 to both subtrahend and minuend, making the minuend $4\frac{15}{12}$ and the subtrahend $2\frac{4}{12}$.

$4\frac{1}{4}-1\frac{1}{3}=3\frac{5}{4}-1\frac{1}{3}$
$=3\frac{15}{12}-1\frac{4}{12}=2\frac{11}{12}$

Second Solution: First change the minuend $4\frac{1}{4}$ into $3\frac{5}{4}$.

Subtract each of the following:

1. $12\frac{2}{3}-6\frac{1}{2}$	4. $6\frac{1}{2}-2\frac{3}{4}$	7. $8\frac{1}{4}-2\frac{2}{3}$
2. $8\frac{1}{7}-4\frac{2}{3}$	5. $8\frac{1}{3}-3\frac{4}{5}$	8. $12\frac{1}{2}-9\frac{7}{12}$
3. $9\frac{1}{4}-4\frac{2}{9}$	6. $14\frac{3}{5}-6\frac{2}{9}$	9. $41\frac{7}{8}-27\frac{9}{10}$

58. Fractions in Common Use. Fractions that are in common use are quite simple, and can be added and subtracted without the aid of pencil and paper.

ORAL EXERCISES

See how long it takes to add the following. Write down the results only. Also see how long it takes to find the difference between the two fractions given in each example.

1. $\frac{1}{2}+\frac{1}{4}$	8. $\frac{1}{2}+\frac{7}{8}$	15. $\frac{1}{3}+\frac{1}{6}$	22. $\frac{1}{4}+\frac{1}{16}$	29. $\frac{2}{3}+\frac{1}{12}$
2. $\frac{1}{2}+\frac{3}{4}$	9. $\frac{1}{2}+\frac{1}{6}$	16. $\frac{2}{3}+\frac{1}{6}$	23. $\frac{3}{4}+\frac{1}{16}$	30. $\frac{1}{3}+\frac{5}{12}$
3. $\frac{1}{2}+\frac{1}{3}$	10. $\frac{1}{2}+\frac{5}{6}$	17. $\frac{1}{3}+\frac{5}{6}$	24. $\frac{1}{4}+\frac{3}{16}$	31. $\frac{2}{3}+\frac{5}{12}$
4. $\frac{1}{2}+\frac{2}{3}$	11. $\frac{1}{3}+\frac{1}{4}$	18. $\frac{2}{3}+\frac{5}{6}$	25. $\frac{3}{4}+\frac{3}{16}$	32. $\frac{1}{4}+\frac{1}{12}$
5. $\frac{1}{2}+\frac{1}{8}$	12. $\frac{2}{3}+\frac{1}{4}$	19. $\frac{1}{2}+\frac{1}{16}$	26. $\frac{1}{4}+\frac{5}{16}$	33. $\frac{1}{8}+\frac{1}{12}$
6. $\frac{1}{2}+\frac{3}{8}$	13. $\frac{1}{3}+\frac{3}{4}$	20. $\frac{1}{2}+\frac{3}{16}$	27. $\frac{3}{4}+\frac{5}{16}$	34. $\frac{3}{8}+\frac{1}{12}$
7. $\frac{1}{2}+\frac{5}{8}$	14. $\frac{2}{3}+\frac{3}{4}$	21. $\frac{1}{2}+\frac{5}{16}$	28. $\frac{1}{3}+\frac{3}{12}$	35. $\frac{5}{8}+\frac{1}{12}$

ORAL REVIEW

1. What is an odd number? An even number?

2. What is a prime number? A composite number?

3. What numbers are divisible by 2?

4. Give a test for divisibility by 4, by 8.

5. Give a test for divisibility by 5.

6. Give a test for divisibility by 3, by 9.

7. What is meant by "a prime factor of a number"?

8. If in a division problem both dividend and divisor are divided by the same number, what is the effect on the quotient? How do the quotients compare in $89000\overline{)43820000}$ and $89\overline{)43820}$?

9. If in a division problem both dividend and divisor are multiplied by the same number, what is the effect on the quotient? How do the quotients compare in $2.98\overline{)76.452}$ and $298\overline{)7645.2}$?

10. What is meant by a fraction in the lowest terms? How may a fraction be reduced to its lowest terms?

11. What is meant by a "proper" and an "improper" fraction? Give examples of each.

12. What is a mixed number? What kind of fraction may be reduced to a mixed number?

13. May a mixed number be reduced to a proper or an improper fraction? Give an example.

14. Give examples of large numbers which are never known exactly. Discuss why they are never known exactly.

15. Give examples of large numbers which are known exactly. Is the number of students enrolled in a school on a certain day known exactly?

59. Multiplication of Fractions.

Example 1. Multiply $\frac{3}{32}$ by 6.

Solution: $6\times\frac{3}{32}=\frac{18}{32}=\frac{9}{16}$.

The work can be shortened by cancelling the factor 2 before multiplying, as in the second process.

Thus: $\overset{3}{\cancel{6}}\times\frac{3}{\underset{16}{\cancel{32}}}=\frac{9}{16}$

Example 2. Multiply $\frac{7}{16}$ by 8.

Solution: $\cancel{8}\times\frac{7}{\underset{2}{\cancel{16}}}=\frac{7}{2}=3\frac{1}{2}$.

Example 3. Multiply 5 by $\frac{3}{16}$.

Solution: $\frac{3}{16}\times5$ is the same as $5\times\frac{3}{16}$. (See page 17.) Hence, $\frac{3}{16}\times5=\frac{15}{16}$.

All possible cancellations should be performed first.

Thus, $\overset{2}{\cancel{6}}\times\frac{2}{\underset{5}{\cancel{15}}}=2\times\frac{2}{5}=\frac{4}{5}$.

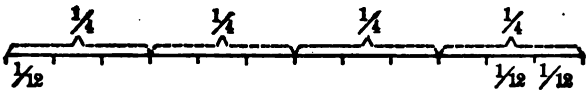
ORAL EXERCISES

Find the products of the following, and reduce each result to its simplest form. Cancel common factors before multiplying.

1. $3\times\frac{2}{3}$	5. $5\times\frac{4}{5}$	9. $9\times\frac{3}{7}$	13. $6\times\frac{4}{9}$
2. $4\times\frac{2}{3}$	6. $3\times\frac{5}{8}$	10. $12\times\frac{3}{4}$	14. $12\times\frac{3}{4}$
3. $4\times\frac{2}{5}$	7. $8\times\frac{4}{9}$	11. $10\times\frac{3}{16}$	15. $9\times\frac{5}{6}$
4. $6\times\frac{3}{7}$	8. $5\times\frac{7}{8}$	12. $10\times\frac{7}{15}$	16. $8\times\frac{5}{12}$

To multiply the following, notice Example 3 above.

17. $\frac{3}{4}\times6$	20. $\frac{5}{6}\times9$	23. $\frac{3}{5}\times7$	26. $\frac{5}{12}\times8$
18. $\frac{4}{5}\times8$	21. $\frac{2}{5}\times6$	24. $\frac{7}{8}\times10$	27. $\frac{5}{12}\times9$
19. $\frac{2}{3}\times9$	22. $\frac{5}{8}\times12$	25. $\frac{1}{12}\times36$	28. $\frac{3}{8}\times6$



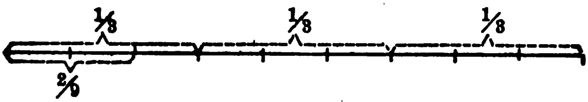
From this figure we see that $\frac{1}{3}$ of $\frac{1}{4}=\frac{1}{12}$

EXERCISES

Draw lines and mark them off to show the following:

1. $\frac{1}{2}$ of $\frac{1}{2}$
2. $\frac{1}{2}$ of $\frac{1}{3}$
3. $\frac{1}{2}$ of $\frac{1}{5}$
4. $\frac{1}{2}$ of $\frac{1}{6}$
5. $\frac{1}{3}$ of $\frac{1}{2}$
6. $\frac{1}{3}$ of $\frac{2}{3}$
7. $\frac{1}{4}$ of $\frac{1}{6}$
8. $\frac{1}{2}$ of $\frac{1}{7}$
9. $\frac{1}{2}$ of $\frac{2}{3}$
10. $\frac{1}{2}$ of $\frac{1}{4}$
11. $\frac{1}{3}$ of $\frac{1}{5}$
12. $\frac{1}{3}$ of $\frac{1}{6}$

From the figure above we also see that $\frac{2}{3}$ of $\frac{3}{4}=\frac{2}{4}=\frac{1}{2}$.



From this figure we see that $\frac{2}{3}$ of $\frac{1}{3}=\frac{2}{9}$.

By studying these examples we shall find that all of them may be solved by the rule:

To multiply one fraction by another, cancel all factors common to numerators and denominators, and then multiply the numerators together and also the denominators.

ORAL EXERCISES

Find the products of the following:

13. $\frac{5}{9}\times\frac{2}{3}$
14. $\frac{7}{8}\times\frac{3}{4}$
15. $\frac{3}{7}\times\frac{5}{6}$
16. $\frac{4}{9}\times\frac{3}{5}$
17. $\frac{2}{3}\times\frac{3}{5}$
18. $\frac{3}{8}\times\frac{3}{5}$
19. $\frac{2}{5}\times\frac{3}{7}$
20. $\frac{2}{7}\times\frac{4}{5}$
21. $\frac{4}{5}\times\frac{5}{9}$
22. $\frac{3}{8}\times\frac{5}{7}$
23. $\frac{5}{9}\times\frac{5}{15}$
24. $\frac{8}{9}\times\frac{3}{16}$

60. Product of a Mixed Number and an Integer. To multiply a mixed number by a whole number, multiply the integer and the fraction separately, and then add the products.

Thus, $4\times3\frac{2}{3}=12\frac{8}{3}=14\frac{2}{3}$.

ORAL AND WRITTEN EXERCISES

Multiply the following and reduce to simplest form. Do as many as you can on this page without using pencil and paper.

1. $3\times1\frac{1}{4}$	6. $8\times8\frac{3}{8}$	11. $10\times3\frac{1}{3}$
2. $6\times8\frac{5}{6}$	7. $12\times14\frac{1}{3}$	12. $9\times4\frac{1}{4}$
3. $8\times3\frac{3}{16}$	8. $44\times2\frac{3}{4}$	13. $7\times4\frac{3}{5}$
4. $4\times8\frac{1}{9}$	9. $8\times9\frac{4}{5}$	14. $16\times3\frac{2}{5}$
5. $6\times3\frac{1}{5}$	10. $12\times3\frac{2}{9}$	15. $3\times18\frac{5}{16}$

Since $2\frac{1}{3}\times2=2\times2\frac{1}{3}$, the product $2\frac{1}{3}\times2$ may be found as in the previous case.

That is, $2\frac{1}{3}\times2=4\frac{2}{3}$.

16. At 40 cents a yard, what is the cost of $12\frac{3}{4}$ yards of cloth?

17. A motor boat goes $8\frac{2}{3}$ miles per hour. How far will it go in 4 hours?

18. At 32 cents a pound, what is the cost of $2\frac{1}{4}$ pounds of meat?

19. At 45 cents a pound, what is the cost of $1\frac{3}{8}$ pounds of boiled ham?

20. A man walks $3\frac{5}{8}$ miles per hour. How far will he walk in 3 hours?

21. A box of crackers weighs $\frac{7}{8}$ of a pound (14 ounces). How many pounds of crackers are there in 12 such boxes?

22. A board of siding is $\frac{5}{8}$ inches thick. What is the height of a pile of 24 such boards?

61. The Product of Two Mixed Numbers.

Example. Multiply $2\frac{1}{2}$ by $3\frac{4}{5}$.

First Solution: Reduce each number to an improper fraction. Then, $2\frac{1}{2}\times3\frac{4}{5}=\frac{5}{2}\times\frac{19}{5}=\frac{19}{2}=9\frac{1}{2}$.

Second Solution (four-step method):

$$
\begin{aligned}
2\times3&=6\\
2\times\tfrac{4}{5}&=1\tfrac{3}{5}\\
\tfrac{1}{2}\times3&=1\tfrac{1}{2}\\
\tfrac{1}{2}\times\tfrac{4}{5}&=\ \tfrac{2}{5}\\
&\ 8\tfrac{15}{10}=9\tfrac{1}{2}
\end{aligned}
$$

First multiply the integers; then multiply each integer by the fraction in the other factor, and finally multiply the fractions. The sum of these products is the product of the mixed numbers.

The first solution is shorter when the numbers are small, and the second when they are large. To find a product like $48\frac{3}{4}\times45\frac{2}{3}$, the second method should always be used.

WRITTEN EXERCISES

Multiply and reduce each result to its simplest form:

1. $5\frac{1}{2}\times2\frac{1}{3}$
2. $12\frac{1}{3}\times2\frac{3}{4}$
3. $9\frac{2}{3}\times4\frac{5}{8}$
4. $5\frac{1}{4}\times17$
5. $8\frac{1}{2}\times3\frac{1}{2}$
6. $48\ \times17\frac{1}{4}$
7. $35\ \times46\frac{2}{3}$
8. $\frac{3}{7}\times43\frac{1}{2}$
9. $1\frac{3}{5}\times1\frac{1}{2}$
10. $12\frac{1}{2}\times390$
11. $15\frac{3}{4}\times2\frac{1}{2}$
12. $53\ \times1\frac{3}{8}$
13. $1\frac{3}{4}\times2\frac{1}{3}$
14. $6\frac{3}{4}\times8\frac{4}{5}$
15. $10\frac{3}{5}\times2\frac{4}{9}$
16. $3\frac{2}{7}\times5\frac{3}{4}$
17. $190\times3\frac{1}{4}$
18. $31\frac{1}{2}\times35\frac{2}{3}$
19. $18\ \times196\frac{3}{4}$
20. $2\frac{4}{5}\times2\frac{3}{4}$
21. $6\frac{2}{3}\times2\frac{3}{4}$
22. $8\frac{1}{2}\times3\frac{4}{5}$
23. $10\frac{2}{3}\times6\frac{1}{2}$
24. $24\ \times16\frac{1}{4}$
25. $6\frac{1}{2}\times3\frac{1}{4}$
26. $1\frac{2}{5}\times2\frac{3}{7}$
27. $8\frac{3}{7}\times2\frac{1}{3}$
28. $12\frac{1}{3}\times240$
29. $36\ \times3\frac{7}{8}$
30. $45\ \times84\frac{1}{3}$
31. $13\frac{3}{8}\times8\frac{1}{4}$
32. $470\times1\frac{1}{8}$
33. $9\frac{1}{3}\times496$

62. Division of Fractions. The following examples lead to a general rule for dividing fractions.

Example 1. Divide 4 by $\frac{1}{2}$.

Solution: Since $\frac{1}{2}$ is contained 2 times in 1, it must be contained $4\times2=8$ times in 4. That is, $4\div\frac{1}{2}=8$.

Example 2. Divide $\frac{1}{2}$ by $\frac{1}{3}$.

$\frac{1}{2}\div\frac{1}{3}=\frac{3}{6}\div\frac{2}{6}$
$=3\div2=\frac{3}{2}=1\frac{1}{2}$.

Solution: Reduce both dividend and divisor to 6ths.

Example 3. Divide $\frac{2}{5}$ by $\frac{3}{4}$.

$\frac{2}{5}\div\frac{3}{4}=\frac{8}{20}\div\frac{15}{20}$
$=8\div15=\frac{8}{15}$.

Solution: Reduce both dividend and divisor to 20ths.

In this way any problem in division of fractions may be solved by reducing both dividend and divisor to like fractions.

All problems in division of fractions may also be solved directly by means of the following rule:

To divide by a fraction invert the terms of the divisor and then multiply.

That is, $4\div\frac{1}{2}=4\times\frac{2}{1}=8$, $\frac{1}{2}\div\frac{1}{3}=\frac{1}{2}\times\frac{3}{1}=\frac{3}{2}$, and $\frac{2}{5}\div\frac{3}{4}=\frac{2}{5}\times\frac{4}{3}=\frac{8}{15}$. Compare results found above.

63. Reciprocal of a Number. The reciprocal of a number is 1 divided by that number.

Thus, $\frac{1}{2}$ is the reciprocal of 2, and $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$.

Instead of dividing by a number we may multiply by the reciprocal of that number.

That is, $\frac{3}{5}\div2=\frac{3}{5}\times\frac{1}{2}=\frac{3}{10}$, and $\frac{2}{3}\div\frac{3}{4}=\frac{2}{3}\times\frac{4}{3}=\frac{8}{9}$.

ORAL EXERCISES

1. $\frac{1}{2}\div\frac{1}{4}$	3. $\frac{2}{3}\div\frac{3}{5}$	5. $\frac{1}{4}\div\frac{1}{2}$	7. $\frac{1}{5}\div\frac{1}{6}$
2. $\frac{1}{3}\div\frac{1}{2}$	4. $\frac{2}{3}\div\frac{5}{7}$	6. $\frac{3}{4}\div\frac{1}{3}$	8. $\frac{1}{6}\div\frac{1}{7}$

Example. Divide $\frac{9}{32}$ by $\frac{15}{28}$.

Solution: $\frac{9}{32}\div\frac{15}{28}=\frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{32}}}\times\frac{\overset{7}{\cancel{28}}}{\underset{5}{\cancel{15}}}=\frac{3\times7}{8\times5}=\frac{21}{40}$

After inverting the terms of the divisor care should be taken to cancel all common factors before multiplying.

WRITTEN EXERCISES

1. $\frac{7}{9}\div\frac{3}{8}$
2. $\frac{5}{12}\div\frac{6}{27}$
3. $\frac{9}{11}\div\frac{24}{35}$
4. $\frac{2}{5}\div\frac{15}{16}$
5. $\frac{15}{16}\div\frac{2}{5}$
6. $\frac{24}{35}\div\frac{8}{15}$
7. $\frac{8}{27}\div\frac{5}{12}$
8. $\frac{3}{8}\div\frac{7}{9}$
9. $\frac{5}{9}\div\frac{6}{25}$
10. $\frac{8}{11}\div\frac{7}{16}$
11. $\frac{4}{17}\div\frac{16}{21}$
12. $\frac{15}{14}\div\frac{5}{7}$

64. **Division of Mixed Numbers.** Mixed numbers may be divided by reducing them to improper fractions.

Example. Divide $4\frac{1}{3}$ by $2\frac{3}{4}$.

Solution: $4\frac{1}{3}\div2\frac{3}{4}=\frac{13}{3}\div\frac{11}{4}=\frac{13}{3}\times\frac{4}{11}=\frac{52}{33}=1\frac{19}{33}$.

WRITTEN EXERCISES

1. $2\frac{4}{5}\div1\frac{2}{3}$
2. $6\frac{7}{8}\div3\frac{1}{4}$
3. $3\frac{5}{6}\div4\frac{1}{4}$
4. $5\frac{2}{3}\div3\frac{4}{5}$
5. $4\frac{3}{5}\div2\frac{4}{7}$
6. $2\frac{2}{7}\div3\frac{4}{5}$
7. $6\frac{1}{9}\div2\frac{2}{3}$
8. $2\frac{3}{7}\div4\frac{4}{7}$
9. $3\frac{2}{5}\div2\frac{2}{5}$
10. $4\frac{5}{6}\div6\frac{3}{4}$
11. $7\frac{1}{3}\div2\frac{1}{4}$
12. $6\frac{1}{2}\div3\frac{1}{5}$
13. $4\frac{5}{6}\div7\frac{2}{3}$
14. $6\frac{7}{8}\div3\frac{3}{4}$
15. $8\frac{2}{5}\div6\frac{3}{4}$
16. $3\frac{2}{3}\div4\frac{1}{6}$
17. $5\frac{3}{4}\div7\frac{4}{5}$
18. $9\frac{1}{2}\div5\frac{3}{8}$
19. $15\frac{3}{8}\div2\frac{3}{4}$
20. $24\frac{3}{4}\div2\frac{3}{8}$
21. $42\frac{5}{6}\div7\frac{3}{4}$
22. $14\frac{1}{2}\div3\frac{2}{3}$
23. $8\frac{2}{3}\div5\frac{1}{4}$
24. $15\frac{1}{4}\div6\frac{1}{2}$
25. $9\frac{3}{4}\div3\frac{1}{3}$
26. $16\frac{1}{5}\div4\frac{1}{2}$
27. $18\frac{1}{3}\div6\frac{2}{3}$

65. Short Cuts in Multiplication of Fractions. The following table will suggest numerous short cuts, some of which have already been given:

$1\frac{1}{4}=\frac{10}{8}$	$8\frac{1}{3}=\frac{100}{12}$	$333\frac{1}{3}=\frac{1000}{3}$
$1\frac{2}{3}=\frac{10}{6}$	$12\frac{1}{2}=\frac{100}{8}$	$25=\frac{100}{4}$
$2\frac{1}{2}=\frac{10}{4}$	$16\frac{2}{3}=\frac{100}{6}$	$125=\frac{1000}{8}$
$3\frac{1}{3}=\frac{10}{3}$	$33\frac{1}{3}=\frac{100}{3}$	$250=\frac{1000}{4}$
$6\frac{1}{4}=\frac{100}{16}$	$166\frac{2}{3}=\frac{1000}{6}$	$75=\frac{300}{4}$

66. Short Cuts in Division. This table also suggests short cuts in division. Thus, to divide by $1\frac{1}{4}$, divide by $\frac{10}{8}$, or, what is the same, multiply by $\frac{8}{10}$.

ORAL AND WRITTEN EXERCISES

1. From the above tables give rules for multiplying by $1\frac{1}{4}$, by $1\frac{2}{3}$, by $2\frac{1}{2}$, by $3\frac{1}{3}$.

2. Give rules for multiplying by $6\frac{1}{4}$, by $8\frac{1}{3}$, by $12\frac{1}{2}$, by $16\frac{2}{3}$, by $33\frac{1}{3}$.

3. Give rules for multiplying by $166\frac{2}{3}$, by $333\frac{1}{3}$, by 25, by 125, by 250, by 75.

4. Give rules for dividing by $1\frac{1}{4}$, by $1\frac{2}{3}$, by $2\frac{1}{2}$, by $3\frac{1}{3}$, by $6\frac{1}{4}$.

5. Give rules for dividing by $8\frac{1}{3}$, by $12\frac{1}{2}$, by $16\frac{2}{3}$, by $33\frac{1}{3}$.

6. Give rules for dividing by $166\frac{2}{3}$, by $333\frac{1}{3}$, by 25, by 125, by 250, by 75.

7. Find the products: $563\times1\frac{1}{4}$, $793\times1\frac{2}{3}$, $8100\times2\frac{1}{2}$, $9368\times3\frac{1}{3}$, $2480\times6\frac{1}{4}$, $97500\times8\frac{1}{3}$.

8. Find the quotients: $563\div1\frac{1}{4}$, $793\div1\frac{2}{3}$, $8100\div2\frac{1}{2}$, $9368\div3\frac{1}{3}$, $2480\div6\frac{1}{4}$, $97500\div8\frac{1}{3}$, $5160\div12\frac{1}{2}$, $12500\div16\frac{2}{3}$, $1800\div166\frac{2}{3}$, $7840\div333\frac{1}{3}$.

WRITTEN EXERCISES

1. At $12\frac{1}{2}$¢ a pound, what is the value of 34 bales of cotton, weighing on an average 485 pounds?

2. At $66\frac{2}{3}$ ¢ a bushel, what is the value of 696 bushels of corn?

3. At $8\frac{1}{3}$¢ a yard, what is the cost of 45 bolts of cotton goods, averaging 48 yards to the bolt?

Suggestion: First indicate the solution thus,

$$8\tfrac{1}{3}\times45\times48=\frac{100}{12}\times45\times48.$$

4. At $1.20 per yard, what is the cost of 85 yards of silk?

Suggestion: To the number of yards add $\frac{1}{5}$ of the number. This will give the result in dollars. Explain.

5. At $3.75 a day, how much does a laboring man earn in 96 days?

Suggestion: Multiply 96 by 2 and deduct $\frac{1}{8}$. Explain.

6. A motor boat travels $8\frac{1}{3}$ miles per hour. How many hours will it take it to go 96 miles?

7. An ocean liner averages $16\frac{2}{3}$ knots per hour. How many hours will it take it to reach New York when it is 780 knots out?

8. A fast freight train averages $33\frac{1}{3}$ miles an hour. How many hours will it take this train to go from Chicago to Boston, a distance of 980 miles?

9. A man's salary is $166\frac{2}{3}$ dollars per month. In how many months will he earn $5000?

10. A pile of $1\frac{1}{4}$ inch boards is 8 feet high. How many boards deep is the pile?

11. At $6\frac{1}{4}$¢ a yard, how many yards of print cloth can a dealer buy for $480.00?

67. Decimal Fractions. Fractions whose denominators are 10, 100, 1000, and so on, are called decimal fractions. In writing such fractions only the numerators are written, the denominators being indicated by the decimal point.

68. Reducing Common Fractions to Decimals. A common fraction may be reduced to a decimal by carrying out the indicated division.

Thus, $\frac{1}{2}=\frac{1.0}{2}=.5$, and $\frac{1}{4}=\frac{1.00}{4}=.25$, and $\frac{1}{8}=\frac{1.000}{8}=.125$.

In some cases there will be a remainder, no matter how far the division is carried.

Thus, in $\frac{5}{6}$ there will always be a remainder 2. Hence $\frac{5}{6}$ cannot be reduced exactly to a decimal. In such cases the remainder may be disregarded after carrying the division to a certain number of places, or a common fraction may be used at the right of the decimal.

Thus, $\frac{5}{6}=.833$ (nearly), or $\frac{5}{6}=.833\frac{1}{3}$ (exactly).

Rule: To reduce a common fraction to a decimal, divide the numerator by the denominator, carrying the quotient to as many decimal places as may be desired.

There are certain important fractions whose decimal equivalent we should recognize at sight. Such are:

$\frac{1}{2}=.5$	$\frac{1}{3}=.33\frac{1}{3}$	$\frac{5}{8}=.62\frac{1}{2}$
$\frac{1}{4}=.25$	$\frac{2}{3}=.66\frac{2}{3}$	$\frac{7}{8}=.87\frac{1}{2}$
$\frac{3}{4}=.75$	$\frac{1}{8}=.12\frac{1}{2}$	$\frac{1}{12}=.08\frac{1}{3}$
$\frac{1}{5}=.2$	$\frac{3}{8}=.37\frac{1}{2}$	$\frac{1}{16}=.06\frac{1}{4}$

WRITTEN EXERCISES

Reduce the following to four-place decimals:

1. $\frac{5}{16}$	4. $\frac{12}{21}$	7. $\frac{3}{32}$	10. $\frac{17}{35}$	13. $\frac{49}{152}$
2. $\frac{7}{16}$	5. $\frac{16}{27}$	8. $\frac{9}{64}$	11. $\frac{23}{47}$	14. $\frac{14}{19}$
3. $\frac{3}{16}$	6. $\frac{9}{14}$	9. $\frac{7}{48}$	12. $\frac{35}{64}$	15. $\frac{27}{46}$

69. Fractions Exactly Reducible to Decimals. The fraction $\frac{1}{3}$, for example, cannot be reduced exactly to a decimal, because no denominator of a decimal fraction is a multiple of 3. Since the decimal denominators 10, 100, 1000, etc., have no prime factors, except 2 and 5, it follows that a fraction whose denominator has a prime factor other than 2 and 5 cannot be reduced exactly to a decimal.

Thus, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{5}$ can be reduced exactly to decimals, while $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{9}$, $\frac{1}{11}$ cannot be so reduced.

Since 10 is a multiple of 2, $\frac{1}{2}$ can be reduced to tenths.

Since 100 is a multiple of 4, $\frac{1}{4}$ can be reduced to hundredths.

Since 1000 is a multiple of 8, $\frac{1}{8}$ can be reduced to thousandths.

EXERCISES

1. Can $\frac{1}{16}$ be reduced exactly to a decimal, and if so how many places will the decimal have?

2. Following are the thicknesses in inches of commercial iron and steel plate:

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{1}{32}$, $\frac{3}{32}$, $\frac{5}{32}$, $\frac{7}{32}$, $\frac{9}{32}$, $\frac{11}{32}$, $\frac{15}{32}$, $\frac{1}{20}$, $\frac{1}{40}$, $\frac{1}{64}$, $\frac{5}{64}$, $\frac{7}{64}$, $\frac{9}{64}$, $\frac{11}{64}$, $\frac{13}{64}$, $\frac{15}{64}$, $\frac{17}{64}$, $\frac{1}{80}$, $\frac{3}{80}$, $\frac{9}{128}$, $\frac{1}{160}$, $\frac{3}{160}$, $\frac{7}{160}$, $\frac{9}{160}$, $\frac{3}{320}$, $\frac{11}{320}$, $\frac{5}{640}$, $\frac{7}{640}$, $\frac{11}{640}$, $\frac{9}{1280}$, $\frac{11}{1280}$, $\frac{13}{1280}$, $\frac{17}{2560}$.

Which of the fractions given above can be reduced exactly to decimals? Answer this question without actually reducing the fractions to decimals.

3. Reduce the fractions given above to decimals, carrying those that do not reduce exactly to decimals to five places. Make a list of these fractions, and the equivalent decimals, arranging them in the order of their magnitude.

Such lists of decimals will be found in catalogues issued by manufacturers of iron and steel plate, and also in engineers' hand-books.

70. Reducing Fractions to Approximately Equal Decimals. The process of reducing a fraction to the nearest approximation in a decimal of a certain number of places is shown by the following examples:

Example 1. Reduce $\frac{1}{3}$ to a four-place decimal.

Solution: $\frac{1}{3} = .3333\frac{1}{3}$. Since the fraction $\frac{1}{3}$ is less than $\frac{1}{2}$, it follows that .3333 is the nearest approximation of $\frac{1}{3}$ in a four-place decimal.

Example 2. Reduce $\frac{2}{3}$ to a four-place decimal.

Solution: $\frac{2}{3} = .6666\frac{2}{3}$. Since $\frac{2}{3}$ is greater than $\frac{1}{2}$, it follows that .6667 is the nearest approximation of $\frac{2}{3}$ in a four-place decimal.

WRITTEN EXERCISES

Reduce the following to the nearest approximation in four-place decimals.

1. $\frac{3}{7}$	4. $\frac{2}{9}$	7. $\frac{5}{19}$	10. $\frac{8}{17}$	13. $\frac{12}{27}$
2. $\frac{4}{9}$	5. $\frac{5}{13}$	8. $\frac{3}{14}$	11. $\frac{5}{21}$	14. $\frac{8}{31}$
3. $\frac{5}{11}$	6. $\frac{7}{15}$	9. $\frac{7}{32}$	12. $\frac{4}{23}$	15. $\frac{17}{55}$

71. Reducing Decimals to Common Fractions. A decimal may be reduced to a common fraction by writing it in the form of a common fraction and then reducing to the simplest form.

Thus, $.5 = \frac{5}{10} = \frac{1}{2}$, $.15 = \frac{15}{100} = \frac{3}{20}$.

WRITTEN EXERCISES

Reduce the following to common fractions:

1. .4	5. .65	9. .485	13. .1444	17. .548
2. .12	6. .85	10. .248	14. .1728	18. .268
3. .35	7. .250	11. .964	15. .2150	19. .128
4. .45	8. .375	12. .732	16. .378	20. .375

72. **Tables of Reciprocals.** Tables giving the reciprocals (see page 62) of whole numbers are used by engineers. In these tables the reciprocals are always expressed as decimals, usually to five or six places.

WRITTEN EXERCISES

1. Find the reciprocal of 135 and of 1960 correct to the fifth decimal place.

2. The average thickness of gold leaf is 1/282000 of an inch. Express as a decimal, correct to the 8th place.

3. Make a table giving the reciprocals of the numbers, 11, 12, 13, 14, 15, 16, 17, 18, 19, correct to five decimal places.

Reduce each fraction to a four-place decimal and then add:

4. $\frac{1}{3}+\frac{1}{7}+\frac{1}{12}$
5. $\frac{3}{7}+\frac{5}{8}+\frac{3}{16}$
6. $\frac{5}{7}+\frac{3}{8}+\frac{5}{12}$
7. $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}$
8. $\frac{5}{9}+\frac{3}{7}+\frac{4}{5}$
9. $\frac{8}{9}+\frac{7}{16}+\frac{5}{12}$
10. $\frac{1}{5}+\frac{1}{7}+\frac{1}{12}$
11. $\frac{4}{9}+\frac{3}{7}+\frac{8}{13}$
12. $\frac{4}{5}+\frac{5}{8}+\frac{7}{9}$

Find the products of the following:

13. $1\frac{1}{2}\times4\frac{2}{3}$
14. $3\frac{1}{2}\times4\frac{2}{5}$
15. $3\frac{1}{3}\times5\frac{3}{4}$
16. $5\frac{2}{3}\times4\frac{3}{5}$
17. $3\frac{4}{7}\times4\frac{3}{7}$
18. $5\frac{2}{3}\times6\frac{1}{2}$
19. $8\frac{1}{2}\times4\frac{1}{3}$
20. $7\frac{2}{3}\times3\frac{1}{4}$
21. $6\frac{5}{6}\times3\frac{4}{7}$
22. $2\frac{3}{4}\times2\frac{1}{4}$
23. $6\frac{1}{3}\times7\frac{4}{5}$
24. $8\frac{2}{5}\times6\frac{1}{3}$

Find the quotients of the following:

25. $4\frac{2}{3}\div1\frac{1}{2}$
26. $3\frac{1}{2}\div4\frac{2}{5}$
27. $5\frac{3}{4}\div3\frac{1}{3}$
28. $5\frac{2}{3}\div4\frac{3}{5}$
29. $3\frac{4}{7}\div4\frac{3}{7}$
30. $6\frac{1}{2}\div5\frac{2}{3}$
31. $8\frac{1}{2}\div4\frac{1}{3}$
32. $7\frac{2}{3}\div3\frac{1}{4}$
33. $6\frac{5}{6}\div3\frac{4}{7}$
34. $8\frac{2}{5}\div6\frac{1}{3}$
35. $1\frac{1}{2}\div4\frac{2}{3}$
36. $4\frac{2}{5}\div3\frac{1}{2}$
37. $3\frac{1}{3}\div5\frac{3}{4}$
38. $4\frac{1}{3}\div8\frac{1}{2}$
39. $3\frac{3}{4}\div6\frac{5}{6}$

73. Aliquot Parts. Review of Short Cuts. A number which exactly divides another number is called an *aliquot part* of that number.

Thus, 5 is an aliquot part of 25, because $25 \div 5 = 5$. Similarly, $2\frac{1}{2}$ is an aliquot part of 25, because $25 \div 2\frac{1}{2} = 10$.

Articles are frequently sold, so many for a dollar, so many for 50¢, or for 25¢. "Two for a quarter," "Three for 50 cents," etc., are familiar expressions. This gives rise to prices which are aliquot parts of a dollar, though they contain fractional parts of a cent.

ORAL EXERCISES

1. Two for a quarter is how many cents apiece?
2. Three for a quarter is how many cents apiece?
3. Three for a half-dollar is how many cents apiece?
4. Two for a quarter is how many for a dollar?
5. Three for a quarter is how many for a dollar?
6. Three for a half-dollar is how many for one dollar?
7. How many for one dollar at each of the following prices: $12\frac{1}{2}$ cents apiece? $16\frac{2}{3}$ cents apiece? $6\frac{1}{4}$ cents apiece? $8\frac{1}{3}$ cents apiece?

Following are aliquot parts of a dollar, which are in common use:

$\frac{1}{2}$ of \$1.00 = 50¢	$\frac{1}{6}$ of \$1.00 = $16\frac{2}{3}$¢	$\frac{1}{16}$ of \$1.00 = $6\frac{1}{4}$¢
$\frac{1}{3}$ of \$1.00 = $33\frac{1}{3}$¢	$\frac{1}{8}$ of \$1.00 = $12\frac{1}{2}$¢	$\frac{1}{20}$ of \$1.00 = 5¢
$\frac{1}{4}$ of \$1.00 = 25¢	$\frac{1}{10}$ of \$1.00 = 10¢	$\frac{1}{24}$ of \$1.00 = $4\frac{1}{6}$¢
$\frac{1}{5}$ of \$1.00 = 20¢	$\frac{1}{12}$ of \$1.00 = $8\frac{1}{3}$¢	$\frac{1}{25}$ of \$1.00 = 4¢

Example. At $6\frac{1}{4}$ cents a yard, how many yards of gingham can be bought for \$2.00?

Solution: We know that $6\frac{1}{4}$¢ is $\frac{1}{16}$ of \$1.00. Hence we can buy 16 yards for \$1.00, or 32 yards for \$2.00. This is simpler than to divide 200 by $6\frac{1}{4}$, unless the short cut on page 64 is used.

The aliquot parts of a dollar are unit fractions of it, such as $\frac{1}{2}$, $\frac{1}{3}$, etc. There are certain other fractions of a dollar which are much in use.

Thus, $\frac{2}{3}$ of \$1.00 = $66\frac{2}{3}$ cents, is the price when 3 are sold for \$2.00.

Other such fractions are:

$\frac{3}{4}$ of \$1.00 = 75¢	$\frac{5}{8}$ of \$1.00 = $62\frac{1}{2}$¢
$\frac{3}{8}$ of \$1.00 = $37\frac{1}{2}$¢	$\frac{7}{8}$ of \$1.00 = $87\frac{1}{2}$¢

From these we have:

75 cents apiece is 4 for \$3.00.
$37\frac{1}{2}$ cents apiece is 8 for \$3.00.
$62\frac{1}{2}$ cents apiece is 8 for \$5.00.
$87\frac{1}{2}$ cents apiece is 8 for \$7.00.

Example 1. At 75¢ a yard, what is the cost of 24 yards?

Solution: 75¢ a yard is 4 yards for \$3.00. Since 4 is contained 6 times in 24, we have $6 \times 3 = 18$ as the cost in dollars.

Example 2. At $87\frac{1}{2}$¢ a yard, what is the cost of 60 yards?

Solution: $87\frac{1}{2}$¢ a yard, is 8 yards for \$7.00. Since 8 is contained $7\frac{1}{2}$ times in 60, we have $7\frac{1}{2} \times 7 = 52\frac{1}{2}$ as the cost in dollars.

ORAL EXERCISES

1. At 75¢ a yard, what is the cost of 16 yards?
2. At $37\frac{1}{2}$¢ apiece, what is the cost of 32 pieces?
3. At $62\frac{1}{2}$¢ apiece, what is the cost of 48 pieces?
4. At $87\frac{1}{2}$¢ apiece, what is the cost of 56 pieces?
5. At $62\frac{1}{2}$¢ apiece, what is the cost of 72 pieces?

You should be on the lookout for possible short cuts in your work. *Remembering* a few short cuts will not be of much value. You must understand *how they are made possible.* Every intelligent computer invents short cuts adapted to his particular work.

		A	B	C	D	E	F
	1.	$4\frac{2}{3}$	$1\frac{4}{5}$	$12\frac{1}{2}$	$8\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$
	2.	$7\frac{4}{5}$	$3\frac{3}{10}$	$14\frac{3}{4}$	$7\frac{1}{8}$	$\frac{1}{3}$	$\frac{1}{4}$
I.	3.	$5\frac{3}{8}$	$9\frac{1}{3}$	$12\frac{1}{8}$	$9\frac{3}{16}$	$\frac{2}{3}$	$\frac{3}{8}$
	4.	$3\frac{3}{4}$	$2\frac{5}{8}$	$18\frac{3}{8}$	$7\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{12}$
	5.	$2\frac{5}{8}$	$1\frac{3}{4}$	$24\frac{5}{8}$	$8\frac{9}{16}$	$\frac{3}{4}$	$\frac{5}{8}$
	6.	$5\frac{7}{8}$	$4\frac{3}{4}$	$14\frac{7}{8}$	$7\frac{3}{32}$	$\frac{1}{8}$	$\frac{1}{16}$
	7.	$4\frac{3}{16}$	$2\frac{1}{4}$	$16\frac{3}{16}$	$4\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{4}$
II.	8.	$7\frac{5}{16}$	$5\frac{3}{8}$	$18\frac{7}{16}$	$7\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{16}$
	9.	$8\frac{7}{16}$	$5\frac{5}{8}$	$19\frac{3}{8}$	$8\frac{5}{16}$	$\frac{7}{8}$	$\frac{2}{3}$
	10.	$9\frac{9}{16}$	$8\frac{7}{8}$	$20\frac{2}{3}$	$9\frac{5}{6}$	$\frac{1}{6}$	$\frac{3}{32}$
	11.	$8\frac{3}{8}$	$2\frac{3}{4}$	$25\frac{3}{5}$	$8\frac{7}{15}$	$\frac{5}{6}$	$\frac{2}{3}$
	12.	$9\frac{7}{8}$	$6\frac{3}{8}$	$17\frac{4}{5}$	$4\frac{3}{5}$	$\frac{1}{16}$	$\frac{1}{32}$
III.	13.	$5\frac{2}{3}$	$2\frac{5}{6}$	$18\frac{7}{8}$	$8\frac{15}{16}$	$\frac{3}{16}$	$\frac{1}{8}$
	14.	$8\frac{4}{5}$	$5\frac{2}{3}$	$12\frac{9}{16}$	$8\frac{7}{32}$	$\frac{5}{16}$	$\frac{3}{32}$
	15.	$9\frac{5}{8}$	$8\frac{1}{4}$	$14\frac{3}{4}$	$9\frac{7}{8}$	$\frac{5}{32}$	$\frac{3}{32}$
	16.	$8\frac{1}{3}$	$5\frac{1}{6}$	$40\frac{7}{8}$	$25\frac{3}{4}$	$\frac{5}{12}$	$\frac{1}{4}$
	17.	$6\frac{1}{4}$	$2\frac{1}{8}$	$10\frac{5}{6}$	$7\frac{2}{3}$	$\frac{7}{16}$	$\frac{1}{3}$
IV.	18.	$7\frac{1}{5}$	$4\frac{1}{6}$	$7\frac{3}{8}$	$2\frac{1}{3}$	$\frac{9}{16}$	$\frac{3}{8}$
	19.	$9\frac{2}{3}$	$6\frac{1}{6}$	$9\frac{5}{8}$	$4\frac{7}{8}$	$\frac{9}{16}$	$\frac{5}{8}$
	20.	$10\frac{3}{4}$	$7\frac{5}{8}$	$10\frac{7}{16}$	$8\frac{3}{4}$	$\frac{7}{12}$	$\frac{1}{3}$
	21.	$5\frac{3}{8}$	$8\frac{2}{3}$	$12\frac{5}{8}$	$16\frac{3}{4}$	$\frac{5}{8}$	$\frac{4}{5}$
	22.	$8\frac{2}{3}$	$9\frac{3}{8}$	$12\frac{3}{4}$	$8\frac{5}{8}$	$\frac{9}{16}$	$\frac{7}{8}$
V.	23.	$10\frac{1}{5}$	$2\frac{1}{3}$	$8\frac{1}{4}$	$9\frac{5}{6}$	$\frac{5}{6}$	$\frac{4}{5}$
	24.	$7\frac{1}{3}$	$8\frac{1}{4}$	$9\frac{1}{5}$	$10\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$
	25.	$15\frac{2}{3}$	$7\frac{3}{5}$	$8\frac{5}{8}$	$7\frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{12}$
	26.	$9\frac{3}{4}$	$5\frac{2}{3}$	$9\frac{3}{5}$	$12\frac{5}{6}$	$\frac{3}{4}$	$\frac{5}{6}$
	27.	$12\frac{4}{5}$	$7\frac{2}{5}$	$12\frac{3}{4}$	$16\frac{3}{8}$	$\frac{11}{16}$	$\frac{5}{8}$
VI.	28.	$9\frac{2}{3}$	$10\frac{3}{4}$	$7\frac{1}{5}$	$6\frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{6}$
	29.	$9\frac{9}{16}$	$8\frac{7}{16}$	$7\frac{5}{16}$	$4\frac{3}{16}$	$\frac{7}{8}$	$\frac{2}{3}$
	30.	$1\frac{4}{5}$	$3\frac{1}{3}$	$9\frac{2}{3}$	$2\frac{5}{8}$	$\frac{5}{8}$	$\frac{7}{16}$

		A	B	C	D	E
	1.	840.28	392.96	86.54	36.81	79.46
	2.	639.29	390.41	73.92	24.96	24.80
I.	3.	317.84	170.40	29.26	17.80	1.96
	4.	562.38	256.83	78.59	54.27	71.51
	5.	859.73	598.37	34.80	26.91	34.64
	6.	1240.07	1024.80	128.94	89.34	9.36
	7.	4937.81	3978.18	657.31	278.68	71.96
II.	8.	8641.92	6148.29	547.68	349.78	5.81
	9.	5100.64	1500.46	971.45	837.87	3.88
	10.	1500.46	1084.20	179.92	146.79	7.9
	11.	1240.07	1024.80	128.94	89.34	9.36
	12.	4290.45	2940.54	710.84	649.79	273.91
III.	13.	7940.80	4279.80	617.42	176.52	314.64
	14.	3429.60	2439.60	799.55	549.64	127.67
	15.	5910.30	3190.50	169.10	27.84	187.76
	16.	3728.60	2378.64	378.67	249.84	58.89
	17.	4290.45	2940.54	710.84	649.79	73.91
IV.	18.	2409.54	1782.45	884.95	499.97	81.20
	19.	3182.20	2138.18	161.35	89.85	24.62
	20.	6228.32	3286.59	745.97	488.98	38.91
	21.	1986.51	1568.91	814.10	148.47	29.64
	22.	2240.60	1427.98	799.80	564.89	37.81
V.	23.	7365.25	5256.37	711.10	638.56	49.64
	24.	3766.52	2565.73	711.10	576.39	27.80
	25.	6573.13	3137.56	410.11	256.51	84.70
	26.	3971.42	8190.81	371.84	351.75	39.64
	27.	6543.95	8540.25	173.48	513.50	52.37
VI.	28.	9472.62	4580.65	980.60	441.60	12.60
	29.	7846.25	9421.14	598.72	124.40	82.40
	30.	1496.72	2419.40	275.85	410.20	64.52

BILLS AND ACCOUNTS

74. **Sales Slips. Bills.** When goods are sold, a sales slip is made out and sent to the buyer, who checks it up by comparing it with the goods received. These slips are kept to compare with the monthly statement. The monthly statement, or bill, is sent to the buyer, the first of each month, and it is this monthly bill which the buyer is supposed to pay.

Such bills may have a variety of forms, but they all contain the following essential elements:

(1) Name of the seller and location of his business.
(2) Name and address of buyer.
(3) Name and quantity of each article, with date of sale and price.
(4) Total cost of each item and total amount of bill.
(5) Terms of sale, such as cash, credit 30 days, etc.

In the bill below, the price of each chair is given, and also the total for the chairs. The total for the chairs is called the *extended* amount for the chairs. Adding the extended amounts to find the total of the bill is called *footing* the bill.

CHARLES D. WARNER & Co. Boston, Mass., May 1, 1918.

Sold to NORMAN J. LEWIS,
Brookline, Mass.

April	7	One rug	$85	00
	10	Six chairs at $12.50	75	00
	"	One dining table	55	00
	"	One sideboard	80	00
	18	One set dishes	45	75
	"	One dozen glasses	9	50
		Total	$350	25

Let the pupils bring in various forms of bills. Many non-essentials will appear on these, such as unpaid balances from preceding months, the business of the seller, and so on.

75. **The Adding Machine.** In large establishments bills are made out on adding machines, which do the work of the ordinary typewriting machine, and extend and foot the bills at the same time. Such machines, however, are expensive, and it does not pay to use them in small concerns. In spite of the extensive use to which such machines are now put, the business accountant must still possess skill in such operations as extending bills and footing columns of figures.

The work of extending bills should be done orally whenever possible.

The following are abbreviations used in business:

Company = Co.	Account = Acct.	Paid = Pd.
Credit (or) = Cr.	Balance = Bal.	Received = Rcd.
Debit (or) = Dr.	Merchandise = Mdse.	Payment = Pay't.

"6 chairs at \$12.50" means "6 chairs at \$12.50 *each.*"

WRITTEN EXERCISES

Make out a bill for each of the following:

1. James R. Speyer of Philadelphia, Pa., sold to Mrs. R. D. Somers, also of Philadelphia, 24 yards muslin at 12½¢, 8 spools of thread at 5¢, 18 yards gingham at 8⅓¢, 12 yards dress goods at 75¢, 18 yards sateen lining at 33⅓¢, 10 yards chambray at 16⅔¢, 6 yards marabout trimming at 37½¢, 12 yards silk braid at 9¢, 7 yards braid border at 35¢, 15 braid loops at 19¢. Date, Aug. 18, 1919.

2. S. T. Hamilton of Chicago, Ill., sold A. F. Langmaid of Decatur, Ill., 6 chest locks at 43¢, 16 drawer locks at 23¢, 32 drawer pulls at 45¢ per dozen, 6 pairs chest handles at 90¢ a pair, 6 dozen hat hooks at 45¢ a dozen, 14 pairs door butts at 18¢ a pair, 2 pairs spring hinges at \$1.68 a pair, 2 door handles and locks at \$3.50, 3 door locks at \$1.80, 2 cylinder night latches at 98¢, one front door lock at \$5.10. Date, Sept. 3, 1919.

76. Accounts. Business people keep a great many accounts; and regular courses in keeping accounts, that is, in bookkeeping, are given in high schools and in business colleges. To become an expert bookkeeper requires considerable study in these higher schools, but we can learn enough in the grades to keep simple accounts. Simple accounts are needed by all who wish to know how their affairs stand from time to time.

77. The Cash Account. The cash account shows every item of cash paid out, and every item taken in. By finding the difference between the sums of these we shall know at any time how much cash should be on hand.

Dr. CASH Cr.

1916				1916			
Apr. 15	To balance..........	$165	35	Apr. 17	By suit of clothes....	$35	00
15	" salary..........	30	00	18	" petty expenses...	2	80
21	" S. B. James.....	5	50	20	" board...........	7	00
22	" salary..........	30	00	22	" laundry..........	1	20
29	" salary..........	30	00	22	" petty expenses...	4	65
				27	" board...........	7	00
				29	" laundry.........		95
					" balance..........	202	25
		$260	85			$260	85
May 1	To balance.........	$202	25				

In the above account the word "cash" indicates that it is a *cash account.* The word "Dr." (debit) on the left indicates that the cash on hand and all cash taken in are entered on that side. The word "Cr." (credit) on the right indicates that all cash paid out is entered on that side. By "To balance," or "On hand," is meant all cash actually in the owner's possession, and also any amount deposited in a checking account in a bank. (See page 123.)

Two horizontal lines are drawn to show that the account is closed. It is reopened by writing "To balance" on the left side, and the amount opposite it.

Rule blanks, as on the opposite page, and make a cash account from each of the following, finding the balance in each case and closing the account.

1. Cash on hand June 21st, 1919—$246.20. Received: June 2, A. B. Braden, $12.80; C. R. Smith, $21.60; R. L. Moore, $13.70; R. F. Keith, $18.70; D. R. Landen, $32.70; Carl Johnson, $21.95. Paid out: Drayage, $14.20; Grocer's bill, $49.60; Butcher's bill, $18.72; Rent, $64.00; Coal, $49.30; Milk, $6.30; Petty expenses, $9.45.

2. Cash on hand May 17th, 1919—$34.16. Received for work: May 18th, $3.25; May 19th, $2.60; May 20, $2.95; May 21st, $3.10; May 22d, $3.80; May 23d, $3.35. Paid out: May 17th, Car tickets, $1.00; May 18th, gave wife $10 for household expenses; May 21, $2.85, for show; May 23d, $1.35 for amusement.

3. Cash on hand Dec. 6th, 1919—$58.40; Dec. 11th, received $25.00 in Salary; Dec. 18th, $25.00 in Salary. Paid out: Dec. 11th, $5.50 for board; Petty expenses, for the week, Dec. 12th, Church contribution, $.50; Dec. 14, Christmas presents, $7.50; Dec. 18th, Christmas presents, $9.35; Board, $5.50; Petty expenses for the week, $1.90; Overcoat, $27.50.

4. Cash on hand March 1st, 1919—$47.00. Received allowance for month, $150.00. Paid out: March 1st, Rent, $27.50; Grocery bill, $21.45; Meat bill, $11.67; Milk, $.60; Electric light, $2.25; Gas, $1.85; Petty expenses, $3.15; Help, $5.00; Laundry, $.65; March 8, Help, $5.00; Petty expenses, $1.75; Laundry, $1.40; March 15, Help $5.00; Laundry, $.86; Petty expenses, $4.50; March 22d, Help, $5.00; Laundry, $.95; Petty expenses, $3.90; March 29th, Help, $5.00; Petty expenses, $7.30; Laundry, $1.05; Flowers, $1.75; Clothing, $21.60.

5. Make up a cash account for yourself, showing all money received and spent by you in one month.

78. Personal Accounts. Storekeepers and others frequently sell goods on credit to people whom they know to be honest and able to pay their bills. Below is an account kept with one of his customers by a man who sells farm implements and also buys grains and other farm products.

Dr. GEORGE ROWLAND Cr.

1918				1918			
May 1	To 1 farm wagon....	$45	00	May 10	By 50 bu. oats at 45c..	$22	50
1	" 1 pair harness....	50	00	10	" 80 bu. corn at 57c..	45	60
1	" 1 plow...........	11	50	29	" $3\frac{1}{2}$ tons hay at		
1	" repairs on seeder..	8	40		$14.50	50	75
15	" 3 hoes at 45c.....	1	35	June 1	" cash to balance....	...	..
15	" 1 garden rake....		75				
15	" 1 shovel.........	1	40				
15	" 1 spade..........		95				
20	" repair on harness.	2	80				
29	" 1 cultivator......	35	00				
29	" 2 forks at 70c....	1	40				

George Rowland, whose name appears at the top of the account, is the man with whom the dealer is trading. Items sold to Mr. Rowland are placed on the left or debit (Dr.) side of the account. Items received from him are placed on the right or credit (Cr.) side. An account of this sort is called a ledger account.

Ledger accounts are kept for two reasons:

(1) It would be impossible to remember the various items sold to many customers, and the payments made by them.

(2) The dealer wants to keep track of his business in such a way that when he looks over his books he can tell who have been his good customers for months, and even years.

ORAL EXERCISES

Discuss reasons for selling goods on credit and for keeping personal accounts.

WRITTEN EXERCISES

1. In the account on the opposite page, find how much cash is needed to balance the account. Make up a ledger account of each of the following. Balance and close each account.

2. Sold to Mrs. John A. Dutton, Nov. 13, 1918, one pair of slippers \$1.75; 2 suits of underwear at \$1.00, 3 pairs of stockings at 33⅓¢; 3 yards of muslin at 45¢ a yard; one dress pattern 15¢; 5 yards lace at 6¢ a yard; 6 yards lace at 8¢ a yard; Nov. 20, 1 dozen handkerchiefs \$3.00; 2 yards velvet ribbon at 65¢ a yard; hooks, eyes, buttons 95¢; 1 pair gloves \$1.50; 1 pair gloves \$2.00; 1 pair corsets \$4.50. Dec. 1st, Mrs. Dutton paid cash to close the account.

3. Sold to Robert Ward, May 1st, 1918: 2 shirts at \$1.35; 3 shirts at \$1.50; 3 suits of underwear at \$1.25; 1 pair garters 25¢; 6 pairs of socks at 35¢; 1 straw hat \$3.50; 2 ties at 75¢; 3 ties at 50¢; 1 belt for \$1.00. June 1st, Mr. Ward paid cash to balance the account.

4. Sold to L. J. Ayer, May 7, 1918, fishing rod \$3.50; hooks and flies \$1.25; 1 fish basket \$1.50; 1 reel \$5.00; 2 fish lines at 75¢; bait 25¢; May 10th, 1 electric flat iron \$5.50; 1 fire screen \$2.75; 1 granite kettle 60¢; 1 wire strainer 25¢; May 21st, 1 wash boiler \$4.25; 1 clothes wringer \$5.75. June 1st, Mr. Ayer paid cash to balance the account.

5. Sold to C. G. Staehling, May 21st, 1918, 2 chairs at \$6.50; 1 swing \$14.00; 1 table \$5.00; 1 porch rug \$4.50; 1 garden rake 90¢; 1 hoe 65¢; 1 spade 85¢; 1 fork \$1.10; 1 summer dress \$10.50; 1 hat \$8.75; 1 pair shoes \$5.50; 1 pair shoes \$7.25. June 1st, Mr. Staehling paid cash to balance the account.

6. Sold to E. O. Bangs, August 1st, 1918: 1 box cigars \$4.75; 1 bottle malted milk \$3.75; 1 hot water bottle \$1.75; August 11, 1 prescription 75¢; August 13, 1 box candy 65¢. September 5th, Mr. Bangs paid cash to balance the account.

ANALYSIS OF PROBLEMS

79. **Steps in Solving Problems.** It will be helpful in solving problems if each of the following steps is given careful attention:

(1.) *Read the Problem with Care.* You cannot solve a problem unless you know what the problem is.

(2.) *Plan the Solution.*

(3.) *Perform the Computations Required by the Plan.*

(4.) *Test the Correctness of the Result.* Never forget that a result is worthless unless you *know* that it is correct.

80. **Analysis.** The planning of the solution is also called the *analysis* of the problem.

Example. If apples are sold at 2 for 5 cents, how much will one dozen apples cost?

Direct Solution: If 2 apples cost 5 cents, one apple will cost one-half of 5 cents, or $2\frac{1}{2}$ cents. Then 12 apples cost 12 times $2\frac{1}{2}$ cents, or 30 cents.

Indirect Solution: 12 is 6 times 2. Hence the price of 12 apples is 6 times the price of 2 apples, or 6×5 cents $=30$ cents.

Indicated Solution: The computation required in the direct solution may be indicated by the expression $\frac{5}{2}\times 12$.

That is, divide 5 by 2 and then multiply by 12

Since $\frac{5}{2}\times 12=30$, this is the price in cents.

81. **Uses of Different Kinds of Solutions.** In simple problems which can be solved mentally it is often best to devise an indirect solution. In more complicated problems it is usually best to indicate the complete solution and then perform the computations. The direct solution contains a full statement of all the steps, and serves as an explanation of the indicated solution.

In each of the problems on the page opposite state the plan of solution (analysis), and then give the result. See who can devise the simplest analysis.

1. If collars are sold at 2 for 25 cents, how much will 1 dozen collars cost?

2. At 26 cents a pound for hamburger steak, what is the cost of $1\frac{3}{4}$ pounds?

3. If 6 men do a piece of work in 10 days, how long will it take 24 men to do it?

4. If 16 men require 7 days to do a piece of work, how long will it take 4 men to do it?

5. If 14 yards of gingham make 8 aprons for the cooking class, how many yards are needed for 28 such aprons?

6. An automobile goes 23 miles on 2 gallons of gasoline. How far will it go on 9 gallons?

7. A steamer goes 40 miles in 3 hours. How far will it go in 9 hours? In 1 hour? In 2 hours? In 10 hours? In 11 hours?

8. If 20 men dig 1200 yards of ditch in 8 days, how many yards can 5 men dig in 4 days?

9. If 1000 brick cost $14, how much will 4500 cost? How much will 2750 cost?

10. In an audience of 3500, $\frac{2}{5}$ are men. How many women and children are there?

11. If 1000 feet of lumber cost $45, what is the cost of 2500 feet? How much lumber can be bought for $67.50?

12. If pencils are sold at 3 for 10 cents, how much will one dozen cost? How many pencils can be bought for 50 cents?

13. If oranges are sold at 60 cents a dozen, how much will 4 oranges cost? How much will 3 oranges cost?

14. A quart bottle of ink costs $1.00. How many small bottles, each holding $\frac{1}{4}$ of a pint, may be filled from this bottle? How much does it cost to fill 3 such bottles?

82. Solution Indicated in the Form of a Fraction. There are many problems the solutions of which may be indicated in the form of a fraction, whose terms are products of integers.

Example 1. A farmer weighs the hay from a measured piece of land, 12 by 18 rods, and finds it to be $3\frac{1}{5}$ tons. At this rate what is the total yield in a field 60 by 95 rods.

Solution: The area of the measured field is 18×12 square rods. Hence the yield on one square rod is $3\frac{1}{5}\div(18\times 12)$. The area of the whole field is 60×95. Hence the total yield is $3\frac{1}{5}\div(18\times 16)$ multiplied by 60×95. This is indicated as follows:

Reduce $3\frac{1}{5}$ to $\frac{16}{5}$. Since this fraction is to be divided by 18×12, multiply the denominator by 18×12. Then multiply the numerator by 60×95, obtaining: $\dfrac{16\times 60\times 95}{5\times 18\times 12}$.

This is the result in tons.

Example 2. At \$16.50 a ton, what is the value of a load of hay containing 2760 pounds?

Solution: To indicate the number of tons, put 2760 in the numerator and 2000 in the denominator.

$\dfrac{2760\times 16.50}{2000}$ Multiply the numerator by 16.50. This represents the result in dollars.

Example 3. A tank is $6\frac{3}{4}$ feet long, $4\frac{1}{2}$ feet wide, and $2\frac{1}{3}$ feet deep. How many tons of water does it hold if one cubic foot weighs 62.5 pounds?

Solution: Reduce the dimensions to improper fractions. Express the volume as the product $\frac{27}{4}\times\frac{9}{2}\times\frac{7}{3}$ or $\dfrac{27\times 9\times 7}{4\times 2\times 3}$.

$\dfrac{27\times 9\times 7\times 62.5}{4\times 2\times 3\times 2000}$ (tons) Multiply the numerator by 62.5 to get the number of pounds. Divide (multiply the denominator) by 2000 to get the number of tons.

In indicating solutions, the following rules are used:

To multiply a fraction by an integer, multiply the numerator.

To divide a fraction by an integer, multiply the denominator.

To multiply a fraction by a fraction, multiply numerator by numerator, and denominator by denominator.

To divide a fraction by a fraction, invert its terms and multiply.

MISCELLANEOUS PROBLEMS

In each of these problems indicate the complete solution before performing any of the calculations.

1. In attempting to predict how many votes will be cast for a candidate at a certain election, 250 voters are asked to state how they will vote. If 96 of these signify their intention of voting for the candidate, how many votes should he get if 16,800 votes are cast?

2. An automobile is found to run 180 miles on 16 gallons of gasoline. At this rate how much does it cost for gasoline for a season's run of 7650 miles, if gasoline is $20\frac{1}{2}$ cents a gallon?

3. If it cost $56 to excavate a cellar, $25' \times 36' \times 5'$, how much will it cost to make an excavation $40' \times 60' \times 7'$?

4. A farmer sold 5 loads of hay weighing 2160, 2345, 2285, 2370 and 2460 pounds respectively. At $14.75 a ton, how much did he get for his hay?

   Suggestion: First find the total weight of the hay.

5. A farmer sold 6 loads of corn (on the cob) weighing 1870, 2040, 2090, 2300, 2620, and 2130 pounds respectively. At 64 cents a bushel, how much was this corn worth if 72 pounds of corn on the cob was regarded as one bushel?

6. A coal dealer hauled 10 loads of coal weighing 5860, 6480, 6730, 6250, 5930, 6150, 6330, 6470, 5490, and 6870 pounds, respectively. At $6.75 a ton, what was the value of this coal? (See suggestion under example 4.)

83. Further Indicated Solutions. Even when the plan of the solution cannot be shown in as simple form as on the preceding pages, it is sometimes best to indicate all the operations before performing any of them, so as to concentrate on the reasoning before the mind is diverted by the calculations.

Example 1. A herder bought 85 head of cattle for \$42 a head. He sold 65 of them at \$57 a head, and the remaining 20 at \$65 a head. What was his average gain per head?

The solution is indicated by:

$$\frac{20\times65+65\times57-42\times85}{85}\text{ (dollars).}$$

This indicates that 65 is to be multiplied by 20, 57 by 65, and the products added; then 85 is to be multiplied by 42, the product subtracted, and the difference divided by 85. The numerator gives the *total* gain, which divided by 85 gives the *average* gain.

Example 2. In a factory 24 men receive \$3.60 a day each, 32 receive \$3.00, 48 receive \$2.40, 60 receive \$1.80, and 48 receive \$1.60 a day. What is the average wage in this factory?

Suggestion: The average wage in dollars is:

$$\frac{24\times3.60+32\times3+48\times2.40+60\times1.80+48\times1.60}{24+32+48+60+48}.$$

Example 3. A rectangular piece of land $45'\times215'$ was sold for \$4500. What was the price per acre?

Analysis: The price per square foot is $\frac{4500}{45\times215}$ (dollars).

The number of square feet in an acre is $160\times16\frac{1}{2}\times16\frac{1}{2}=160\times\frac{33}{2}\times\frac{33}{2}.$ Hence the required price is

$$\frac{4500\times160\times33\times33}{45\times215\times2\times2}\text{ (dollars).}$$

The analysis indicated in each of these examples is made mentally, and the expressions given above are written down at once.

PROBLEMS ON AVERAGES

1. A fast train covered a distance of 480 miles in 10 hours, and 432 miles in 8 hours. What was the average speed per hour?

Solution: The average speed was $\frac{480+432}{18}=\frac{912}{18}=50\frac{2}{3}$ (miles).

2. A farmer sold 240 bushels of wheat for 80 cents a bushel, and 680 bushels for 90 cents a bushel. What was the average price per bushel?

3. A ship averaged 18 knots per hour for 14 hours, and 16 knots per hour for 10 hours. What was the average speed for the whole time? Find the result to the nearest tenth of a knot.

4. A farmer has corn in 3 fields. One field of 250 acres averages 54 bushels of corn to the acre, another of 40 acres averages 48 bushels, and the third of 36 acres averages 60 bushels to the acre. What is the average per acre for the three fields?

5. A dealer in rugs bought 84 rugs at $85 apiece. He sold 62 of them at $125 apiece, and the remaining 22 at $96 apiece. What was his average gain on these rugs?

6. A farmer sold 8 horses at $160 per head, and 4 at $120 per head. What was the average price per head?

7. In a class of 24, 6 had a grade of 90; 8 had a grade of 85; 5 had a grade of 80; 4 a grade of 70; and 1 a grade of 60. What was the average grade?

Suggestion: Divide $6\times90+8\times85+5\times80+4\times70+60$ by 24, obtaining

$$\frac{6\times90+8\times85+5\times80+4\times70+60}{24}$$

8. A farmer hauled 4 loads of hay weighing respectively 2400, 2640, 2264, 3184 pounds. What was the average weight of these loads?

CHAPTER II

Business Arithmetic

PERCENTAGE

84. Percentage. It has become customary to express a large number of fractions as *hundredths* or *per cent.*

The words "per cent" are derived from the Latin words *per* and *centum,* meaning *in the hundred* or *hundredths.*

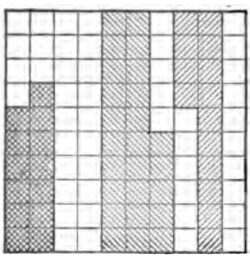
The symbol for per cent is %.

85. Use of per cent. Through long usage people have become so accustomed to fractions expressed as hundredths or per cent, that even in simple cases a clearer meaning is conveyed to the mind by per cents than by common fractions.

Thus, we speak of a rate of interest as 5% and not as $\frac{1}{20}$, and of a rate of gain as $12\frac{1}{2}$% and not as $\frac{1}{8}$.

Nothing new, except the sign "%" is brought into the theory of arithmetic in connection with percentage, and it is important that you should understand that very clearly. It will be useful, however, to gain a direct acquaintance with the amounts represented by various per cents. This we now proceed to do.

EXERCISES



1. How many squares are there in this figure? One square is what per cent of the whole figure? 2 squares are how many per cent of the figure? 3 squares? 4 squares? 10 squares? 25 squares?

2. How many per cent of this figure is covered by each kind of shading?

3. Draw a figure like the above, and shade 10% of it.

4. Draw another square and shade 25% of it.

Example. Reduce $1\frac{1}{4}\%$ to a common fraction.

Solution: $1\frac{1}{4}\% = \dfrac{1\frac{1}{4}}{100} = \frac{5}{4} \div 100 = \dfrac{5}{400} = \frac{1}{80}.$

EXERCISES

In this manner reduce each of the following per cents to common fractions, solving as many as you can orally:

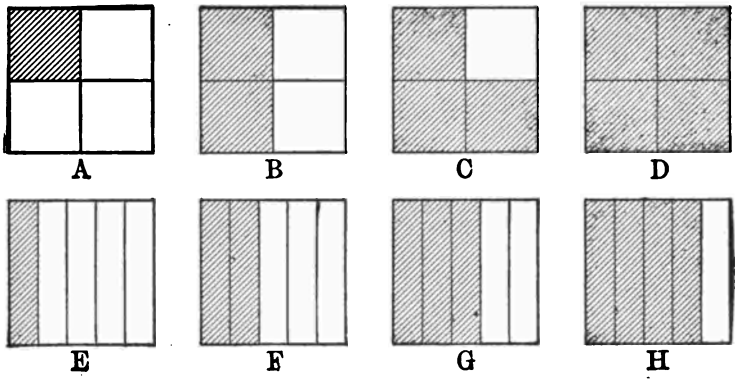
1. $1\frac{1}{3}\%$
2. $2\frac{2}{3}\%$
3. $3\frac{2}{3}\%$
4. $3\frac{1}{3}\%$
5. 4%
6. 5%
7. $6\frac{1}{4}\%$
8. $6\frac{2}{3}\%$
9. $8\frac{1}{3}\%$
10. 10%
11. $12\frac{1}{2}\%$
12. $13\frac{1}{3}\%$
13. $16\frac{2}{3}\%$
14. $18\frac{3}{4}\%$
15. 20%
16. 25%
17. $26\frac{2}{3}\%$
18. $31\frac{1}{4}\%$
19. $33\frac{1}{3}\%$
20. 35%
21. $37\frac{1}{3}\%$
22. 40%
23. $41\frac{2}{3}\%$
24. $43\frac{3}{4}\%$
25. 45%
26. $46\frac{2}{3}\%$
27. 50%
28. $53\frac{1}{3}\%$
29. 55%
30. $56\frac{1}{4}\%$
31. $58\frac{1}{3}\%$
32. $62\frac{1}{2}\%$
33. 65%
34. $66\frac{2}{3}\%$
35. $68\frac{3}{4}\%$
36. $73\frac{1}{3}\%$
37. 75%
38. $81\frac{1}{4}\%$
39. $83\frac{1}{3}\%$
40. 85%
41. $87\frac{1}{2}\%$
42. $91\frac{2}{3}\%$
43. $93\frac{2}{3}\%$
44. $93\frac{3}{4}\%$
45. 95%
46. 125%
47. 150%
48. 175%

The common fraction equivalents of the following should be remembered:

$2\frac{1}{2}\%$, $6\frac{1}{4}\%$, $8\frac{1}{3}\%$, $12\frac{1}{2}\%$, $16\frac{1}{3}\%$, $33\frac{1}{3}\%$, $37\frac{1}{2}\%$, $62\frac{1}{2}\%$, $66\frac{2}{3}\%$, $87\frac{1}{2}\%$,

Make a list of these and memorize them.

There are certain other per cents whose common fraction equivalents you will know at a glance without remembering them. Such are: 4%, 5%, 10%, 20%, 25%, 50%, 75%.



ORAL EXERCISES

1. State what per cent of each of the above figures is shaded?

2. The shaded area in A is how many per cent of the shaded area in B?

3. The shaded area in E is how many per cent of the shaded area in F?

4. The shaded area in E is how many per cent of the shaded area in H?

5. The shaded area in F is how many per cent of the shaded area in H?

6. The number 2 is how many per cent of 1?

Since $1 = \frac{100}{100} =$ of 1, or 100% of 1, it follows that $2 = 200\%$ of 1.

7. The shaded area in B is how many per cent of the shaded area in A?

8. The shaded area in C is how many per cent of the shaded area in A?

9. The shaded area in H is how many per cent of the shaded area in E?

ORAL AND WRITTEN EXERCISES

Read and supply the missing numbers in the following:

1. 1 is ? % of 1
2. 2 is ? % of 1
3. 3 is ? % of 1
4. 2 is ? % of 2
5. 24 is ? % of 24
6. 4 is ? % of 2
7. 6 is ? % of 2
8. 8 is ? % of 2
9. 10 is ? % of 2
10. 12 is ? % of 2
11. 6 is ? % of 3
12. 8 is ? % of 4
13. 12 is ? % of 6
14. 12 is ? % of 3
15. 12 is ? % of 8
16. 16 is ? % of 8
17. 16 is ? % of 4
18. 16 is ? % of 2
19. 20 is ? % of 5
20. 20 is ? % of 4

Give the equivalent in per cents of each of the following:

21. $\frac{1}{4}$
22. $1\frac{1}{4}$
23. $2\frac{1}{4}$
24. $\frac{1}{2}$
25. $1\frac{1}{2}$
26. $2\frac{1}{2}$
27. $1\frac{1}{5}$
28. $2\frac{1}{5}$
29. $3\frac{1}{5}$
30. $\frac{1}{10}$
31. $1\frac{1}{10}$
32. $2\frac{1}{10}$
33. $\frac{1}{3}$
34. $1\frac{1}{3}$
35. $2\frac{1}{3}$
36. $\frac{1}{8}$
37. $1\frac{1}{8}$
38. $2\frac{1}{8}$
39. $\frac{1}{6}$
40. $1\frac{1}{6}$
41. $2\frac{1}{6}$
42. $1\frac{1}{12}$
43. $\frac{1}{12}$
44. $2\frac{1}{12}$
45. $\frac{2}{3}$
46. $\frac{3}{8}$
47. $\frac{5}{8}$
48. $\frac{7}{8}$
49. $\frac{5}{6}$
50. $\frac{5}{12}$

Find:

51. 25% of 20
52. 25% of 32
53. 33⅓% of 24
54. 20% of 16
55. 150% of 8
56. 125% of 4
57. 250% of 12
58. 75% of 16
59. 66⅔% of 18
60. 133⅓% of 9
61. 12½% of 32
62. 37½% of 24
63. 140% of 260
64. 15% of 200
65. 49% of 500

86. The Three Numbers in Percentage. There are three numbers involved in every problem in percentage. These are the *base*, the *rate*, and the *percentage*.

The *base* is a number of which a certain per cent is taken.

The *rate* is the fraction, expressed as hundredths or per cent, which indicates the part that is taken of the base.

The *percentage* is the product of the base and rate.

The base is *always* a whole of which a part called the *percentage* is taken. The rate indicates what part is taken, that is, how many hundredths of the whole are taken.

The fundamental equation on percentage is:

$$\textit{rate} \times \textit{base} = \textit{percentage} \qquad \text{(P)}$$

87. The Three Problems in Percentage. In every problem in percentage, two of the three numbers, base, rate, percentage, are given, and the third is to be found.

ORAL EXERCISES

1. If the base and rate are given, how do you find the percentage?

2. If the percentage and base are given, how do you find the rate?

3. If the percentage and rate are given, how do you find the base?

The answers to these questions contain complete information about arithmetic so far as percentage is concerned. There is no need of remembering these answers, as they can be reproduced at any time from equation (P). This equation is fundamental, and must be remembered.

In problems involving percentage, the chief difficulty lies in deciding which number is the base, which is the rate, and which is the percentage. When that has been decided, the number that is missing may always be found by using the answer to one of the three questions given above.

WRITTEN EXERCISES

Find the percentages in the following:

1. 12% of 568	7. 20% of 7920	13. 50% of 9370
2. 16% of 8660	8. 25% of 7340	14. 60% of 5430
3. 5% of 7894	9. 33% of 7350	15. 87% of 792
4. 7% of 9470	10. 15% of 48.50	16. 45% of 6500
5. 6% of 380	11. 18% of 7640	17. 145% of 240
6. 8% of 916.50	12. 35% of 490	18. 350% of 74

88. Fractional Per Cents. Frequently fractions of 1 per cent are used. Thus, 5½% is very common as a rate of interest.

Example. Find 5½% of 685.90.

To take 5½% of the number we may multiply by .055, or we may proceed as follows:

Find 5% of the number in the usual way.

Then find ½ of 1 per cent by noticing that it is one-tenth of 5%.

$$\begin{array}{r} 685.90 \\ \underline{.05\tfrac{1}{2}} \\ 34.2950 \\ \underline{3.4295} \\ 37.7245 \end{array}$$

To take 5¼% of a number, we may proceed in a similar way. To take 5¾% it is best to multiply by .0575.

WRITTEN EXERCISES

Find the following per cents:

1. 5½% of 890.50	7. 4⅝% of 3190	13. 6⅝% of 25000
2. 5¼% of 450	8. 4⅕% of 3190	14. 6¼% of 3840
3. 5¾% of 3760	9. 5⅜% of 18500	15. 6¾% of 5670
4. 4½% of 9340	10. 5⅞% of 16000	16. 7½% of 1440
5. 4⅜% of 127000	11. 6½% of 8150	17. 6⅝% of 64300
6. 4⅞% of 24,600	12. 6⅛% of 4260	18. 7⅞% of 3760

89. Fractional Per Cents Expressed as Decimals. Fractional parts of one per cent are often written as decimals.

Thus, we say that three samples of milk contain 3.4%, 3.8%, and 4.2% of butter fat, respectively.

To find 3.4% of a number multiply it by .034. Why?

WRITTEN EXERCISES

Find the percentages in the following:

1. 3.4% of 760	4. 6.7% of 4960	7. 62.3% of 640
2. 3.8% of 824	5. 12.4% of 8960	8. 37.8% of 284
3. 4.2% of 536	6. 9.3% of 1820	9. 57.4% of 835

10. In one year a cow gives 10480 pounds of milk containing 4.4% butter fat. How many pounds of butter fat are there in this milk?

90. Selling Coal. Hard coal is sorted at the mine into different sizes. *Egg* coal consists of lumps not less than 1.75 inches in diameter; *stove* coal is between 1.25 inches and 1.75 inches; *chestnut* between .75 and 1.25 inches; *pea*, between .50 and .75 inches; and *buckwheat*, between .25 and .50 inches. The carbon in the coal is the substance which burns, and the value of coal, therefore, depends upon the amount of carbon it contains. Coal consisting of smaller pieces contains more dirt and less carbon. The following is from an analysis of hard coal of various sizes taken from the same mine:

	Carbon	Ash
Egg.................	88.49%	5.66%
Stove................	83.67%	10.17%
Chestnut............	80.73%	12.67%
Pea..................	79.05%	14.66%
Buckwheat..........	76.92%	16.62%

WRITTEN EXERCISES

Find the number of tons of carbon and of ash in a carload of 50 tons (2000 lbs. a ton) of each of these kinds of coal.

WRITTEN EXERCISES

1. One cubic foot of water weighs 62.5 pounds. What is the weight per cubic foot of each of the following kinds of wood, the weight being given as so many per cent of the weight of water. Find each result to the nearest pound.

Beach	73%	Oak (red)	74%
Hemlock	59%	Pine (white)	45%
Linden (basswood)	60.4%	Pine (yellow)	61%
Oak (white)	77%	Maple	68%

The weight of different specimens of the same kind of wood varies considerably. The numbers given here are the averages for well seasoned wood.

2. A cord of wood is a pile 4 feet wide, 8 feet long, and 4 feet high. For average wood, about 56% of a pile is solid wood, and the rest crevices. How many cubic feet of solid wood are there in a cord?

3. Find the weight of one cord of seasoned wood of each kind given in the above table.

4. In drying, a pine board shrinks 2.6% of its width and thickness, and 0.06% of its length. If 10-inch boards are laid down green, what will be the width of the cracks between the boards when they are dried? By how much will these cracks differ from a quarter of an inch.

5. Oak boards shrink 3.5% of their width in drying. If oak flooring 3 inches wide is laid down green, what will be the width of the cracks when the boards are dried? By how much will these cracks differ from one-tenth of an inch?

6. In changing from a temperature of 40° below zero to a temperature of 100 above zero, a steel rail expands 0.09% of its length. How much does a 30-foot steel rail expand by this change of temperature? By how much will this expansion differ from a third of an inch?

91. Finding the Rate. When the base and the percentage are given the base may be found.

From the equation

$$rate \times base = percentage, \qquad \text{(P)}$$

it is clear that the rate may be found by dividing the percentage by the base and stating the result in hundredths.

Thus, 3 is 75% of 4 because $\frac{3}{4} = .75 = 75\%$.
Again, 2 is $27\frac{4}{7}\%$ of 7 because $\frac{2}{7} = .28\frac{4}{7} = 28\frac{4}{7}\%$.

ORAL EXERCISES

Find the rates in the following:

	Base	Percentage		Base	Percentage
1.	100	8	15.	50	24
2.	100	46	16.	500	25
3.	100	84	17.	500	50
4.	100	106	18.	1000	100
5.	100	180	19.	32	16
6.	100	240	20.	48	12
7.	200	12	21.	1	$\frac{1}{4}$
8.	200	24	22.	3	1
9.	200	36	23.	6	4
10.	200	50	24.	6	1
11.	400	100	25.	$\frac{1}{2}$	$\frac{1}{4}$
12.	400	20	26.	60	12
13.	400	16	27.	80	16
14.	50	8	28.	2500	500

92. **Purpose of Finding the Rate.** The chief purpose of finding the rate when the base and the percentage are given is to compare different rates. If one knows that in Massachusetts, with a population of 3,366,416, 1,059,245 are foreign born, while in Rhode Island, with a population of 345,506, 106,305 are foreign born, it is not very easy to say off-hand which state has the larger fraction of its population foreign born. When reduced to per cents, we find that Massachusetts has 31.5% of foreign born, and Rhode Island has 30.8%.

Example 1. A school baseball team played 17 games and won 11 of them. What per cent of the games played were won?

Solution: The team won $\frac{11}{17}$ of the games played, and

$$11 \div 17 = .647 = 64.7\%$$

This result is correct to the nearest tenth of one per cent.

Example 2. In 1909 (census of 1910) there were on an average 6,615,046 wage earners in the United States. Of these 311,704 worked in establishments employing from one to five wage earners. What per cent of the wage earners of the United States worked in such small establishments?

Solution: $311704 \div 6615046 = .047 = 4.7\%$.

WRITTEN EXERCISES

In the following find results to the nearest tenth of one per cent.

Size of establishment.	Total number of wage earners in each kind of establishment.	Per cent of total.
6 to 20 wage earners..........	640,793	....
21 to 50 " "	764,408	....
51 to 100 " "	782,298	....
101 to 250 " "	1,258,639	....
251 to 1000 " "	1,006,457	....
501 to 1000 " "	837,473	....
Over 1000 " "	1,013,274	....

93. **Graphs.** On the opposite page is a graphical representation of the values of the products of different manufacturing industries in the United States. From the data given on this page, find the per cent of the total value of the product of all industries produced by each industry. Give results to the nearest tenth of one per cent.

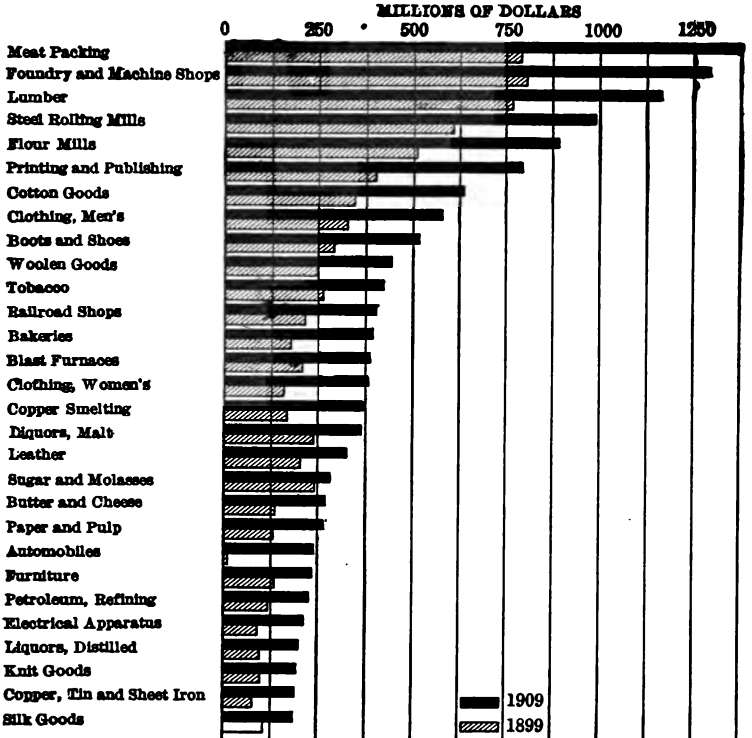
Industry	Amount	% of Total	Industry	Amount	% of Total
All industries....	20,672,052,000	100	Copper smelting..	384,572,000	....
Meat packing....	1,370,568,000	....	Liquors, malt....	374,730,000	....
Foundry and machine shops....	1,228,475,000	....	Leather..........	327,874,000	....
Lumber.........	1,156,129,000	....	Sugar and molasses	279,294,000	....
Steel rolling mills.	985,723,000	....	Butter, cheese....	274,558,000	....
Flour mills.......	883,584,000	....	Paper and pulp...	267,657,000	....
Printing and publishing.........	737,876,000	....	Automobiles......	249,202,000	....
Cotton goods....	628,392,000	....	Furniture........	239,887,000	....
Clothing, men's..	568,077,000	....	Petroleum, refining	236,998,000	....
Boots and shoes..	512,798,000	....	Electrical apparatus...........	221,309,000	....
Tobacco.........	416,695,000	....	Liquors distilled..	204,699,000	....
Railroad shops (cars, etc.).....	405,601,000	....	Knit goods.......	200,144,000	....
Bakeries.........	396,865,000	....	Copper, tin, sheet iron...........	199,824,000	....
Blast furnaces....	391,429,000	....	Silk goods........	196,912,000	....
Clothing, women's	384,752,000	....	All other industries...........	6,517,260,000	....

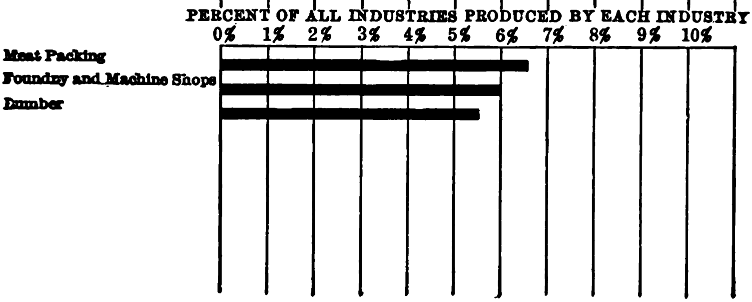
To find how many per cent of 20,672,052,000 there are in 1,370-568,000 it is sufficiently accurate to find how many per cent of 20,672 there are in 1371.

1. After computing the per cents required to fill in the table above, make a graph representing these per cents. Get a large strong sheet of paper and make the graph with care. It will contain much interesting information. (See page 97.)

2. On a large strong sheet of paper make a copy of the first graph on the opposite page.

The best copies should be hung in the schoolroom. Ask and answer questions which can be answered from these graphs.





The first graph given above is taken from the United States census reports for 1910. Such graphs occur frequently and should be understood.

94. Finding the Base. In some problems the rate and percentage are given, and it is required to find the base.

Example 1. 8 is 20% of what number?
Solution: 20% of a number is $\frac{1}{5}$ of the number. But 8 is $\frac{1}{5}$ of 40. Hence 40 is the required number.

Example 2. 60 is 5% of what number?
Solution: If 60 is 5% of a number then 1% of the number is $\frac{1}{5}$ of 60, or 12. Hence the number is $100\times 12=1200$.

These two examples show how simple examples of this kind may be solved orally.

ORAL EXERCISES

Find the bases in the following:

	Percentage	Rate		Percentage	Rate		Percentage	Rate
1.	50	5%	7.	500	2%	13.	36	4%
2.	200	4%	8.	600	10%	14.	64	6%
3.	150	6%	9.	1200	8%	15.	125	5%
4.	75	3%	10.	2500	20%	16.	80	20%
5.	80	4%	11.	6000	25%	17.	140	25%
6.	400	8%	12.	180	20%	18.	150	$33\frac{1}{3}$%

From the equation *base*$\times$*rate*=*percentage*, it is obvious that the base equals the percentage divided by the rate.

Example. The milk brought to a certain creamery averages 4% of butter fat. How many pounds of milk must be brought to yield 350 pounds of butter fat?
Solution: The required number of pounds of milk is:

$$350\div .04=35000\div 4=8750.$$

In special cases, such as when the rate is $12\frac{1}{2}$% or $16\frac{2}{3}$%, it is more convenient to reduce the per cents to common fractions.

WRITTEN EXERCISES AND PROBLEMS

Find the base in each of the following:

Percentage	Rate	Percentage	Rate	Percentage	Rate
1. 480	15%	6. 4720	6¾%	11. 16200	16⅔%
2. 6785	25%	7. 5970	8%	12. 18500	25%
3. 4190	6½%	8. 264	4⅛%	13. 49.80	33⅓%
4. 2140	4¾%	9. 640	7%	14. 37.50	66⅔%
5. 1280	6¼%	10. 8910	2¼%	15. 258.40	87½%

16. A beef weighed 840 pounds dressed. What was the weight of the live animal if its dressed weight was 54% of its live weight?

17. A farmer sold 460 bu. of corn in January. How many bushels was this in October if the corn shrank 7.5% from October to January?

18. A farmer sold 756 bu. of potatoes in April at 86¢ a bushel. He could have sold them in October for 70¢ a bushel. If the potatoes shrank 15% from drying, rotting, etc., would they have brought more or less if sold in October?

19. A certain grade of coal contains 87.6% of its weight in carbon. How many tons of such coal will contain 50 tons of carbon?

20. A farm yields a permanent net income of $350 by renting it. For how much would the farmer be willing to sell it if he could invest the money at 7%?

21. A farmer wishes to have 50 bushels of seed corn in the spring. How many bushels must he save in the fall if corn shrinks 14.7% from October until May?

22. If ice shrinks 35% from the time it is harvested until it is sold, how many tons of ice must be put up in the winter in order to sell 1500 tons?

PROBLEMS

1. The weight of flour required to make bread is about 65% of the weight of the bread. How many pound loaves of bread can be made from a bushel of wheat, weighing 60 pounds, if 72% of the weight of the wheat is flour?

2. Hogs shipped from Iowa to Chicago lost 5.6% in weight. The weight of the pork killed and dressed was 74% of the weight of the live animals just before killing. How many pounds of pork were obtained in Chicago from a hog weighing 328 pounds when shipped from Iowa?

3. The New York Association for Improving the Condition of the Poor estimated at one time that the lowest average daily expenditure of a family of five living in the city of New York, to preserve health, mind, character, self-respect, and proper conditions of family life was: 65¢ for rent, $1.29 for food, 43¢ for clothing, and 60¢ for miscellaneous expenditure. What was the total expenditure for a year of 365 days? What per cent of the total expenditure was used for each of these?

4. Hogs were sold at $12.25 a hundred pounds live weight. How much was this per pound of pork if the pork weighed 69.5% of the live weight.

5. How many pounds of wheat are required to make one barrel of flour weighing 196 pounds, if 72% of the wheat is flour?

6. A cow yields 9840 pounds of milk in one year, 3.9% of which is butter fat. If 85 pounds of fat make 100 pounds of butter, how many pounds of butter can be made from this cow in one year. At 32¢ a pound, what is the value of this butter?

7. A farmer sold 24 horses at $160 per head, and 12 at $140 per head. What was the average price per head? If the horses cost the farmer on an average $145 per head, what was his gain or loss per cent on the whole deal?

1. A dealer in rugs bought 104 rugs at $36 apiece. He sold 74 of them at $60 apiece, and the remaining 30 at $45 apiece. What was the average gain per rug? What was his gain per cent on all the rugs?

2. A car load of beef cattle weighed 24,000 pounds at the time of shipping. When they reached Chicago, they had shrunk 4.7%. What was their weight on reaching Chicago? If the weight of dressed beef was 58.5% of the weight when they reached Chicago, how many pounds of meat were obtained from this car load of cattle?

3. A certain alloy of copper and zinc contains 58.49% copper and 41.10% zinc, while the rest consists of other metal. How many pounds of copper, zinc, and other metals are there in one ton of this alloy?

4. If 56% of a pile of wood is solid wood, how many cubic feet of solid wood are there in one cord of wood?

5. A load of yellow pine cord-wood weighs 3740 pounds. If one cubic foot of solid yellow pine weighs 38 pounds, and if 56% of the 128 cubic feet of a cord is solid wood, what would be the value of this load of wood at $4.25 a cord?

6. A silver dollar weighs 412½ grains. It contains 41¼ grains of copper, while the rest is pure silver. What per cent of the silver dollar is copper, and what per cent is pure silver?

7. A five-cent piece weighs 77.16 grains, 19.299 grains of which is nickel, and the rest copper. What per cent of the five-cent piece is nickel, and what per cent is copper?

8. A company will put up a creamery plant in a certain neighborhood provided that they can get enough milk to turn out 600 pounds of butter daily. If 85% of butter is butter fat, and if the average milk they get contains 3.9% of butter fat, how many pounds of milk must they get daily? At 8.6 pounds to a gallon, how many gallons is this?

SIMPLE INTEREST

95. **Interest.** Anyone who borrows money on a business basis is obliged to pay something for the use of it. The amount paid for the use of money is called *interest.*

96. **Reasons for Paying Interest.** Those who have saved money are not willing to let others have the use of their savings without deriving some benefit from making the loan. Hence those who borrow are obliged to pay interest.

If a man owning a good house could borrow money without interest to beautify his grounds he would then enjoy these grounds at no expense to himself. Would this be right?

97. **Principal. Rate of Interest.** *The rate of interest* is the *rate* per cent of the amount borrowed which is paid as interest for one year.

The amount loaned is called the *principal.* The principal plus the interest is called the *amount.*

The prevailing rate of interest differs widely in different countries and in different localities within the same country. It also varies with the kind of securities offered, the time of the loan, etc.

98. **Legal and Maximum Rate.** When no agreement is made as to the rate of interest to be paid on a loan, except that it is to *bear interest,* then the law in each state specifies what the rate is to be. This is called the *legal rate.* In all but a few states, the law also specifies that interest above a certain rate is illegal, and cannot be collected. This is called the *maximum rate.* An attempt to charge more than the maximum rate is called *usury.*

99. **Simple Interest.** At the end of a certain period the interest becomes due and payable, but if no agreement is made to the contrary the interest becomes an ordinary debt and does not draw interest. Such interest is called *simple interest.*

100. **Fundamental Rule on Interest.** The rate of interest per year is always expressed as so many per cent of the principal. This is often expressed by saying that the rate is so many per cent *per annum.*

The fundamental rule on interest is expressed by the equation,

$$principal \times rate \times time = interest. \qquad \text{(I)}$$

Time must be expressed in terms of years when the interest is given at a certain rate per year. The common method of counting time is to regard 360 days as one year, and 30 days as one month.

Thus, 4 months, 12 days is regarded as 132 days, or $\frac{132}{360}$ of a year.

101. **Time One or More Years.** When the time is one or more whole years the problem of finding the interest is very simple.

To find the interest for one year is a problem in ordinary percentage when the base and rate are given, and the percentage is to be found.

To find the interest for several whole years, find the interest for one year and multiply by the number of years.

Thus, the interest on \$4800 at 6% for one year is $.06 \times 4800 =$ \$288, while the interest on \$4800 at 6% for 3 years is 3 times the interest for one year, or $3 \times \$288 = \$864$.

102. **Time Less than One Year.** The great majority of loans are made for periods of less than one year, and the principal difficulties in computing interest arise in connection with such loans.

103. **Different Methods of Computing Interest.** There are several methods in general use for computing interest. The most important of these are given on the following pages. The pupil should study one of these methods with care, and use it in solving the examples that follow. He should, however, study the other methods sufficiently to obtain a general idea of them.

104. Cancellation Method. The so-called cancellation method is perhaps most easily learned, since it follows directly from equation (I), page 103. The following examples show this method.

Example 1. Find the interest on \$4200 for 90 days at 6%.
Solution: 90 days $=\frac{90}{360}$ years $=\frac{1}{4}$ of a year.

Hence the interest is $\overset{21}{\cancel{4200}}\times\frac{\overset{3}{\cancel{6}}}{\cancel{100}}\times\frac{1}{4}=63$ (dollars)

Example 2. Find the interest on \$2800 for 4 months, 17 days at 7%.
Solution: 4 months, 17 days $=137$ days or $\frac{137}{360}$ years.
Hence the interest is

$$\overset{7}{\cancel{2800}}\times\frac{7}{\cancel{100}}\times\frac{137}{\underset{90}{\cancel{360}}}=\frac{7\times7\times137}{90}=74.59 \text{ (dollars)}$$

Example 3. Find the interest on \$12450 for 2 months, 21 days at $5\frac{1}{2}$%.
Solution: 2 months, 21 days $=81$ days or $\frac{81}{360}$ years $=\frac{9}{40}$ years. Also $5\frac{1}{2}\%=\frac{11}{2}\%=\frac{11}{200}$.
Hence the interest is

$$\overset{249}{\cancel{12450}}\times\frac{9}{\underset{4}{\cancel{40}}}\times\frac{11}{\underset{40}{\cancel{200}}}=\frac{249\times9\times11}{4\times40}=154.07 \text{ (dollars)}$$

In case the time is more than one year, find the interest for the whole years separately. Then find the interest for the days and the months, and add the two results.

In some cases the whole time may be written as an improper fraction of a year. Thus, one year 8 months may be written $1\frac{2}{3}$ years $=\frac{5}{3}$ years. This should be done in case the fractional part of the year is a very simple fraction.

105. **The Bankers' Method.** The bankers' method is based on the fact that 6% for one year is the same as 1% for 60 days. If the rate is other than 6% the interest at 6% is found first, and then the interest at the required rate. The following examples show the method.

Example 1. Find the interest on $4200 for 90 days at 6%.

Solution:

Interest for 60 days (1% of $4200)	$42.00
Interest for 30 days (½ of 60 days)	21.00
Interest for 90 days (60 days+30 days)	$63.00

Example 2. Find the interest on $8940 for 2 months and 17 days at 6%.

Solution: The time is 77 days.

Interest for 60 days	$89.40
Interest for 15 days (¼ of 60 days)	22.35
Interest for 2 days (1/30 of 60 days)	2.98
Interest for 77 days	$114.73

Example 3. Find the interest on $265.40 at 6% for 49 days.

Solution:

Interest for 60 days	$2.654
Interest for 10 days	.442
Interest for 50 days (60 days—10 days)	2.212
Interest for 1 day (1/10 of 10 days)	.044
Interest for 49 days (50 days—1 day)	$2.17

Example 4. Find the interest on $124.50 for 2 months, 21 days at 5½%.

Solution: The time is 81 days.

Interest 6% for 60 days	$1.2450
Interest 6% for 20 days	.4150
Interest 6% for 1 day	.02075
Interest 6% for 81 days	$1.68075
To find interest at 5½% deduct 1/12	.140
	$1.54

106. The Six Per Cent Method. The method consists in first finding the interest on one dollar at 6% for the given time, and then multiplying by the number of dollars in the principal.

It is easily seen that the interest on one dollar at 6% for

1 year is	\$.06	1 day is	\$.0001⅔
1 month is	.005	2 days is	.0003⅓
6 days is	.001	3 days is	.0005
12 days is	.002	4 days is	.0006⅔
18 days is	.003	5 days is	.0008⅓
24 days is	.004		

We can now write down the interest on \$1.00 at 6% for any number of months and days.

Thus, the interest for 7 months, 11 days is obtained as follows:

Interest for 7 months = 7 × .005	\$.035
Interest for 9 days = .001 + .0005	.0015
Interest for 2 days	.0003⅓
Interest for 1 dollar for the given time	\$.0368⅓

Example 1. Find interest on \$265.40 at 6% for 49 days.

Solution:

Interest on \$1.00 for 30 days	\$.005
Interest on \$1.00 for 18 days	.003
Interest on \$1.00 for 1 day	.0001⅔
Interest on \$1.00 for 49 days	\$.0081⅔

Interest on \$265.40 = 265.40 × .0081⅔ = 2.167 (dollars).

Example 2. Find the interest on \$12,450 for 2 months, 21 days at 5½%.

Solution: Using the method shown above, we find that the interest on one dollar for the given time at 6% is \$.0135.

Interest on \$12,450 = \$12,450 × .0135	\$168.08
To find interest at 5½% deduct $\frac{1}{12}$	14.01
	\$154.07

107. Preliminary Work for Cancellation Method. In case you are using the cancellation method (see page 104), solve the examples in this paragraph, otherwise not.

WRITTEN EXERCISES

Express each of the following as a fraction of a year, using 360 days as one year. One month, 2 months, and so on up to 12 months. One day, 2 days, 3 days, and so on up to 29 days.

108. Preliminary Work for Bankers' Method. In case you are using the bankers' method (see page 105), solve the examples in this paragraph, otherwise not.

Notice that the interest for 60 days, 30 days, 20 days, 15 days, 12 days, 10 days, 6 days, 5 days, 3 days, 2 days, and 1 day, can be found very easily in the order in which they are given here:

Thus, $30 = \frac{1}{2}$ of 60	$10 = \frac{1}{2}$ of 20 or $\frac{1}{6}$ of 60
$20 = \frac{1}{3}$ of 60	$6 = \frac{1}{10}$ of 60
$15 = \frac{1}{4}$ of 60	$5 = \frac{1}{12}$ of 60 or $\frac{1}{2}$ of 10
$12 = \frac{1}{5}$ of 60	$1 = \frac{1}{6}$ of 6, or $\frac{1}{5}$ of 5, etc.

Hence it is important to be able to select convenient combinations of these to equal any given number of days.

Thus, $4 = 3 + 1$, $8 = 6 + 2$ or $10 - 2$, $7 = 6 + 1$, $5 = 6 - 1$.

ORAL EXERCISES

1. Find convenient combinations for all integers from 1 to 30.

2. Find convenient combinations for all integers from 31 to 59.

109. Preliminary Work for Six Per Cent Method. In case the six per cent method is used, solve the examples in this paragraph, otherwise not. (See page 106.)

WRITTEN EXERCISES

Examples 1-36. Find the interest on \$1.00 at 6% for the periods given in the exercises in § 107.

WRITTEN EXERCISES

Solve the following problems by your chosen method, the rate being 6% in all cases.

	Principal	Time		Principal	Time
1.	$8650	3 mo. 12 days	24.	$29,460	11 mo. 8 days
2.	$175	4 mo. 18 days	25.	$1280	2 mo. 12 days
3.	$3940	2 mo. 24 days	26.	$1490	10 mo. 27 days
4.	$370.50	7 mo. 6 days	27.	$4910	7 mo. 28 days
5.	$480	9 mo. 3 days	28.	$5460	4 mo. 24 days
6.	$670	11 mo. 9 days	29.	$7880	3 mo. 15 days
7.	$590.40	8 mo. 15 days	30.	$3960	2 mo. 24 days
8.	$7610	10 mo. 21 days	31.	$1540	7 mo. 21 days
9.	$370	9 mo. 21 days	32.	$6700	4 mo. 15 days
10.	$730	3 mo. 8 days	33.	$8420	8 mo. 12 days
11.	$1670	5 mo. 10 days	34.	$4600	9 mo. 15 days
12.	$4090	7 mo. 11 days	35.	$1500	10 mo. 17 days
13.	$7500	4 mo. 13 days	36.	$500	9 mo. 5 days
14.	$3700	5 mo. 14 days	37.	$760	11 mo. 4 days
15.	$7670	8 mo. 16 days	38.	$1780	7 mo. 13 days
16.	$7600	7 mo. 20 days	39.	$2860	5 mo. 6 days
17.	$3120	2 mo. 26 days	40.	$9370	2 mo. 4 days
18.	$320	8 mo. 28 days	41.	$2170	8 mo. 7 days
19.	$2130	10 mo. 2 days	42.	$15,692	1 mo. 17 days
20.	$4920	5 mo. 6 days	43.	$46,570	1 mo. 24 days
21.	$810	4 mo. 3 days	44.	$84,280	9 mo. 15 days
22.	$3280	9 mo. 7 days	45.	$95,910	7 mo. 8 days
23.	$16,640	7 mo. 9 days	46.	$76,400	5 mo. 9 days

110. Finding Interest at Rates Other than Six Per Cent. In case either the bankers' method or the six per cent method is used, care should be taken to pass from the interest at 6% to the interest at the given rate in the simplest manner.

Thus: To find the interest at 5% deduct $\frac{1}{6}$ of the interest at 6%, and to find the interest at 7% add $\frac{1}{6}$ of the interest at 6%.

The following table suggests other such combinations:

$6\frac{1}{2}=6+\frac{1}{12}$ of 6	$5\frac{3}{4}=6-\frac{1}{24}$ of 6
$7\frac{1}{2}=6+\frac{1}{4}$ of 6	$5\frac{1}{4}=6-\frac{3}{24}$ of 6, or $6-\frac{1}{8}$ of 6
$4\frac{1}{2}=6-\frac{1}{4}$ of 6	$8\ =6+\frac{1}{3}$ of 6

WRITTEN EXERCISES

Solve the following problems by any method you choose:

1. $790, 2 months, 12 days, at 6%.
2. $240, for 4 months, 6 days, at 5%.
3. $80.50 for 9 months, 18 days, at $5\frac{1}{2}$%.
4. $160.40 for 3 months, 24 days, at 7%.
5. $230.50 for 10 months, 9 days, at $4\frac{3}{4}$%.
6. $296 for 5 months, 8 days, at $5\frac{1}{4}$%.
7. $340 for 8 months, 27 days, at $5\frac{3}{4}$%.
8. $145 for 6 months, 15 days, at 6%.
9. $495 for 12 days, at $6\frac{1}{4}$%.
10. $1390 for 11 months, 27 days, at $5\frac{1}{2}$%.
11. $2460 for 7 months, 21 days, at $5\frac{3}{4}$%.
12. $4890 for 5 months, 18 days, at $4\frac{1}{2}$%.
13. $3170 for 3 months, 15 days, at $4\frac{3}{4}$%.
14. $2640 for 10 months, 18 days, at $6\frac{3}{4}$%.

111. Exact Interest. In some problems the so-called exact interest is required. This is obtained by counting a year as 365 days, and taking the exact number of days between dates.

Table of Time. To find the exact number of days between the date when the loan is made and the date when it is due, use is sometimes made of a table like the following:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from March 10th to July 18th is obtained from this table by noticing that from March to July is 122 days, and then adding 8 days, getting 130 days.

ORAL EXERCISES

Using the table, find the exact number of days from:

1. January 7 to April 9
2. February 3 to August 27
3. July 6 to November 4
4. September 3 to December 17
5. February 24 to July 3
6. March 3 to October 7
7. October 13 to January 27
8. November 24 to March 3
9. January 9 to September 1
10. May 27 to October 3
11. June 18 to November 7
12. August 14 to December 5

112. Cancellation Method Used in Finding Exact Interest. The bankers' method and the six per cent method are not adapted to finding exact interest, since these methods are based on a year of 12 months of 30 days each. Hence the cancellation method is used. In actual practice bankers who compute exact interest use tables to shorten the work.

Example. Find the exact interest on \$2460 at $5\frac{1}{2}\%$ from May 7th to October 3d.

Solution: From the table we find that there are 153 days from May to October. Subtracting 4 days, we have 149 days, as the exact time. Then the required interest is:

$$2460 \times \frac{11}{200} \times \frac{149}{365} = 55.23 \text{ (dollars).}$$

WRITTEN EXERCISES

In this manner find the exact interest in each of the following:

1. \$3970, May 1st to August 6th, at 5%.
2. \$4380, June 9th to September 14th, at 6%.
3. \$2160, January 14th to May 24th, at $5\frac{1}{2}\%$.
4. \$480, February 3d to April 15th, at 5%.
5. \$1230, March 17th to May 15th, at $5\frac{1}{4}\%$.
6. \$6290, December 12th to April 3d, at $5\frac{1}{2}\%$.
7. \$458.60, November 30th to March 12th, at $4\frac{3}{4}\%$.
8. \$1440, September 12th to February 3d, at $5\frac{1}{4}\%$.
9. \$1980, October 18th to January 25th, at 5%.
10. \$1690, August 20th to January 15th, at $5\frac{1}{4}\%$.
11. \$8830, July 29th to November 4th, at 5%.
12. \$5970, July 29th to December 14th, at 5%.

113. Comparison of Common and Exact Interest. In case the time for which interest is computed is given in days, instead of days and months, then the common interest is always greater than the exact interest.

Thus, in finding the exact interest for 90 days, the time in terms of years is $\frac{90}{365}$, while for common interest it is $\frac{90}{360}$.

Clearly $\frac{90}{360}$ is greater than $\frac{90}{365}$. The same would be true for any number of days.

For this reason exact interest is usually used when large sums are involved. The United States Government uses exact interest, and obviously any institution or individual *paying* interest would do so if permitted to choose.

Banks, on the other hand, *receive* interest, and usually use common interest. In the case of small amounts the difference between common and exact interest is very small, and the common interest is more readily computed. This may also influence the banks in their choice of method.

WRITTEN EXERCISES

Find both the exact and the common interest for the following:

	Time	Principal	Rate		Time	Principal	Rate
1.	75 days	$280	5%	9.	120 days	$390	6%
2.	45 days	$360	5½%	10.	85 days	$670	5¾%
3.	90 days	$850	5¼%	11.	75 days	$1900	4¼%
4.	80 days	$600	5¾%	12.	50 days	$3400	5%
5.	30 days	$1200	6%	13.	82 days	$2600	6½%
6.	60 days	$1500	6½%	14.	93 days	$3650	7%
7.	35 days	$2400	4¾%	15.	51 days	$8400	5½%
8.	24 days	$480	5½%	16.	64 days	$1680	5%

114. The Amount. When the interest is added to the principal, the sum is called the *amount.*

Thus, if $100 is invested at 6%, the amount at the end of one year is $106.

To find the amount, find the interest for the given time, and add this to the principal.

Example 1. Find the amount of $3800, at the end of one year, the rate of interest being 6%.

Solution:

$3800	A slightly different form of solution is ob-	$3800
.06	tained by noticing that the principal is	1.06
228.00	100% of itself, and that when 6% interest	22800
3800	is added to it, we have in all 106% of the	3800
$4028.00	principal.	$4028.00

Example 2. Find the amount at the end of 6 months, of $450, the rate being 6%.

In this case the interest is 3% of $450. Hence multiply $450 by 1.03. If the time were 4 months, we should multiply by 1.02. Why?

Example 3. What is the principal if the rate is 6%, and if the amount at the end of the year is $1650?

Suggestion: $1640 = 1.06 × principal. Find the principal.

WRITTEN EXERCISES

1. Find the amount if the principal is $500, the rate 8%, and the time 6 months.

2. Find the amount if the principal is $2100, the rate 8%, and the time 4 months.

3. Find the amount if the principal is $800, the rate 6%, and the time 3 months.

4. Find the principal if the amount is $480, the rate 6%, and the time 4 months.

1. On October 20th a man was offered \$1580 for a city lot. He sold it July 2d, the following year, for \$1675. In the meantime he paid \$45 in taxes. If he figured interest at 6½% per annum on the \$1580, did he gain or lose by keeping the lot?

2. A business man needs the use of \$35,000 from June 1st to October 15th. At 5¾% how much interest does he pay?

3. A farmer sells his farm for \$160 an acre, and invests the money at 5½%. How much per acre does he get as yearly income from his investment?

4. A prospective tenant offers to pay a cash yearly rent of \$6.50 per acre and all taxes, or to buy the farm for \$110 an acre. If the money can be invested at 5¼%, which will bring the greater income per acre, and how much?

5. A man buys an automobile for \$2250. The machine depreciates 30% the first year. The money could be loaned at 6½%. What is the loss due to depreciation and loss of interest?

6. A piano can be rented for \$5 a month, while kept tuned at the owner's expense. It may be bought for \$350. Keeping it tuned costs \$8.00 a year. The depreciation of the instrument is 10% a year. If money can be loaned at 6%, which pays better, to rent the piano or to buy it.

7. A furnished house may be rented for \$45 a month, and an unfurnished house of equal grade for \$30 a month. It costs \$1200 to furnish the house. If money can be loaned at 5½%, and if furniture depreciates 8⅓% in one year, which pays better, to rent the house furnished or unfurnished?

8. A man building a house in a city considers whether to buy a lot 30 feet wide, or 60 feet wide. If the wider lot costs \$3500 more, and if taxes on it are \$30 a year more than on the narrower one, what is the extra yearly expense on the larger lot if he must borrow the money at 5½%?

115. **Rates on Large Loans.** When large sums of money are involved, the rate of interest is frequently not expressed in such simple numbers as we have been using. An example will illustrate: On June 5, 1913, the State of New York borrowed $27,000,000. Following were some of the successful bids, and the rates of interest per annum:

	Amount	Rate per cent
Folson & Adams, N. Y.	$275,000	4.625
J. W. Griggs, N. Y.	100,000	4.7
L. Van Hoffman & Co., N. Y.	100,000	4.748
L. Van Hoffman & Co., N. Y.	100,000	4.798
L. Van Hoffman & Co., N. Y.	100,000	4.848
Albert L. Judson, Albany	10,000,000	4.85
Kissel, Kinnicut & Co., N. Y.	250,000	4.875
L. Van Hoffman & Co., N. Y.	100,000	4.898
Bankers' Trust Co., N. Y.	500,000	4.95
Bond & Goodwin, N. Y.	500,000	4.97
J. P. Morgan & Co., N. Y.	2,000,000	4.975
Guaranty Trust Co., N. Y.	2,568,000	4.985

1. Compute the yearly interest on each of these amounts.

2. The money was received on June 10, 1913, and was paid back February 2, 1914. What was the exact time of this loan?

3. Compute the exact interest from June 10 to February 2, on the three largest loans.

4. Compute the common interest on the $10,000,000 loan of the above. What is the difference in the interest on this loan when computed according to these two methods?

5. In a recent year a large university got an income of 4.47% on its investments. At this rate how much income would it get by investing a gift of $2,750,000? After studying the next page find what was the total investment of this university if the income was $1,875,000.00.

116. Finding the Principal. From the equation,

$$principal \times rate \times time = interest, \qquad \text{(I)}$$

we see at once how to find any one of the four numbers involved when the other three are given. The problem which occurs the most frequently is to find the interest, when the principal, rate and time are given, and this is the one we have considered thus far.

In problems where the rate or principal is to be found, the time is usually one year. In that case the equation becomes:

$$principal \times rate = interest$$

People sometimes wish to make an investment to produce a certain yearly income.

Example. How much money must be invested at $5\frac{1}{2}\%$ interest to yield a yearly income of $250?

Solution: We have given the rate and the interest and are required to find the principal. But principal = interest ÷ rate = $250 \div \frac{11}{200} = 250 \times \frac{200}{11} = \frac{50000}{11} = 4545.45$ (dollars).

Compare this problem with those on page 98.

WRITTEN EXERCISES

Find the principal in each of the following, the time in each case being one year.

	Rate	Interest		Rate	Interest
1.	5%	$640	8.	6¾%	$810.30
2.	4¾%	$860	9.	5.12%	$3500
3.	4¼%	$235	10.	4.8%	$4000
4.	5¼%	$360	11.	4.7%	$4500
5.	5¼%	$186.40	12.	4.6%	$2500
6.	6%	$1240	13.	5.1%	$5000
7.	6¼%	$496.80	14.	5%	$3500

117. **Rates of Income from Securities.** Various interest-bearing notes and securities are constantly bought and sold. In this case the income or interest is known because there is a fixed amount paid each year. The principal is the price paid for such note or security. The problem is to find the rate of interest yielded by the investment. That is, given the principal and the interest, to find the rate. Such problems are of frequent occurrence.

Example. Find the rate of interest if $108 yields $5.00 yearly income.

Solution: The rate = interest ÷ principal = 5 ÷ 108 = .0463. That is, the rate is 4.63%. The result is given to the nearest one-hundredth of one per cent.

Compare this problem with those studied on page 94. In nearly all practical problems of this kind the rate of interest fails, as in this case, to come out an exact decimal.

WRITTEN EXERCISES

Find the rate in each of the following:

	Principal	Income		Principal	Income
1.	$112.25	$4.50	11.	$92.00	$4.50
2.	$175.75	$8.00	12.	$219.00	$10.00
3.	$77.50	$3.50	13.	$107.00	$6.00
4.	$53.00	$3.00	14.	$135.00	$7.00
5.	$108.00	$5.50	15.	$32.00	$2.00
6.	$36.25	$2.00	16.	$148.75	$8.00
7.	$122.25	$7.00	17.	$114.50	$6.00
8.	$251.00	$12.00	18.	$134.00	$7.00
9.	$99.25	$5.00	19.	$155.25	$8.00
10.	$45.50	$5.00	20.	$94.75	$5.00

MISCELLANEOUS PROBLEMS

1. A man bought a lot for $1500 and built a house on it, costing $4200. He rents the house for $45 a month. What rate per cent does he get on his investment if he pays $83.50 a year in taxes, and $100 on an average each year for repairs.

2. A family are renting a house for $55 a month. How much money could they borrow and put into a new house without increasing their expenses, if interest is 6% and if they figure taxes at $70 a year, and repairs, etc., at $125 a year?

3. A man buys a farm containing 280 acres at $95 an acre. He rents it for $6.50 an acre. What rate per cent income does he get on his investment if he pays $190 yearly in taxes?

4. A manufacturer has machinery worth $290,000. How much will it be worth one year from now if it depreciates 8%?

5. A manufacturer invests $75,000 in additional machinery, thereby saving $18,000 a year in wages. He borrows money at 6%, and counts 10% depreciation on the machinery each year. Does he gain or lose by putting in the machinery, and how much?

6. A manufacturer, who has antiquated machinery, figures that by replacing it he can increase his yearly output by $80,000. If it costs $875,000 to put in the machinery, and if he borrows the money at 5¾%, does he gain or lose by replacing the old machinery, and how much?

7. A railway company figures that it can save $2,490,000 a year in operating expenses by investing $35,000,000 in improving the track. If the company pays 5⅛% interest, does the company lose or gain by making the improvement, and how much?

8. A lady bought a washing machine for $125, thereby saving $2.00 a week in wages. What is the rate of income on the investment if the machine depreciates $25.00 a year?

MISCELLANEOUS PROBLEMS

1. A city borrows $50,000 at 4.85% interest, and $300,000 at 5.05%. Find the average rate on these two loans.

Suggestion: It would be wrong to take the average of 4.85 and 5.05. Find the total interest paid for both loans, and then find what per cent that is of the total loan.

2. What is the average rate of interest of the two largest loans described on page 115?

3. A city borrows money at different times and at different rates of interest. Its total debt is $1,385,000, on which it pays $68,400 yearly in interest. Find the average rate to the nearest hundredth of 1 per cent.

4. A man borrows money at 6½%, and pays $110,000 for a property yielding a yearly net income of $8600. What is his yearly loss or gain on the transaction?

5. A man offers to sell for $65,000 a property yielding a net yearly income of $4800. What is the rate per cent of income on this investment?

6. A power company figures that it will cost $1,730,000 to build an electric plant, using water power, while it will cost only $860,000 to build a steam plant supplying the same amount of current. The steam plant costs $50,000 a year more to operate than does the electric plant. Which is the more economical, if the company borrows money at 5¾%?

7. A stone road costs $8300 per mile to build, and $130 a year per mile to keep in repair. If money is borrowed at 5¼%, how much does this road cost per mile per year?

8. A young man takes 4 years in which to learn a profession, and thereby increases his average yearly income for the rest of his life by $2300. To what investment of capital is this equal if money is worth 5⅛% per year?

		A	B	C	D	E	F
	1.	6480	3560.00	1260.00	25%	8%	5%
	2.	12690	6940.00	6400.00	30%	7%	6%
I.	3.	13560	5870.00	4790.00	40%	6%	5%
	4.	2360	496.80	2470.00	10%	4%	6%
	5.	8960	785.00	260.00	20%	5%	10%
	6.	490	53.76	40.40	15%	6%	5%
	7.	830	394.83	98.79	20%	8%	8½%
II.	8.	4940	2765.80	1890.00	15%	10%	4%
	9.	8995	8498.00	2380.90	12%	8%	5%
	10.	24360	6490.50	3790.80	10%	6%	5%
	11.	17860	985.60	8195.00	18%	15%	7%
	12.	9345	2559.80	1960.00	25%	10%	8%
III.	13.	21700	1758.60	1340.00	22%	12%	6%
	14.	95400	6540.60	5745.00	35%	8%	5%
	15.	47890	4060.75	2160.00	33%	20%	10%
	16.	2980	698.70	270.00	40%	20%	10%
	17.	3760	356.50	342.00	24%	20%	10%
IV.	18.	8500	957.38	243.00	35%	20%	8%
	19.	3700	1970.00	786.00	15%	10%	5%
	20.	15900	5378.00	783.00	10%	8%	5%
	21.	26580	8700.00	380.00	18%	10%	7%
	22.	3970	3500.00	470.00	20%	15%	5%
V.	23.	18360	7260.00	260.00	25%	10%	5%
	24.	8590	5190.00	630.00	28%	15%	6%
	25.	7650	1950.00	286.00	40%	15%	10%
	26.	59246	3900.00	198.00	25%	12%	8%
	27.	84930	7640.00	741.00	15%	10%	4%
VI.	28.	14810	1200.00	864.50	24%	8%	5%
	29.	24290	5120.00	211.90	17%	9%	6%
	30.	62370	7142.40	841.70	14%	12%	7%

		A	B	C	D
	1.	5%	5½%	3 mo. 6 days	5 mo. 27 days
	2.	6%	6½%	4 mo. 12 days	9 mo. 18 days
I.	3.	7%	5¾%	8 mo. 18 days	7 mo. 17 days
	4.	8%	5¼%	6 mo. 15 days	3 mo. 14 days
	5.	4%	5½%	10 mo. 24 days	2 mo. 12 days
	6.	7%	8%	3 mo. 6 days	2 mo. 2 days
	7.	6¼%	9%	2 mo. 9 days	3 mo. 28 days
II.	8.	6¾%	4¼%	3 mo. 4 days	4 mo. 28 days
	9.	5½%	4¾%	5 mo. 8 days	5 mo. 15 days
	10.	4%	5¼%	7 mo. 3 days	1 mo. 24 days
	11.	6%	8%	5 mo. 3 days	5 mo. 18 days
	12.	7%	4%	4 mo. 8 days	3 mo. 12 days
III.	13.	5%	5%	3 mo. 7 days	2 mo. 24 days
	14.	6%	6½%	2 mo. 24 days	3 mo. 21 days
	15.	6%	5½%	3 mo. 17 days	3 mo. 27 days
	16.	10%	6%	8 mo. 9 days	10 mo. 14 days
	17.	9%	7%	4 mo. 7 days	7 mo. 16 days
IV.	18.	7½%	7½%	7 mo. 12 days	8 mo. 4 days
	19.	6%	5½%	3 mo. 15 days	11 mo. 8 days
	20.	5%	6%	4 mo. 18 days	3 mo. 10 days
	21.	6%	5½%	8 mo. 9 days	7 mo. 5 days
	22.	6½%	6%	2 mo. 24 days	3 mo. 7 days
V.	23.	6%	6½%	8 mo. 27 days	5 mo. 9 days
	24.	5%	5½%	7 mo. 3 days	8 mo. 12 days
	25.	5½%	5%	5 mo. 6 days	11 mo. 19 days
	26.	6½%	5½%	3 mo. 8 days	4 mo. 8 days
	27.	4¾%	6½%	2 mo. 7 days	3 mo. 12 days
VI.	28.	5%	7%	7 mo. 9 days	6 mo. 24 days
	29.	7%	8%	8 mo. 15 days	10 mo. 18 days
	30.	8%	10%	9 mo. 21 days	7 mo. 4 days

BANKING

118. The Commercial Bank. A commercial bank is an institution whose business it is to receive money on deposit, and to make loans. We will now consider a commercial bank.

Mr. Sam Jenkins keeps a clothing store in a small town in Wisconsin. In the afternoon he takes all the cash out of the cash drawer (except what he wishes to keep to make change) and carries it to the FIRST NATIONAL BANK to deposit it.

119. The Deposit Slip. On a blank provided by the bank Mr. Jenkins makes out a deposit slip, like the one on this page, and gives it to the *receiving teller* (the man who receives cash for the bank), together with the cash and his *pass book.*

THE FIRST NATIONAL BANK
BARABOO, WISCONSIN

DEPOSITED BY

Sam Jenkins

May 8, 1919

Please list each check separately.

Bills	$146.00
Gold	20.00
Silver	14.45
Checks	24.80
	49.60
	37.00
Total	$291.85

The deposit slip must contain an accurate statement of the amount of each kind of money and of the checks offered for deposit.

In any bank you can find deposit slips on the tables for the use of depositors. Get some real deposit slips and study them. Use them for making out the deposit slips required on page 124.

120. The Pass Book. Balance. Bank Account. The pass book is a small book in which the receiving teller writes the amount he receives. This is given to Mr. Jenkins as a receipt for his money. The teller counts the money with care, to make sure that he gives receipt for the amount actually received. Mr. Jenkins deposits his money this way every day. The amount of money which a person has in a bank is called his balance in the bank, or his bank account.

121. Paying by Check. On Saturday Mr. Jenkins pays his clerks their weekly salary. He gives each a check on the FIRST NATIONAL BANK. A check is an order to the bank to pay a certain amount to a certain person.

Stub	Check
No. 486 May 17, 1919 To James Patten for salary Brought forward...$1386.40 Am't of this check. 25.00 Balance ....$1361.40 Carried forward....$1361.40	THE FIRST NATIONAL BANK. No. 486 Baraboo, Wisconsin, May 17, 1919. Pay to the order of James Patten $25.00 Twenty-five and ~~~~~~ $\frac{no}{100}$ DOLLARS. SAM JENKINS.

Mr. Patten takes this check to the bank, and signs his name on the back of it. The check is stamped "paid," Mr. Patten is given $25, and that amount is deducted from Mr. Jenkins' account with the bank.

122. Identification. It is necessary that the *paying teller* in the bank (the man who pays out money) should know Mr. Patten personally, or that some one should *identify* him. The bank keeps on file a copy of Mr. Jenkins' signature, so the paying teller can tell whether the check is genuine or not.

In the larger cities, more than 90% of all payments are now made by checks.

WRITTEN EXERCISES

Make out a deposit slip for Mr. Jenkins for each of the following, and find total on each slip:

1. Monday, May 12, 1919. Bills, $115; gold, $5; silver, $25.80; checks, $18.50, $34.50, $12.40.

2. Tuesday, May 13, 1919. Bills, $84; gold, $15; silver, $18.65; checks, $8.60, $15.60, $80.00.

3. Wednesday, May 14, 1919. Bills, $67; gold, $15; silver, $27.40; checks, $24.30, $12.00, $17.50.

4. Thursday, May 15, 1919. Bills, $167; silver, $19.35; checks, $35.60.

5. Friday, May 16, 1919. Bills, $93; gold, $5.00; silver, $34.90; checks, $5.30, $21.50, $15.90.

6. Saturday, May 17, 1919. Bills, $218; silver, $48.30; gold, $15; checks, $39.20, $6.40, $18.90, $13.60.

7. What were the total deposits for this week?

8. Monday, May 19, 1919, the following items of cash were received in Mr. Jenkins' store. Find the total for this day.

$.15	$1.50	$.49	$3.80	$.12	$.74
.25	10.60	.57	.43	.43	10.60
1.20	.34	1.64	.27	.98	.45
.35	.17	3.81	.45	.65	.39
.05	3.60	12.67	.83	1.50	.90
.10	1.90	3.59	.38	1.43	.15
.75	2.85	.04	1.63	.24	.17
5.00	.75	1.63	.24	2.50	2.30
.40	.38	.14	.89	6.41	1.60
.04	.56	.29	.34	.51	.57
2.00	1.39	2.38	1.40	.32	1.30
.75	4.67	5.46	1.51	.57	3.00

123. The Bank Balance. During the month of May, 1919, Mr Jenkins' deposits were:

$194.60	$402.00	$241.30	$124.16	$268.40
381.80	321.50	272.73	381.45	319.60
267.40	235.65	273.82	297.37	
235.52	249.16	291.54	410.70	
306.00	132.67	267.18	256.90	
409.50	504.23	401.67	407.25	

During the same month he drew out money by checks as follows: Was his *balance* in the bank increased or decreased this month, and how much?

$45.00	$11.42	$13.00	$2.91
27.00	18.45	1070.00	4.83
18.00	860.00	439.30	18.00
18.00	120.00	60.00	18.00
18.00	18.00	18.00	15.00
37.00	18.00	18.00	15.00
75.00	15.00	15.00	12.00
15.00	15.00	12.00	24.00
15.00	12.00	24.00	25.00
12.00	24.00	54.00	64.00
24.00	25.00	29.00	120.00
69.00	50.00	80.00	3400.00
31.20	42.70	6.35	41.60
3.80	29.60	3.47	276.40
16.65	24.70	31.20	14.40

Write a check like the one on page 123, payable to one of your schoolmates, and sign it yourself.

Be sure to fill out the stub first (see page 123). The stub remains in the check book when the check is torn out. It shows how much is in the bank, the amount of the check, the name of the person to whom it is payable, and the purpose of the payment.

124. Borrowing Money from a Bank. Running a bank would not pay if it consisted only in receiving and keeping money, and paying it out when demanded. The banker's source of profit is in lending money, and charging interest on it. It is not necessary for a bank to keep in its vaults all the money deposited with it. In practice, only from 25% to 30% of the deposits are kept in cash in the bank, or in other banks. That is, if the bank has $100,000 deposits, it may have $25,000 in cash. The remaining $75,000 is loaned to responsible persons for short periods—say 30, 60, or 90 days. If Mr. Jenkins wishes to borrow $10,000 for 90 days he makes out a promissory note like the following:

$10,000.00	Aug. 15, 1919
Ninety days after date I promise to pay	
to the order of FIRST NATIONAL BANK	
Ten Thousand and ~~~~~~ no/100 DOLLARS	
at The First National Bank, Baraboo, Wisconsin	
Value received.	
No. 46. due Nov. 13, 1919.	SAM JENKINS

The amount written in the note is called the *face* of the note. The man who signs the note is called the *maker* of the note.

In this note nothing is said about interest. The bank gets its interest by deducting from the face of the note interest at a certain rate for 90 days. If the rate is 6%, the amount deducted is

$100.00	= int. for 60 days
$50.00	= int. for 30 days
$150.00	= int. for 90 days

Mr. Jenkins gets $10,000 − $150 = $9850.00. The note is due 90 days from Aug. 15, or Nov. 13, and must be paid on that date. Mr. Jenkins then pays $10,000 as specified in the note.

125. **Bank Discount.** In ordinary business, other than banking, interest is payable yearly, and at the *end of the year;* or if the period of the loan is less than one year, then at the *end of the period.* The banker, however, usually deducts the interest or bank discount at the *beginning* of the period. The bank is said to *discount* the note, and the amount deducted, as interest, is called the *discount.* The amount left after the discount has been deducted is called the *proceeds* of the note. Any one discounting a note at a bank has the proceeds added to his balance, and draws out the money by means of checks.

126. **The Bank Draft.** Mr. Jenkins has bought his fall stock in Chicago, and wishes to pay his bills there. Therefore he buys a draft on Chicago, and uses part of the proceeds of his note to pay for it. This draft is a check on a Chicago Bank, drawn by Mr. Jenkins' local bank.

A BANK DRAFT

$8540.00 Baraboo, Wis., Aug. 22, 1919. No. 34

THE FIRST NATIONAL BANK

Pay to the order of Sam Jenkins $8540.00

Eight thousand five hundred forty, and ~~~~~~ $\frac{\text{no}}{100}$ DOLLARS

TO CORN EXCHANGE NATIONAL BANK
Chicago

ALBERT E. NOYES
Cashier

On the back of this check Mr. Jenkins writes:

Pay to the order of
Marshall Field & Co.,

SAM JENKINS.

The check is said to be endorsed to Marshall Field & Co., and no bank will advance money on it except on the order of that firm.

127. Reasons for Borrowing Money. Even the richest business houses borrow large sums from the banks. One reason for this is that they need much more money for a few months each year than they need the rest of the time.

This can be shown by an example: A company publishing school books makes nearly all its collections in the fall. The books are printed, bound, and shipped during the spring and summer months. Such a firm is likely to borrow hundreds of thousands of dollars during the summer months, repaying the loans as the proceeds of the sales come in.

Suppose such a firm borrows $200,000 for four months at 6%. The interest charge is $4000. To avoid borrowing, the firm would have to increase its capital by $200,000, which would involve a yearly charge in the way of interest of about $12,000—a loss of $8,000 yearly, because the $200,000 would be idle two-thirds of the year. During these two-thirds of the year the bank loans the same money to other concerns which are in special need of cash at that time. In this way the capital is kept busy the whole year. So, we see that the bank performs a real service to the business world. It would be difficult, if not impossible, to transact modern business without the aid of banks.

WRITTEN EXERCISES

Find the proceeds of each of the following notes:

Face of note	Time	Rate of discount
1. $5800	60 days	5½%
2. $15000	75 days	5¾%
3. $7000	45 days	6%
4. $4300	30 days	5⅞%
5. $50000	90 days	5⅛%
6. $35000	80 days	5%

128. Call Loans. A loan made for a definite time is called a *time loan*. There is another class of loans called *call loans*, on which payment may be demanded any time without notice. Call loans are usually made at a lower rate of discount (interest) than time loans.

Banks try to avoid calling their call loans; for to do so would make it very difficult to borrow money at all. A call loan may be paid as soon as the borrower wishes to do so.

WRITTEN EXERCISES

Find the bank discount on the following call loans:

	Face of note	Time	Rate of discount
1.	$8000	18 days	3½%
2.	$75000	21 days	3¼%
3.	$40000	42 days	3%
4.	$56000	35 days	3¼%
5.	$85000	60 days	2¾%
6.	$15000	63 days	3¾%
7.	$7600	17 days	3¼%
8.	$25000	38 days	3¾%
9.	$60000	49 days	3%
10.	$35000	52 days	3¼%
11.	$6000	52 days	2⅞%
12.	$12000	57 days	2⅝%
13.	$7400	59 days	3⅛%
14.	$18000	32 days	3⅜%
15.	$50000	41 days	3%

129. Compound Interest. In some cases the interest is not paid when due, but is added to the principal, after which the new principal draws interest until the next interest day, and so on. Interest accumulating in this manner is called *compound interest.* The interest is said to be compounded at the time when it falls due and is added to the principal.

Example. Find the amount of $500 at 4% compounded annually for 4 years.

Solution:

(a) 1.04×$500=$520 is the amount due at the end of one year. This is the second principal.

(b) 1.04×$520=$540.80 is the 3d principal.

(c) 1.04×$540.80=$562.43, is the 4th principal.

(d) 1.04 × $562.43 = 584.93 is the amount due at the end of 4 years.

1st principal	$500
	1.04
	2000
	500
2d principal	$520.00
	1.04
	2080
	520
3d principal	$540.80
	1.04
	21632
	5408
4th principal	$562.43
	1.04
	224972
	56243
Amount due in 4 years	$584.93

130. Interest Compounded Semi-annually or Quarterly. On deposits in savings banks interest is usually compounded semi-annually, and sometimes quarterly. When interest is compounded semi-annually, it is computed every six months and added to the principal instead of every year, as in the example above. When interest is compounded quarterly it is computed and added to the principal every three months.

Example. Find the amount of \$1500 at 4%, compounded, semi-annually for 2 years:

The interest for 6 months is 2% of the principal.
Second principal = 1.02 × \$1500 = \$1530.
Third principal = 1.02 × \$1530 = \$1560.60.
Fourth principal = 1.02 × \$1560.60 = \$1591.81.
Amount due in two years = 1.02 × \$1591.81 = \$1623.65.

Notice that this problem is the same as finding the amount due in four years at 2% compounded annually.

WRITTEN EXERCISES

Find the amount of each of the following:

1. \$200 at 4% for 3 years, compounded semi-annually.
2. \$160 at 3% for 2 years, compounded annually.
3. \$150 at 4% for 4 years, compounded semi-annually.
4. \$360 at 3½% for 3 years, compounded annually.
5. \$450 at 3¾% for 4 years, compounded annually.
6. \$640 at 4% for 4 years, compounded semi-annually.

SAVINGS BANKS

131. **Savings Deposits.** Savings banks do not accept deposits to be withdrawn by means of checks. Usually there is a rule that money can be withdrawn only after a notice of, say, 30 days. Interest is allowed on deposits, but only when left in the bank for a certain time, as three or six months.

132. **Interest on Savings Deposits.** The savings banks are compelled by law to invest their deposits in very high-grade securities. Since such securities pay a low rate of interest, the savings banks are obliged to pay a low rate. The rate paid by them varies from 3% to 5%, the average being under 4%.

COMPOUND INTEREST TABLE

133. Amount of $1.00 compounded annually.

Years	2%	3%	4%	4½%	5%
1	1.02000	1.03000	1.04000	1.04500	1.05000
2	1.04040	1.06090	1.08160	1.90203	1.10250
3	1.06121	1.09273	1.12486	1.14117	1.15763
4	1.08243	1.12551	1.16986	1.19252	1.21551
5	1.10408	1.15927	1.21665	1.24618	1.27628
6	1.12616	1.19405	1.26532	1.30226	1.34010
7	1.14869	1.22987	1.31593	1.36086	1.40710
8	1.17166	1.26677	1.36857	1.42210	1.47746
9	1.19509	1.30477	1.42331	1.48610	1.55133
10	1.21899	1.34392	1.48024	1.55297	1.62889
11	1.24337	1.38423	1.53945	1.62285	1.71034
12	1.26824	1.42576	1.60103	1.69588	1.79586
13	1.29361	1.46853	1.66507	1.77220	1.88565
14	1.31948	1.51259	1.73168	1.85194	1.97993
15	1.34587	1.55797	1.80094	1.93528	2.07893
16	1.37279	1.60471	1.87298	2.02237	2.18287
17	1.40024	1.65285	1.94790	2.11338	2.29202
18	1.42825	1.70243	2.02582	2.20848	2.40662
19	1.45681	1.75351	2.10685	2.30786	2.52695
20	1.48595	1.80611	2.19112	2.41171	2.65330
21	1.51567	1.86029	2.27877	2.52024	2.78596
22	1.54598	1.91610	2.36992	2.63365	2.92526
23	1.57690	1.97359	2.46472	2.75217	3.07152
24	1.60844	2.03279	2.56330	2.87601	3.22510
25	1.64061	2.09378	2.66584	3.00543	3.38635

The above is part of a compound interest table, such as is used in computing compound interest. To find the amount of $500 at 5% compound interest for 6 years, compounded annually, multiply the amount of one dollar for that rate and time by 500. If the rate is 4%, compounded semi-annually for 6 years, we take 2% for 12 years.

Example. Find the amount at compound interest of \$340 for 5 years and 4 months at 4% compounded semi-annually.

From the table, amount of \$1 at 2% for 10 years =	\$1.21899
	340
	4875960
	365697
Amount of \$340 for 5 years	\$414.45660
Interest on \$414.46 at 4% for 4 months	5.53
Required amount	\$419.99

The savings banks compute interest only on whole dollars. Thus on a balance of \$162.85 the bank computes interest on \$162. For this reason the accumulation in a savings bank is not exactly as if computed by the table.

WRITTEN EXERCISES

Find the amount of each of the following, using the table opposite:

1. \$450 at 4½% compounded annually for 15 years.
2. \$75 at 4% compounded semi-annually for 8 years.
3. \$320 at 5% compounded annually for 8 years, 6 months.
4. \$280 at 6% compounded semi-annually for 7 years, 10 months.
5. \$800 at 5% compounded annually for 7 years, 10 months.
6. \$1200 at 4% compounded semi-annually for 10 years, 2 months.

Find the amount of each of the following:

	Principal	Rate	Time	Compounded
7.	\$820	4½%	2 years 8 months	annually
8.	\$360	3%	12 years 6 months	annually
9.	\$500	4%	7 years 3 months	semi-annually
10.	\$1150	5%	8 years 9 months	annually
11.	\$680	4½%	5 years 7 months	annually

134. Promissory Notes. In modern business, promissory notes are used a great deal, and every well informed person should know something about the laws and usages that govern them.

A PROMISSORY NOTE

$500.00	May 14, 1919
Two years after date	I promise to pay
to the order of James A. Stuart	
Five hundred and ~~~~~~~~~~ no/100	DOLLARS
at CHASE NATIONAL BANK, NEW YORK CITY	
Value received with interest at 6%.	
	ARTHUR E. PECK

Any written promise made by one person or firm to pay to another a definite sum of money at a definite time or on demand, is called a *promissory note*, or simply a *note*.

The essential things about a note are:

(a) *It must specify a definite sum to be paid.*

To promise to pay enough to defray one year's expenses at college might be a binding contract, but such a written promise would not be a promissory note.

(b) *The person to whom it is to be paid, that is, the payee, must be designated in the note. It must be payable to bearer or to some definite person.*

It is not sufficient to write: "I promise to pay $500.00."

(c) *The note must be signed by the maker.*

It is convenient to insert a place of payment, but this is not necessary. In case no place of payment is inserted, the note must be presented for payment at the maker's place of business, or to him personally.

135. Maker and Payee of Note. The person who signs the note is the *drawer* or *maker* of the note.

Thus, in the note opposite, Arthur E. Peck is the maker.

The one in whose favor the note is made is the *payee*.

In the note opposite, Mr. James A. Stuart is the payee.

The sum of money specified in the note is the *face* of the note.

136. Negotiable Notes. If the note reads, "I promise to pay to the order of James A. Stuart" the note may be sold by Mr. Stuart. To do so he must write his name on the back of it. In doing this Mr. Stuart is said to *endorse* the note. The endorsement is regarded as Mr. Stuart's order to pay.

If the note reads, "I promise to pay to bearer," the note can be sold without endorsement, inasmuch as any person having it in his possession may present it for payment.

If the note reads, "I promise to pay to James A. Stuart," omitting the words "the order of," then the note cannot be sold.

A note which can be sold is said to be *negotiable*. A note which cannot be sold is *non-negotiable*.

137. Maturity. The date on which a note falls due is called its date of *maturity*.

It was an old custom to allow a note to be paid three days after the date of maturity. These days were called *days of grace*. The days of grace have been abolished by law in most states. If a note falls due on a Sunday or legal holiday it is payable on the next following business day, or on the next preceding business day, according to local custom.

ORAL EXERCISES

1. What is necessary in order that a written promise to pay money shall be a promissory note?

2. What is the difference between a negotiable and a non-negotiable note? Can money be obtained on a negotiable note before it is due? On a non-negotiable note?

138. Time Notes and Demand Notes. A note due at a definite time is called a *time note.*

If the note reads: "On demand I promise to pay," it is called a *demand note.*

139. Non-interest-bearing Notes. If interest is not specified in the note, no interest is payable on it until it is due; but if the note is not paid at maturity, it draws interest at the legal rate from that date, whether it reads "with interest" or not.

140. Different Kinds of Endorsements. A note may be endorsed in several different ways:

James A. Stuart

Pay to C. Smith or order James A. Stuart

Pay to C. Smith James A. Stuart

Without recourse James A. Stuart

The first endorsement shown in the left margin is called endorsement in bank. This endorsement makes the note payable to bearer.

The second endorsement makes the note negotiable, but not payable to bearer.

The third endorsement makes the note non-negotiable. This endorsement is called *restrictive.*

In case the maker of the note fails to pay it when due, any one of the above described endorsements renders the endorser liable for payment of the note.

If James A. Stuart wishes to make himself not liable for payment of the note in case Mr. Peck fails to pay it, he writes "*without recourse*" above his signature.

141. Protesting Note. If a note is not paid at maturity, the holder should *protest* it. That means sending a legal notice to the maker and the endorsers of the note, that the note has not been paid. If this is not done promptly the endorsers escape liability on the note.

WRITTEN EXERCISES

1. Make a promissory note in which you are the maker, one of your schoolmates the payee, the face of the note $280, the rate of interest 5%, and the time 90 days. Write the word "specimen" across the face of this note so your schoolmate cannot collect it from you.

2. Make a note like that of Example 1 payable to bearer.

3. Make a note like that of Example 1 payable on demand.

4. Make a note like that of Example 1 not bearing interest.

5. Make a note and endorse it in blank.

6. Make a note and endorse it to one of your schoolmates, making it negotiable.

7. Make a note and endorse it, making it non-negotiable.

8. Endorse a note to one of your schoolmates in such a manner that you would not be responsible in case the maker should fail to pay the note.

9. A note is made by Arthur E. Peck to James A. Stuart, payee. It bears endorsement as follows:

James A. Stuart

Pay to John M. Shepherd
or order
Walter Haskins

Without recourse
John M. Shepherd

Mr. Stuart sold the note to Walter Haskins. How did Mr. Stuart endorse it? To whom did Haskins dispose of the note? How did he sign it? How did Shepherd endorse the note? If Mr. Peck fails to pay the note, when due, what will be the liability of Mr. Shepherd? Of Mr. Haskins? Of Mr. Stuart?

142. **Reasons for Discounts.** Manufacturers prepare catalogues or price lists of their goods, and the prices quoted in them are the highest at which the goods are expected to sell. Reductions from the *list price* are made for a variety of reasons. Some goods are advertised by the manufacturers to sell retail at a certain price. The retail traders receive regular discounts from the advertised retail prices which are sufficient to pay their expenses and besides make a profit. Special discounts are usually made for cash payments. Special discounts may also be given on large orders, or on a first order from a customer.

143. **Discount Series.** A bill may be subject to one, two, three, or more discounts. The amount of a bill before discounts are deducted is called the *gross amount.* The amount after the discounts are deducted is called the *net amount.*

Example. Find the net amount due on a bill for $6540 with discounts of 25%, 10%, and 5%.

Solution:

	$6540
25% (or 1/4) off	1635
	$4905
10% (or 1/10) off	490.50
(Note that 5% is 1/2 of 10%)	$4414.50
5% off	220.725
Net amount	$4193.775

When an additional discount is given, this is invariably computed on the amount after the other discounts have been deducted. There is good reason for this: Suppose the gross amount of a bill is $6540, and a discount of 25% has been allowed to a dealer, because the market price of the goods is really below the list price. Then the bill is really $4905, and not $6540. If now 10% is allowed for cash payment, this 10% must be computed on the real amount of the bill—that is, on $4905, and not on $6540.

144. **Order of Discounts.** The *order* in which two or more successive discounts are taken is immaterial.

Thus, \$800 less 25% is \$600, and \$600 less 5% is \$570. Also \$800 less 5% is \$760, and \$760 less 25% is \$570.

Instead of finding 25% of \$800, and subtracting it, we may find 75% of \$800.

Thus,
$$\begin{array}{r} 800 \\ .75 \\ \hline 4000 \\ 5600 \\ \hline 600.00 \\ .95 \\ \hline 3000 \\ 5400 \\ \hline 570.00 \end{array}$$

We now see at a glance that it is immaterial whether we multiply by .75, and then by .95, or by .95 and then by .75. (See page 17.)

WRITTEN EXERCISES

Find the net amount of each of the following. Use the method described on this page.

Gross amount	Discounts			
1. \$590	10%	12½%	15%	
2. \$740	20%	15%	10%	
3. \$8600	25%	16⅔%	10%	
4. \$3280	40%	15%	5%	
5. \$6430	25%	20%	12½%	
6. \$10,000	30%	10%	5%	
7. \$1260	20%	10%	10%	5%
8. \$2860	35%	10%	5%	5%
9. \$1940	45%	16⅔%	10%	5%
10. \$4500	50%	15%	5%	2%

COMMISSION

145. The Commission Merchant. The cut represents the place of business of J. F. Stephens & Company, who receive all kinds of farm produce and sell it to people in the city.

A farmer, by name H. V. Collins, sends a consignment of eggs to Stephens & Co., who sell the eggs at the best price they can get. They then deduct from the selling price a certain per cent of it, and send the rest to Mr. Collins.

Stephens & Co. perform a valuable service for Mr. Collins, in finding a purchaser for his eggs, and hence are entitled to the compensation they get.

Every day there is a tremendous stream of foodstuffs passing from the country into the great cities, to feed the millions that live there. A large share of this passes through the hands of the commission merchants, who serve to direct it where it is needed. This is especially true of perishable goods that must reach the consumer in a short time.

To appreciate the magnitude of this business, one may recall that more than half of the population of the state of New York live in the city of New York, where scarcely any foodstuffs at all can be raised.

If you live in the country find what articles are sent to the city from your neighborhood to be sold by commission merchants.

146. **Commissions.** The amount which a commission merchant charges for his service is called his *commission*. Commission is computed as *a certain per cent of the sales price.*

PROBLEMS

1. A commission merchant received a consignment of 50 dozen eggs, and 250 lbs. of butter. He sold the eggs at 32¢ a dozen, and the butter at 28¢ a pound. How much did he remit if his commission was 8%?

2. A commission merchant sells goods for $324.50, charging 8½% commission. How much does he remit to his principal? (The man for whom a commission merchant sells goods is called his principal or client.)

3. A commission merchant charged 6% commission for selling farm produce. At the end of a month he remitted $485.60 to his client. What was the total sales if he deducted $34.50 for expenses?

4. During the last five years a certain commission merchant has established a reputation for honesty and for skill in selling at good prices, which enables him to charge on an average 2½% higher commission than he did at the beginning of this period. If his sales during one year amount to $168,500, how much does he make because of this higher commission?

If money can be invested at 4¾%, how much money would have to be invested to yield a yearly interest equal to the value of this merchant's goodwill? ("Goodwill" is a term used by business men to indicate the good reputation they have and the business connections they have formed, which have a real value in their business.)

5. During the last 3 years a certain commission merchant has increased his average monthly sales from $6800 to $18,300. His average rate of commission is 6¼%. By how much has his gross yearly income increased?

147. The Real Estate Agent. This is a copy of an advertisement seen on a vacant lot in the city of Chicago. A. R. Draper & Co. are acting as agents for the owner of the lot. If they succeed in selling it, they get a certain per cent of the selling price as their *commission*. Draper & Co. have many means of finding a purchaser that the owner of the lot does not have. In a big city there are always people wanting to sell, and others wanting to buy. The agent's service consists in bringing these parties together, and it is for doing this that he gets his commission.

One who sells land and buildings on commission is a real estate agent.

PROBLEMS

1. An agent sells a city property for $8500. His commission is 5%. Find the commission and the net proceeds of the sale.

2. A real estate agent buys two lots for $470. He spends $240 for paving, $350 for grading the lots, and pays $85 in taxes. He then sells them for $1500. How much does he make on this deal?

3. A real estate agent sells a house and lot, remitting $7990 to the owner. For how much did he sell the property, if his commission was 6%?

4. During one month a real estate firm sold real estate for customers to the amount of $48,300. If the total commission on these sales was $2574.50, find the average rate of commission.

5. I am offered $7500 for a piece of property with no commission for making the sale. A real estate man can get me a purchaser who will pay $500 more for the property. If I have to pay him a commission of 6% on the sale, which offer is better, and how much?

148. The General Agent or Broker. There are general agents, or brokers, who sell a variety of things, such as motor boats, sail boats, etc., and receive as their compensation a percentage of the sales. There are agents for steamship lines who receive a percentage on every ticket sold, and on every ton of freight obtained. There are men with expert knowledge of such things as musical instruments, automobiles, etc., who make it a business to test out and buy such things for other people.

PROBLEMS

1. An expert buys an automobile for Mr. R. L. Moore for $1150. If his charge for commission is 5%, how much does the car cost Mr. Moore?

2. An agent is instructed to sell a sail boat for the best price he can get, and keep as his commission all he gets above $3,000. If the agent's minimum commission is 5%, what is the lowest price for which he will sell?

3. If in the preceding problem the agent sells the boat for $3500, what is his rate of commission?

4. A man instructs his agent to buy a second-hand automobile to cost not more than $1200, including commission. How much can the agent pay for the car if he wants a commission of 6% on the purchase price?

5. An agent bought a motor boat for $160. For how much did he sell the boat if he figures that he made 10% commission both for buying and selling?

6. An agent bought a used victrola for a customer, paying $125, charging 7% commission for buying. One year later the agent sold it for $100, charging 6% commission for selling. How much did the man pay for one year's use of the victrola. counting interest on the money invested at the rate of 7%?

149. **Problems of the Dairy.** Modern dairy men keep strict account of the food which each cow consumes, and of the amount and quality of the milk which she produces.

RECORD OF JOHANNA PRILLY, FOR WEEK ENDING NOV. 22, 1919

Day	Feed				Milk production		
	Grain	Roots	Silage	Hay	Milk	Fat	% Fat
	lb.	lb.	lb.	lb.	lb.	lb.	
Monday......	$13\frac{3}{4}$	41	24	14	59.7	1.417	....
Tuesday......	$14\frac{3}{4}$	44	20	18	62.4	1.507	....
Wednesday...	$15\frac{1}{2}$	44	26	20	62.3	1.567	....
Thursday....	$15\frac{3}{4}$	44	22	20	65.0	1.609	....
Friday.......	18	44	26	18	64.2	1.632	....
Saturday.....	17	44	30	16	65.1	2.232	....
Sunday......	17	44	30	16	65.1	2.040	....
Total for week							

Complete this record by adding the columns and filling in the per cents of butter fat. At 8.6 pounds to the gallon, how many gallons did this cow produce in one week?

1. If the grain is worth $2\frac{1}{4}$ cents a pound, the roots $\frac{1}{2}$ cent a pound, the silage $\frac{1}{2}$ cent a pound, and the hay $\frac{3}{4}$ cents a pound, what is the cost of the feed for this week? If the milk is worth $2\frac{1}{4}$ cents a pound, what is the value of the milk, and what is the difference between the value of the feed and the value of the milk?

2. A certain cow yields 412.5 pounds of milk in one week, 3.72% of which is butter fat. How many pounds of butter can be made from this milk if 85% of the butter is butter fat?

3. Milk delivered at a certain creamery contains on an average 3.9% of butter fat. Find the cost of producing one pound of 85% butter if the milk is delivered at the creamery for 19 cents a gallon, and if the cost of making the butter is $\frac{3}{4}$ cents per pound. (One gallon of milk weighs 8.6 pounds.)

4. One of the high records for producing milk and butter fat made by a cow in one year is 19,639 pounds of milk and 1,060 pounds of butter fat. At 38 cents a pound for 85% butter, what was the value of the butter made from this butter fat?

5. What was the per cent of butter fat contained in the milk produced by the cow in Example 4?

6. At 8.6 pounds to the gallon, what would be the value of the milk produced by the cow in Example 4 provided it was sold at 21 cents a gallon?

7. Of two cows given the same feed, one produced 11,890 pounds of milk in one year, and the other only 6840 pounds. If both cows averaged 3.8% of butter fat, how many pounds of 85% butter would each cow produce? At 36 cents a pound, what was the difference in the values of the yearly product of these cows?

8. Discuss the importance to a farmer of keeping records of the milk production of his cows and of the feed consumed by them.

150. **Skillful Buying.** By watching the markets carefully, much saving may be effected in buying the things necessary to a family. Below are given real instances of special sales for which prices were reduced.

Find the rate per cent of discount on each article given below.

1. FINAL CLOSING PRICES ON WINTER GOODS

Women's and Misses' suits were $35.00, now $15.00.
Women's and Misses' suits were $42.00, now $20.00.

2. OVERCOATS

Overcoats were $15.50, now $10.00.
Overcoats were $60.00, now $27.50.

3. BUDGET OF BARGAINS IN BRIEF

Women's shoes were $7.50, now $3.95.
$1,500 worth of embroideries to be sold for $700.
Women's three-quarter coats were $27.50, now $14.95.

4. MEN'S HIGH GRADE SHIRTS

Pure silk shirts were $6.50, now $4.00.
New Spring negligee shirts were $2.50, now $1.75.
Fine tucked tuxedoes were $3.50, now $2.00.

5. WOMEN'S FRENCH KID GLOVES

Sixteen-button length were $2,85, now $2.10.
Two-clasp overseam were $1.65, now $1.00.
Gloves were $1.35, now $0.85.

6. PORCELAIN DINNER SETS

102 piece sets were $19.98, now $9.98.
100 piece sets were $8.98, now $6.98.
Thin blown tumblers, were 65¢ per dozen, now 39¢.

7. Discuss discounts offered in special advertisements in your local papers, and the probable accuracy of the statements made in these advertisements. Remember that a successful housekeeper must be a skillful buyer.

151. **Buying Liberty Bonds.** Before solving the problems on this page review pages 130-133. The Liberty Bonds sold in the fall of 1917 bear interest at the rate of 4% per annum.

1. John, aged 8, bought a Liberty Bond for $100 with his savings. If he deposits the interest each year in a savings bank which pays 4% compounded yearly, how much will the bond and the interest amount to in 13 years (when John will be 21 years old)? (Notice that this is the same problem as finding the amount of $100 in 13 years at 4% compounded annually. See page 132. Use the table on page 132.)

Banks do not compute interest on fractions of a dollar. Hence the amount computed by means of the table on page 132 will be just a little greater than the amount accumulated in the bank. This remark applies also to the examples following.

2. Eleanor's father bought her a Liberty Bond for $100. How much will her bond and interest amount to when she is 21 years old if she was 6 years old at the time the bond was bought? (Use the same conditions as in Example 1.)

3. Nancy's grandmother bought her a bond for $100. How much will the bond and interest amount to when she is 21 years old if she was 1 year old when the bond was bought? (Use the same conditions as in Example 1.)

4. A boy at the age of 11 years saves his earnings, and at the end of one year has enough to buy one $50 Liberty Bond. If he puts the interest ($2.00 a year) into the savings bank each ear where it is compounded yearly at 4%, how much will he have, including the $50 bond, when he is 21 years old.

5. Under the conditions of example 4, how much will he have if he buys one bond each year until he is 21 years old?

Remarks. In most cases Liberty Bonds are exempt from all taxes. Find out what you can about Liberty Bonds. If you know a banker he will be able to furnish you information on this matter.

152. The Boy Scouts Raising Potatoes. In order to increase agricultural production during the war, companies of boy scouts engaged in truck farming on vacant lots in our towns and cities. The boys in the picture were given the use of a half block of ground in a small city. They decided to put the whole lot into potatoes.

Make up an account with this potato field, using the data given below. Enter all expenditures on the debit side, and the value of the crop on the credit side. Study again pages 38 and 39.

PROBLEMS

1. A company of boy scouts consisting of 32 boys hired a man and team for $7.50 to plow and drag the lot. They bought six bushels of seed potatoes at $3.50 a bushel, 4 loads of manure at $1.75 a load, and hand implements for $5.75. They paid $3.80 for paris green and for an experienced man to help them spray the potatoes the first time. A team to haul the potatoes to the freight-house cost $5.60. The boys had additional petty expenses amounting in all to $7.45. Find the total of these expenses.

2. The crop from this lot amounted to 284 bushels of potatoes, which the boys sold in the fall at 84 cents a bushel. What was the total value of the crop, and how much did the boys make raising potatoes, not counting the value of their labor? How much was there for each boy?

3. If the boys put in 97½ days' work in all on this lot, how much per day did they earn?

Suggestion: Divide the gain found in Problem 2 by the number of days.

4. If potatoes (sold by weight) shrink 18% from the time they are dug to April, and if storage and extra handling costs 5 cents a bushel, what must be the price per bushel in April in order to make it pay to keep the potatoes?

5. In the year 1917 the potato crop of the United States was estimated at 540,000,000 bushels. If 50,000,000 bushels were needed for seed in the spring of 1918, and if the crop shrank on an average 8½% before being used, and if the total exports were 35,000,000 bushels, what was the average per capita consumption of potatoes in the United States during the year 1917-1918, assuming 103,000,000 as the total population? How many persons were supplied with potatoes from the lot tilled by these boys?

6. The average yearly per capita consumption of wheat in the United States has been 4.6 bushels. If this was decreased 17% by food economy during the war, what was the total amount of wheat consumed in the United States during the year 1917–1918?

7. In normal times the boys would have bought their seed potatoes for $1.10, and the rest of their expenses would have been $8.50 less. They would have sold their potatoes in the fall for 49 cents a bushel. Make up an account to show how much the boys would have made.

153. **The Cost of Keeping an Automobile.** In computing the cost of keeping an automobile, gasoline, tires, oil, repairs, depreciation and interest on the money invested must be included.

1. A man bought a new automobile for $1650. If depreciation the first year is 35%, and if interest on the investment is 7%, what is the amount of these two items for one year?

2. The machine averages $12\frac{1}{4}$ miles on a gallon of gasoline. Tires run on an average 6800 miles, and oil costs 75 cents for each 100 miles. The gasoline costs 24 cents a gallon, and the tires $47.50 apiece. If the machine is run 7840 miles in one season what is the cost of gasoline, tires, and oil?

3. If repair work amounted to $135.60 for the season, what was the total cost for one year? What was the average cost per mile of running the machine?

4. At the rate found in Problem 3, what is the cost of a day's outing, going 108 miles, if incidental expenses are $3.95?

5. What sum invested at 6% interest will yield an income sufficient to cover the yearly expenditure found in Problem 3?

6. If the expenditure for the car found in Problem 3 is taken from a family expenditure of $4500, what per cent of this expenditure goes for the car?

7. Discuss the expenditures for various sizes of car, and also the savings which may be effected by having a car. What could a family go without to have a car?

154. **The Cost of Keeping a Servant.** The cost of keeping a servant not only includes the wages paid, but also the cost of the food consumed, possible extra wastefulness in the kitchen, the use of a room in the house, etc.

1. A family pays a servant $30 per month. The cost of food consumed is estimated at $14 a month, and extra wastefulness in the kitchen at $10 per month. The servant occupies a room, which, together with heat and light, is estimated at $6.00 per month. What is the total cost per year of keeping this servant?

2. In the same household if a servant is not kept, outside help is brought in, involving a total weekly expense of $4.75. If the cost of this help is deducted from the cost of the servant as found in Problem 1, what is the net expense of the servant? (Note that a year has 12 months and 52 weeks.)

3. If the cost of service as given in Problem 1 is taken from a total family expenditure of $2800, what per cent of this expenditure goes for service?

4. If the cost of help as given in Problem 2 is taken from a total family expenditure of $2400, what per cent of this expenditure goes for service?

5. If the saving effected during 10 years by bringing in outside help, as in Problem 2, instead of keeping a servant, as in Problem 1, is invested at 6%, what will be the yearly income?

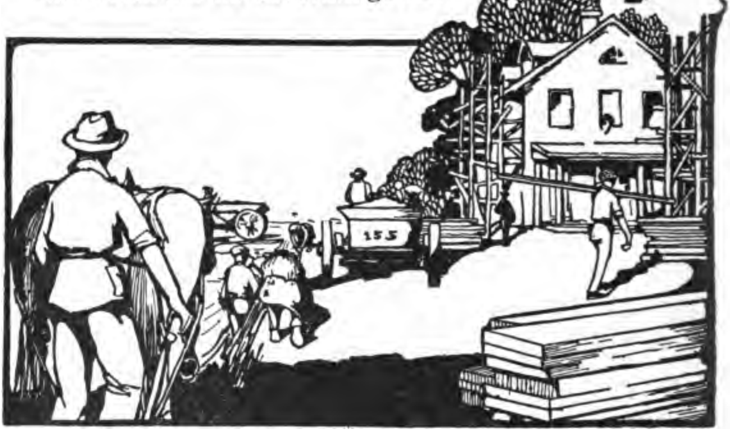
6. A housewife finds that by installing various conveniences in a house at a total expenditure of $480 she can dispense with service costing $104 a year. What will be the rate of income from this investment?

(*Suggestion:* $104 is how many per cent of $480?)

7. Discuss various appliances which may be installed in a house to make housework easier.

155. **Building a House.** Harry Forbes decided to build a house, by buying the material and employing labor by the day instead of letting the work by contract.

1. Two teams and 3 men were employed $3\frac{1}{2}$ days excavating the basement. If a team and man cost \$6.50 per day, and the extra man \$3.25 a day, how much did the excavating cost?



2. To put a concrete floor in the basement, build the foundation and the walks about the house, 65 sacks of cement costing 70 cents a sack, and six loads of sand costing \$1.25 a load, were used. To do the work 3 men worked 5 days at \$4.75 a day, and 2 men worked 5 days at \$2.35 a day. What was the total cost of the basement, foundation and walks?

3. Plumbers' supplies and fixtures cost \$315.50. To put in the plumbing fixtures 2 men worked 9 days at \$5.50 a day. What was the total cost of the plumbing?

4. The work of building the house was done by $108\frac{1}{2}$ days' labor at \$4.75 a day, 114 days' labor at \$2.75, and $24\frac{1}{2}$ days' labor at \$4.50. Find the total cost of this labor.

5. The following bill of lumber was bought:

12 pieces 6″×6″–16′	$28.00 per M.
30 pieces 2″×8″–16′	23.00 per M.
4 pieces 2″×6″–12′	23.00 per M.
84 pieces 2″×4″–16′	23.00 per M.
30 pieces 2″×6″–16′	28.00 per M.
64 pieces 2″×4″–12′	23.00 per M.
24 pieces 2″×4″–16′	23.00 per M.
4500′ boards	23.00 per M.
2200′ flooring	37.00 per M.
2500′ siding	37.00 per M.
4 doors	5.00 each
8 doors	7.25 each
12 windows	3.00 each
10 windows	4.00 each
3½ M. lath	4.50 per M.

Copy on a suitable blank, extend each item, and foot the bill.

6. Find the total cost of building the house if bricks, hardware, paint, and other minor items cost $305.00.

7. Find the total cost of the grounds and buildings if the lots cost $2250, and work on the grounds amounted to $341.

8. Figuring interest on investment in the house at 7%, taxes at $87.50, depreciation and repairs at 10% of the cost of the building (not including cost of lots), and care of the lawn at $75.00 a year, what is the yearly cost of this building?

9. If the building described in the preceding problems is to be rented, what must be the monthly rental to cover current expenses and pay 9% on the investment?

10. Endeavor to obtain data showing cost of building a house in your neighborhood, and make problems similar to those given on these two pages.

156. **Family Budgets.** A certain family, having a yearly income of $1800, decides to spend $300 for rent, $60 for light and telephone, $90 for fuel, $540 for food, $350 for clothing, $50 for amusement, $25 for gifts, and $200 for other expenses, such as life insurance premiums, car-fare, etc. The remainder is to be placed in a savings account. Such a list of proposed expenditures is called a *family budget*.

1. In the case stated above, how many per cent of the total income are used for each of the purposes named?

2. A family having an income of $2750 is paying $40 a month rent. Of the remainder of the income the family decides to spend 22% for food, 20% for clothing, 8% for amusement, 15% for help, and 5% for other purposes. How much does this family propose to spend for each of these purposes? How much does it propose to save?

3. A family spends $720 a year for rent, which is 18% of the total yearly expenditures of the family. The family spends 21% of the total expenditures for food, 15% for clothing, 10% for amusement and travel, 8% for contributions and gifts, 5% for fuel, and 15% for other purposes. How much does the family spend for each purpose, and how much does it save?

4. During the war the family of Problem 3 decided to cut down all expenditures except rent and travel by 15%. The rent remained unchanged, while travel was cut down by 50%. How much did the family save by this change?

5. A laborer's family spends $35.00 a month for food. What is the yearly income of the family if 42% of it is spent for food? This family spends 20% of its income for clothing, 15% for rent, and 12% for other purposes. How much does the family save, and how much does it spend for each of the purposes named?

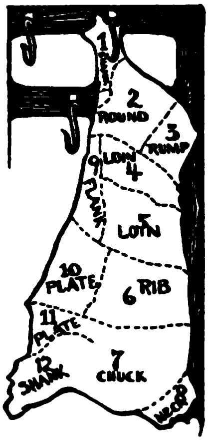
6. Discuss the usefulness of making a family budget.

157. In the Butcher Shop. The so-called "straight" cuts of beef are shown in the figure below.

The average percent of the whole carcass in each "straight" cut of beef is shown in the following table:

Loins	Ribs	Round Rump	Chucks	Plates	Flanks	Shanks	Suet
17%	9%	23%	26%	13%	4%	4%	4%

1. In a certain grade of butcher's cattle the carcass weighs 56.5% of the live weight. If the live weight of a certain animal is 1280 pounds, find the weight of each kind of cut from this animal.



2. If loins are sold for 38 cents a pound; ribs for 32 cents; round for 30 cents; chucks for 24 cents; plates for 18 cents; flanks for 22 cents; shanks for 16 cents; and suet for 18 cents, find the total amount for which the meat of the animal described in Problem 1 was sold.

3. At what price should the butcher buy this carcass if he expected to make a gross profit of 36% on his investment, provided he could sell the hide, etc., for $14.50?

4. A certain grade of hogs dress 72% of their live weight. What must be the average price per pound of pork if hogs sell for $15.50 live weight (the price guaranteed by the Government during the war), and if a profit of 23% is to be made for butchering and selling?

5. Discuss the various items of expense which the retail butcher must meet from his gross profits.

158. **Cost of Running a School.** To find the cost of running a school, account must be taken not only of the current expenses, such as fuel, salaries of teachers, principal and janitors, the cost of repairs, supplies, etc., but also of the interest on the money invested in the building, and of the depreciation of the whole plant.

1. A certain school building, including the grounds, cost $85,400. If the yearly depreciation amounts to 3½% of the whole cost, and if the interest on the money invested is figured at 5½%, what do these two items amount to in one year?

2. The salaries of teachers, principal, and janitors amount to $2820 per month for 10 months, and the janitors' salaries amount to $155 a month for the remaining two months. How much do these items amount to in one year?

3. During one year 98 tons of coal, at $5.90 a ton, and other supplies costing $475.00 are used. Other sundry expenses are $565.80. How much do these items amount to?

4. What is the total of the expenditures enumerated in Problems 1, 2, 3? If the average attendance is 412 pupils, what is the average yearly expense per pupil?

5. If the salaries given in Example 2 are raised 10%, how much will that increase the yearly cost per pupil?

6. A gymnasium is added to this building at a cost of $14,500. Equipment costs $2450.00. Figuring interest and depreciation as in Problem 1, what is the yearly cost per pupil of the gymnasium, including $1050 for a teacher?

7. A playground and athletic field is bought for $5400, and equipment for the playground for $830. Figuring 5½% interest on both these investments, and 12½% depreciation on the equipment of the playground, what is the yearly cost per pupil of the playground and athletic field?

159. **Cost of Manual Training and Household Arts in a School.** Many grade schools are now introducing manual training for the boys, and household arts (cooking, sewing, etc.) for the girls.

1. Rooms in the basement of a school are fitted up for manual training. Fixtures and tools cost \$1870, and installation \$153.80.

Counting interest on the amount invested at the rate of $5\frac{1}{2}\%$, and yearly depreciation at $16\frac{2}{3}\%$, how much do these two items amount to in a year?

2. A teacher whose salary is \$1450 spends $\frac{3}{5}$ of his time teaching manual training. Supplies and other expenditures amount to \$347.03 for the year. What is the amount of these items?

3. What is the total yearly cost of manual training in this school? What is the cost per pupil if 79 boys take manual training?

4. Rooms for cooking and sewing are fitted up at a total expense of \$1385.65. Figuring interest and depreciation as in Problem 1, what do these items amount to in one year?

5. A teacher whose yearly salary is \$1150 spends $\frac{4}{5}$ of her time teaching household arts. Other expenditures amount to \$284.25 during the year. What is the total expense of household arts in this school? What is the expense per pupil if 97 girls take household arts ?

6. Discuss the usefulness of manual training and household arts in a grade school.

CHAPTER III

Review and Extension of Business Arithmetic

160. **Importance of Fundamental Operations.** People engaged in business are constantly adding, subtracting, multiplying and dividing numbers. It is therefore very important that facility should be acquired in performing these operations. This can be done only by constant and long-continued drill.

161. **Copying Figures.** The accurate copying of figures is as important as accurate computation. Salesmen, clerks, bookkeepers are constantly copying figures, and a mistake in copying gives a wrong result just as surely as a mistake in addition or multiplication. In the "Drill Exercises" that follow, mistakes in copying should be regarded just as seriously as mistakes in computing.

162. **Checks.** It is not only necessary that the results are correct. The computer must *know* that they are correct. For this reason checks of various kinds are used. Checks for the various operations are suggested in the proper places in this book. The important point here is that the pupil should *insist* that the work is properly checked. If he does not, he will be poorly prepared for business.

163. **Time Limits.** The pupil should endeavor to work *both rapidly* and *accurately*. The ability to do this comes only by practice. See how long it takes to work a certain set of examples. Then see how much you can improve your record.

164. **Addition and Multiplication.** The operations most frequently used are addition and multiplication, and these should be practiced, to obtain great facility and accuracy.

165. Grouping Numbers in Addition. In adding a column of figures, any numbers whose sum is 10 or less should be grouped, as in the example to the left below.

Add:

$$\begin{array}{l}\left.\begin{matrix}4\\3\end{matrix}\right\}7\\ 6\}6\\ 8\}8\\ \left.\begin{matrix}9\\1\end{matrix}\right\}10\\ \left.\begin{matrix}3\\6\end{matrix}\right\}9\\ \left.\begin{matrix}2\\5\end{matrix}\right\}7\\ \left.\begin{matrix}4\\3\end{matrix}\right\}7\end{array}$$

Instead of saying 4, 7, 13, 21, etc., we see at once that $4+3=7$, etc., and say 7, 13, 21, 31, 40, 47, 54. This is the most important method of shortening the work of addition.

The method may be varied in many ways. Experienced computers group several figures, say any whose sum is less than 20.

Some computers group figures which do not follow each other in order. Thus, they might group 4 and 6, 1, 3, and 6, and 2, 5, 3. This process should not be encouraged. The beginner should confine himself to the grouping suggested above.

ORAL EXERCISES

Add the following by grouping:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
4	4	3	7	5	2	1	3	4	5
8	3	2	3	4	3	4	2	2	3
5	1	5	5	2	1	2	7	4	2
6	5	4	5	2	3	9	5	6	4
7	3	9	7	5	1	1	6	3	5
1	9	5	1	3	6	8	5	3	1
9	7	3	2	1	9	3	1	5	1
2	3	4	7	2	5	3	1	6	4
8	9	5	6	5	3	8	4	3	2
1	5	7	3	4	5	3	7	9	6
3	2	5	4	1	7	8	9	4	3
7	5	1	2	3	4	3	1	7	5
3	4	7	2	4	2	8	9	1	2

Copy and add the numbers on this page. Add down to the first line, and place the sum above it. Then add down to the next line, and so on. To get the whole sum, add these sums. Find the sum of all the columns on the page. Add by grouping.

$434.76		$1400.40	$209.83	$4.75
891.08		670.35	.30	2.45
4329.86		94.16	.50	3.75
1243.47		715.80	1.25	1.45
9427.50		13.45	.30	255.38
1246.75		15.00	1.25	82.10
978.30		24.12	.75	173.28
3874.55		6.00	3.55	24.22
4700.80		8.00	2.45	92.30
678.92	$27805.99	.94	3.75	21.12
384.64		5.78	1.10	37.49
2927.81		46.28	15.00	51.74
8046.49		24.75	2.50	14.15
4001.80		31.92	.65	4.50
276.49		84.61	19.65	61.50
27.91		8.81	5.25	16.12
1.49		93.84	3.40	20.41
28.40		32.16	4.25	507.00
7.92		23.61	.25	12.40
8.42		91.04	12.75	30.00
30.80		37.84	1.25	59.27
17.89		23.72	4.25	60.48
3.46		27.25	251.93	2.47
15.40		19.45	1.75	3.64
672.85		6.74	74.60	7.34
88.90		87.90	3.25	18.75
135.70		21.97	4.25	89.00
468.90		27.45	.25	37.60
468.90		27.45	.25	37.60

166. **Adding Two-figure Numbers.** It is possible to add two or more columns at once. This method is especially useful in adding numbers of two figures. In adding Example 1 below say 21, 61, 63, 83, 89, 119, 126, 166, 172, 192, 199. This process may be shortened in places. Thus, we can say 63 at once instead of 21, 61, 63. You should be on the alert for such possibilities.

EXERCISES

Add two columns at once:

1.	2.	3.	4.	5.	6.	7.	8.
21	46	62	26	54	36	29	37
42	21	81	34	23	43	37	27
26	47	18	14	64	29	24	42
37	32	94	23	23	19	47	51
46	62	32	67	71	47	65	29
27	74	29	62	26	82	51	29

9.	10.	11.	12.	13.	14.	15.	16.
45	26	24	84	67	51	34	43
26	51	31	85	52	78	26	49
83	26	42	26	26	25	84	27
29	78	24	71	78	67	76	24
67	23	34	49	45	26	64	31
34	43	64	27	71	24	62	26

17.	18.	19.	20.	21.	22.	23.	24.
92	31	39	21	38	85	74	37
27	42	64	47	96	92	56	85
34	24	81	37	78	34	27	96
45	71	94	64	23	45	45	35
45	35	64	38	62	27	26	31
27	61	49	51	16	51	73	47
78	37	51	78	35	73	19	31

167. Horizontal Addition. In practice it is often convenient to add numbers horizontally, since that may save copying the figures.

In the case of two-figure numbers this can be done in the same manner as the adding of two columns at once. (See page 161.)

ORAL EXERCISES

Add the following horizontally:

1. 14+16+42+36+38+94+76+23+24+64
2. 26+49+38+46+37+68+94+38+92+46
3. 37+86+96+81+97+26+34+45+56+67
4. 59+84+78+65+59+87+38+41+18+37
5. 65+85+64+95+13+24+32+45+26+54
6. 34+79+34+27+72+42+43+54+62+45
7. 89+46+12+17+81+92+11+23+37+10
8. 95+23+61+45+91+17+82+19+51+72
9. 28+37+24+36+27+42+24+37+45+26
10. 39+19+65+29+41+62+26+53+35+41
11. 65+24+42+73+63+54+33+43+17+11
12. 43+71+92+55+26+72+43+26+71+19
13. 78+46+52+23+36+27+34+62+17+19
14. 91+37+89+64+46+27+26+41+31+81
15. 64+12+34+51+32+61+27+18+52+84
16. 84+56+78+67+45+26+57+83+39+48
17. 19+90+11+87+21+41+49+56+38+59
18. 73+56+68+62+26+21+39+19+43+37
19. 26+56+78+62+26+21+39+19+43+37
20. 38+36+58+62+56+63+72+29+79+67

168. **Horizontal Addition of Large Numbers.** Numbers containing more than two figures are frequently added horizontally. The work is usually done as in ordinary addition, and not as on page 162. Care must be taken to add figures of the same order. Separating the figures in groups of three helps to do this. To add Example 1 add 2, 8, 6, 4, 1, 4, obtaining 25. Place 5 in units' place and carry 2 as usual.

Add 2, 7, 7, 3, 1, 3, 0, obtaining 23, etc.

1. 1,472+2,678+18,936+42,814+128,931+38,604.
2. 49,864+21,476+13,574+6,730+18.970+7,645.
3. 16,940+84,670+47,620+14,386+45,250+9,786.
4. 16,940+84,670+47,620+14,386+45,250+9,786.
5. 32,275+2,450+3,936+364+3917+58,810.

169. **Horizontal and Vertical Addition.** The following are the numbers of men, women, and children admitted to a series of baseball games:

	Wednesday	Thursday	Friday	Saturday	Sums
Men..........	2,685	2,437	3,937	10,680	19,739
Women.......	1,439	1,763	2,184	8,638	14,024
Children......	376	485	765	2,306	3,932
Sums.........	4,500	4,685	6,886	21,624	37,695

1. Find the total number of men by horizontal addition, also the total number of women and the total number of children. Then add these totals to get the grand total for the series.

2. Find the total attendance each day, and add these horizontally to find the total for the series.

In this example the total for the series is found in two ways. Such problems are *self-checking*.

Find the sums and the total in the following without copying the sums:

						Sums
1.	580	1926	4237	2426	2891	
	546	237	3146	1928	6532	
	896	913	1492	578	4236	
	304	46	296	3456	89	
	289	729	854	8276	378	
	457	3291	9370	901	294	
	2913	8364	2194	653	1327	
	1724	427	3846	7931		
	7197	198	8976	3675		
	845	2782	5441	4527		
	297	3943	319	391		
	497	2860	5310	491		
	846	439	728	437		
Sums:						=Total

2. Find the sums and total of the following:

					Sums
2478	3819	1731	1978	1498	
1478	1426	1429	4243	2237	
2562	4567	4928	5520	124	
6489	826	2246	8240	2630	
4667	7459	6472	6289	6200	
1849	8190	9134	2637	4212	
9197	1809	7438	4228	4564	
8264	8460	4783	659	8800	
397	7780	6670	370	7740	
889	9280	9080	7430	3470	
2940	9450	5900	4700	2364	
8190	1860	8200	1240	4578	
1241	8890	2245	5678	8195	
9376	7254	9876	2109	4173	

Sums:

1. How do you find the difference between two numbers?

2. How do you find how much must be added to one number to make the sum equal to another number.

A useful device in subtraction is shown by the following:

Example. Find how much must be added to the sum of $39.47, $24.28, $5.92, and $47.60, to make it equal to $384.15. (See page 15.)

(*Suggestion:* Write these numbers down as shown here.)

$384.15
39.47
24.28
5.92
47.60
$266.88

Adding first column upward: 2, 10, 17. 17+8=25. Write 8 below the line and carry 2. Adding second column 2, 8, 17, 19, 23. 23+8=31. Write 8, carry 3. Again, 3, 10, 15, 19, 28. 28+6=34. Write 6, carry 3. 3, 7, 9, 12. 12+6=18. Write 6, carry 1. 1+2=3. Write 2.

In this manner find by how much the number at the top of each of the following columns of figures exceeds the sum of those placed below it:

3. $492.40	4. $529.63	5. $274.00	6. $290.78
37.60	192.72	19.60	55.40
29.47	23.40	42.20	29.82
164.16	30.50	112.75	127.76
7.82	37.40	3.65	22.15

7. $1249.80	8. $892.30	9. $637.80	10. $314.56
12.70	105.50	94.60	29.40
480.00	156.50	147.35	27.50
394.20	78.50	22.21	108.40
223.70	25.10	44.00	56.50

11. $2540.00	12. $1880.00	13. $1249.60	14. $542.34
360.00	242.50	245.80	151.71
724.70	170.75	135.42	55.00
314.00	381.55	219.61	34.60
55.67	400.00	195.60	29.31
29.00	540.30	19.80	4.26
183.70	6.12	25.00	78.60

170. Horizontal Subtraction. Subtracting without writing the subtrahend below the minuend sometimes saves copying. Thus, in a table giving gross weights and tares, the net weight is found by horizontal subtraction.

In weighing coal, for example, the wagons with the loads and the same wagons without the loads are weighed, and the weights entered in columns. The net weight of the coal is then found by subtraction.

Horizontal subtraction is performed exactly like ordinary subtraction, but care must be taken to subtract figures of the same order.

EXERCISES

Write down the net weight in the following:

	Gross	Tare	Net		Gross	Tare	Net
1.	8490	2130		9.	4960	1840	
2.	9460	2039		10.	4280	1850	
3.	8790	2080		11.	4760	1840	
4.	8280	2070		12.	4730	1810	
5.	9270	2110		13.	4610	1800	
6.	8940	2120		14.	4580	1830	
7.	9190	2110		15.	4570	1810	
8.	8810	2000		16.	4520	1820	

Write down the remainders in the following:

17. 19470 − 8675
18. 34925 − 23790
19. 77840 − 14296
20. 3789 − 2946
21. 8420 − 6796
22. 9284 − 3898
23. 12470 − 9378
24. 9870 − 2464
25. 24708 − 19704
26. 38609 − 27807

WRITTEN EXERCISES

1. Multiply 59 by 84, and show each step, as on page 18.

In finding the products of the following, time yourself to see how long it takes for the first ten, the second ten, and the third ten:

2. 643 29	13. 1498 378	24. 498 328
3. 8964 382	14. 8300 809	25. 4900 8721
4. 843 89	15. 1943 848	26. 8231 749
5. 598 397	16. 724 893	27. 6278 294
6. 784 585	17. 7306 807	28. 4477 688
7. 398 473	18. 468 579	29. 2132 4362
8. 456 789	19. 2378 840	30. 8421 978
9. 1234 123	20. 4973 648	31. 5792 348
10. 890 123	21. 1786 487	32. 3759 684
11. 4576 879	22. 9420 840	33. 1572 937
12. 3927 1427	23. 6007 430	34. 2917 396

EXTENSION OF BILLS

Copy the following bills, extend the items, and foot up to find totals. Put in all items needed to make these bills complete. Read page 74.

1. Great Western Lumber Co.,
St. Paul, Minn.

Sold to Arthur B. Hays.

Terms: Cash 30 days.

		34 M. white pine $34			
		35 M. red oak $57			
		82 M. pine ceiling $48			
		13 M. maple flooring $62			
		64 M. white oak $68			
		864 M. rough boards $28			
		380 M. 2 x 4 (pine) $32			
		160 M. 2 x 8 (pine) $38			

2. Wholesale dealer's bill to retail butcher.

		1860 lbs. of beef . . . . at 18¢			
		3265 lbs. pork . . . . at 23¢			
		1265 lbs. lard . . . . at 18¢			
		868 lbs. mutton . . . . at 17¢			
		2534 lbs. pork . . . . at 22¢			
		3582 lbs. beef . . . . at 19¢			
		1360 lbs. mutton . . . . at 18¢			
		4482 lbs. beef . . . . at 21¢			
		2980 lbs. lard . . . . at 19¢			
		495 lbs. fowl . . . . at 21¢			
		384 lbs. turkey . . . . at 23¢			

1. Wholesale dealer's bill to retail hardware store. Copy and find amount. Supply all items needed to make complete bills. Use your own name as that of the buyer, and the name of a business house you know as that of the seller. Use the date on which you are working as the date of purchase, and the first of the next month as the date of the bills.

	13 doz. hammers	$9.60			
	28 doz. saws	12.50			
	35 doz. locks	18.00			
	6 doz. locks	28.60			
	2460 lbs. iron rods	.07			
	27 doz. pocket knives	8.30			
	6 doz. wash boilers	32.75			
	56 tea kettles	.78			
	65 doz. dippers	2.35			
	18 doz. razors	14.60			

2. Wholesale dealer's bill to retail shoe dealer. Copy and find amount.

	14 doz. pair slippers	$12.60			
	18 doz. pair ladies' shoes	21.40			
	24 doz. pair ladies' shoes	26.80			
	35 doz. pair racer shoes	32.50			
	30 doz. pair men's shoes	42.50			
	45 doz. pair rubbers	9.80			
	6 doz. pair children's shoes	10.50			
	8 doz. pair children's shoes	14.20			
	4 doz. pair fancy shoes	45.50			
	25 doz. pair working shoes	28.50			
	6 doz. pair patent leather	42.60			
	20 doz. pair working shoes	22.80			

171. Explanation of Division. To divide $16+12$ by 4, we may divide 16 and 12 separately, and then add the quotients. This gives $16 \div 4 + 12 \div 4 = 4 + 3 = 7$.

Example 1. Divide 4969 by 134.

$$\begin{array}{r} 37 \\ 134\overline{)4969} \\ \underline{4020} \\ 949 \\ \underline{938} \\ 11 \end{array}$$

In reality the following is what happens:

$$\frac{4969}{134} = \frac{4020+938+11}{134} = \frac{4020}{134} + \frac{938}{134} + \frac{11}{134} = 30 + 7 + \frac{11}{134} = 37\tfrac{11}{134}.$$

Long division is simply a process of separating the dividend into exact multiples of the divisor. Each of these multiples is subtracted from the dividend as it is found.

Example 2. Divide 87690 by 858.

$$\begin{array}{r} 102 \\ 858\overline{)87690} \\ \underline{85800} \\ 1890 \\ \underline{1716} \\ 174 \end{array}$$

The process is explained by the following:

$$\frac{87690}{858} = \frac{85800+1716+174}{858} = \frac{85800}{858} + \frac{1716}{858} + \frac{174}{858} = 100 + 2 + \frac{174}{858} = 102\tfrac{174}{858} = 102\tfrac{29}{143}$$

The importance of our usual method of dividing is apparent when we recall that as late as the year 1000 a writer on mathematics divided 6152 by 15 by making a list of all multiples of 15 up to 6000. This gave a quotient of 400, and a remainder of 152. He then used the list of the multiples of 15 up to 150, which gave him a quotient of 10, and a remainder of 2. In this way the quotient was found to be 410, and the remainder 2. It will be seen that this process is the same as subtracting 15 successively until a remainder less than 15 is left.

The explanation given above will help to gain insight into the nature of the process provided it is understood fully. In practice we proceed as explained on pages 24, 25.

WRITTEN EXERCISES

1. Divide 8493 by 68. Show each step, as on the opposite page.

In finding the quotients and remainders of the following, check each result and time yourself to see how long it takes for the first ten, for the second ten, and for the third ten:

2. 28)107997
3. 87)43476
4. 35)302146
5. 123)94649
6. 98)20848
7. 75)211617
8. 249)255907
9. 54)154219
10. 106)484975
11. 41)91754
12. 67)334532
13. 37)65354
14. 621)92729
15. 86)929002
16. 32)4765
17. 82)85270
18. 154)652700
19. 219)1378600
20. 89)863700
21. 99)87351
22. 801)23569
23. 57)98865
24. 127)98361
25. 96)469264
26. 61)199452
27. 203)39287
28. 185)602008
29. 831)637215
30. 53)49981
31. 183)667801
32. 531)142991
33. 369)782543
34. 602)891854
35. 372)88446
36. 171)84600
37. 263)97800
38. 327)798200
39. 592)765000
40. 237)92480
41. 372)84290
42. 593)45670
43. 751)65432
44. 927)73926
45. 639)29737
46. 581)47632
47. 851)760342
48. 392)243067
49. 472)394521
50. 742)921900
51. 648)593000
52. 937)543900

ORAL EXERCISES

See how long it takes to write sums for Exs. 1–20; Exs. 21–40.

1. $\frac{1}{2}+\frac{1}{3}$	21. $\frac{1}{3}+\frac{5}{8}$	41. $\frac{2}{3}+\frac{1}{6}$	61. $\frac{1}{2}+\frac{3}{16}$
2. $\frac{1}{2}+\frac{2}{3}$	22. $\frac{1}{3}+\frac{7}{8}$	42. $\frac{2}{3}+\frac{5}{6}$	62. $\frac{1}{2}+\frac{5}{16}$
3. $\frac{1}{3}+\frac{1}{4}$	23. $\frac{2}{3}+\frac{1}{8}$	43. $\frac{1}{2}+\frac{1}{4}$	63. $\frac{1}{2}+\frac{7}{16}$
4. $\frac{1}{3}+\frac{3}{4}$	24. $\frac{2}{3}+\frac{3}{8}$	44. $\frac{1}{2}+\frac{3}{4}$	64. $\frac{1}{2}+\frac{7}{16}$
5. $\frac{2}{3}+\frac{1}{4}$	25. $\frac{2}{3}+\frac{5}{8}$	45. $\frac{1}{2}+\frac{1}{12}$	65. $\frac{1}{4}+\frac{3}{16}$
6. $\frac{2}{3}+\frac{3}{4}$	26. $\frac{2}{3}+\frac{7}{8}$	46. $\frac{1}{2}+\frac{5}{12}$	66. $\frac{1}{4}+\frac{5}{16}$
7. $\frac{1}{2}+\frac{1}{8}$	27. $\frac{1}{2}+\frac{1}{5}$	47. $\frac{1}{2}+\frac{7}{12}$	67. $\frac{3}{4}+\frac{3}{16}$
8. $\frac{1}{2}+\frac{3}{8}$	28. $\frac{1}{2}+\frac{2}{5}$	48. $\frac{1}{3}+\frac{1}{12}$	68. $\frac{3}{4}+\frac{5}{16}$
9. $\frac{1}{2}+\frac{5}{8}$	29. $\frac{1}{2}+\frac{3}{5}$	49. $\frac{1}{3}+\frac{5}{12}$	69. $\frac{3}{4}+\frac{7}{16}$
10. $\frac{1}{2}+\frac{7}{8}$	30. $\frac{1}{2}+\frac{4}{5}$	50. $\frac{1}{3}+\frac{7}{12}$	70. $\frac{1}{8}+\frac{1}{16}$
11. $\frac{1}{4}+\frac{1}{8}$	31. $\frac{1}{3}+\frac{1}{5}$	51. $\frac{2}{3}+\frac{1}{12}$	71. $\frac{1}{8}+\frac{5}{16}$
12. $\frac{1}{4}+\frac{3}{8}$	32. $\frac{1}{3}+\frac{2}{5}$	52. $\frac{2}{3}+\frac{5}{12}$	72. $\frac{3}{8}+\frac{1}{16}$
13. $\frac{1}{4}+\frac{5}{8}$	33. $\frac{1}{3}+\frac{3}{5}$	53. $\frac{2}{3}+\frac{7}{12}$	73. $\frac{3}{8}+\frac{5}{16}$
14. $\frac{1}{4}+\frac{7}{8}$	34. $\frac{1}{3}+\frac{4}{5}$	54. $\frac{1}{4}+\frac{1}{12}$	74. $\frac{5}{8}+\frac{3}{16}$
15. $\frac{3}{4}+\frac{3}{8}$	35. $\frac{2}{3}+\frac{1}{5}$	55. $\frac{1}{4}+\frac{5}{12}$	75. $\frac{5}{8}+\frac{5}{16}$
16. $\frac{3}{4}+\frac{3}{8}$	36. $\frac{2}{3}+\frac{2}{5}$	56. $\frac{1}{4}+\frac{7}{12}$	76. $\frac{7}{8}+\frac{3}{16}$
17. $\frac{3}{4}+\frac{5}{8}$	37. $\frac{2}{3}+\frac{3}{5}$	57. $\frac{3}{4}+\frac{1}{12}$	77. $\frac{7}{8}+\frac{5}{16}$
18. $\frac{3}{4}+\frac{7}{8}$	38. $\frac{2}{3}+\frac{4}{5}$	58. $\frac{3}{4}+\frac{5}{12}$	78. $\frac{7}{8}+\frac{7}{16}$
19. $\frac{1}{3}+\frac{1}{8}$	39. $\frac{1}{3}+\frac{1}{6}$	59. $\frac{3}{4}+\frac{7}{12}$	79. $\frac{7}{8}+\frac{9}{16}$
20. $\frac{1}{3}+\frac{3}{8}$	40. $\frac{1}{3}+\frac{5}{6}$	60. $\frac{1}{2}+\frac{1}{16}$	80. $\frac{7}{8}+\frac{11}{16}$

Example 81–160. Find the difference of each pair of fractions in the above examples.

WRITTEN EXERCISES

1. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$
2. $\frac{1}{2}+\frac{1}{3}+\frac{3}{8}$
3. $\frac{1}{4}+\frac{2}{3}+\frac{5}{6}$
4. $\frac{3}{4}+\frac{5}{8}+\frac{5}{12}$
5. $\frac{4}{5}+\frac{3}{10}+\frac{4}{15}$
6. $1\frac{2}{3}+4\frac{1}{2}+5\frac{3}{4}$
7. $6\frac{1}{2}+3\frac{2}{3}$
8. $8\frac{2}{3}+3\frac{3}{4}$
9. $7\frac{1}{2}+3\frac{3}{8}+2\frac{1}{21}$
10. $8\frac{6}{7}+3\frac{5}{9}+2\frac{1}{21}$
11. $8\frac{3}{5}+3\frac{5}{9}+2\frac{1}{3}$
12. $6\frac{3}{4}+2\frac{1}{8}+3\frac{1}{2}$
13. $2\frac{4}{5}+1\frac{1}{3}+2\frac{5}{8}$
14. $1\frac{1}{8}+2\frac{4}{9}+3\frac{1}{2}$
15. $16\frac{1}{2}+14\frac{1}{4}+7\frac{5}{8}$
16. $2\frac{5}{16}+1\frac{1}{2}+1\frac{15}{16}$
17. $3\frac{7}{12}+3\frac{1}{4}+2$
18. $6\frac{5}{9}+3\frac{7}{8}+1\frac{1}{2}$
19. $9\frac{1}{6}+2\frac{1}{3}+4\frac{5}{6}$
20. $8\frac{2}{3}+\frac{4}{5}+2\frac{7}{8}$
21. $12\frac{3}{5}+3\frac{7}{12}+5\frac{3}{4}$
22. $8\frac{3}{7}+\frac{9}{10}+\frac{3}{4}$
23. $\frac{3}{4}+\frac{7}{8}+\frac{7}{12}$
24. $\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$
25. $\frac{4}{5}+\frac{2}{3}+\frac{1}{4}$
26. $\frac{3}{8}+\frac{5}{24}+\frac{7}{9}$
27. $\frac{5}{7}+\frac{3}{8}+\frac{5}{12}$
28. $8\frac{4}{5}+2\frac{3}{4}+1\frac{1}{2}$
29. $7\frac{7}{8}+2\frac{1}{4}+4\frac{3}{16}$
30. $5\frac{5}{8}+7\frac{3}{16}+8\frac{11}{16}$
31. $6\frac{5}{12}+2\frac{1}{8}+3\frac{15}{16}$
32. $9\frac{3}{4}+5\frac{7}{8}+3$
33. $3\frac{5}{16}+2\frac{1}{3}+8\frac{3}{4}$
34. $12\frac{4}{7}+3\frac{3}{4}+1\frac{1}{2}$
35. $\frac{3}{10}+\frac{7}{8}+\frac{5}{12}$
36. $\frac{8}{9}+\frac{5}{18}+\frac{5}{24}$
37. $25\frac{1}{2}+4\frac{3}{4}+8\frac{7}{16}$
38. $15\frac{6}{7}+3\frac{5}{14}+2\frac{1}{21}$
39. $8\frac{5}{8}+2\frac{1}{4}+3\frac{5}{32}$
40. $21\frac{3}{4}+4\frac{5}{16}+\frac{1}{8}$
41. $5\frac{1}{5}+10\frac{1}{10}+15\frac{1}{15}$
42. $7\frac{2}{3}+4\frac{1}{4}+6\frac{3}{8}$
43. $4\frac{2}{5}+6\frac{3}{10}+2\frac{3}{9}$
44. $6\frac{2}{3}+\frac{7}{8}+\frac{15}{16}$
45. $9\frac{1}{7}+24\frac{1}{8}+5\frac{1}{28}$
46. $\frac{3}{7}+\frac{5}{9}+\frac{8}{21}$

ORAL EXERCISES

See how many minutes and seconds it takes you to write down the products of the following:

1. $4\times\frac{3}{5}$	11. $5\times2\frac{1}{3}$	21. $4\frac{2}{3}\times\frac{1}{8}$
2. $7\times\frac{1}{8}$	12. $6\times2\frac{2}{3}$	22. $\frac{2}{3}\times6\frac{3}{4}$
3. $5\times\frac{1}{2}$	13. $\frac{1}{3}\times5$	23. $\frac{3}{4}\times1\frac{1}{2}$
4. $10\times\frac{1}{8}$	14. $\frac{1}{2}\times\frac{1}{3}$	24. $\frac{1}{3}\times2\frac{1}{3}$
5. $8\times\frac{5}{6}$	15. $\frac{2}{3}\times\frac{4}{5}$	25. $\frac{2}{5}\times1\frac{3}{4}$
6. $10\times\frac{3}{5}$	16. $\frac{3}{4}\times\frac{4}{5}$	26. $\frac{3}{5}\times2\frac{1}{4}$
7. $2\times1\frac{1}{2}$	17. $1\frac{1}{3}\times\frac{1}{2}$	27. $\frac{1}{8}\times3\frac{3}{8}$
8. $2\times3\frac{1}{3}$	18. $3\frac{2}{3}\times\frac{1}{3}$	28. $1\frac{1}{4}\times1\frac{1}{3}$
9. $2\times3\frac{2}{3}$	19. $1\frac{1}{2}\times\frac{1}{4}$	29. $2\frac{1}{2}\times2\frac{1}{2}$
10. $3\times2\frac{4}{7}$	20. $3\frac{1}{3}\times\frac{1}{3}$	30. $\frac{3}{8}\times3\frac{3}{4}$

See how many minutes and seconds it takes you to write down the quotients in the following:

31. $\frac{1}{2}\div3$	40. $4\frac{1}{2}\div4$	49. $2\frac{1}{4}\div\frac{2}{3}$
32. $\frac{1}{3}\div2$	41. $4\frac{1}{8}\div3$	50. $2\frac{3}{4}\div\frac{1}{5}$
33. $\frac{2}{3}\div3$	42. $5\frac{1}{3}\div3$	51. $3\div\frac{2}{5}$
34. $\frac{3}{4}\div2$	43. $\frac{1}{2}\div\frac{1}{3}$	52. $4\div\frac{3}{4}$
35. $1\frac{1}{2}\div2$	44. $\frac{1}{3}\div\frac{1}{2}$	53. $2\frac{1}{3}\div\frac{1}{4}$
36. $1\frac{1}{3}\div3$	45. $\frac{2}{3}\div\frac{1}{4}$	54. $3\frac{4}{5}\div\frac{2}{3}$
37. $2\frac{1}{2}\div4$	46. $\frac{2}{3}\div\frac{2}{5}$	55. $\frac{3}{4}\div1\frac{1}{2}$
38. $2\frac{2}{3}\div3$	47. $1\frac{1}{2}\div\frac{1}{2}$	56. $\frac{2}{3}\div2\frac{1}{3}$
39. $3\frac{1}{2}\div2$	48. $1\frac{1}{3}\div\frac{1}{3}$	57. $\frac{3}{5}\div1\frac{4}{5}$

WRITTEN EXERCISES

See how long it takes you to solve the first 21 of the examples below. Also see how long it takes to solve the last 21.

1. $5\frac{3}{8} \div \frac{5}{8}$
2. $7\frac{2}{3} \div 2\frac{1}{2}$
3. $21\frac{3}{4} \times 2\frac{3}{8}$
4. $41\frac{1}{2} \div \frac{3}{4}$
5. $1\frac{1}{3} \div 2\frac{1}{4}$
6. $4\frac{3}{4} \div 2\frac{3}{8}$
7. $2\frac{1}{3} \div 4\frac{1}{4}$
8. $5\frac{1}{6} \div 2\frac{2}{3}$
9. $1\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$
10. $2\frac{1}{2} \div 1\frac{1}{3}$
11. $36\frac{1}{3} \div 5\frac{1}{2}$
12. $28\frac{1}{2} \div 7\frac{3}{8}$
13. $28\frac{1}{3} \div 7\frac{3}{8}$
14. $28\frac{1}{2} \times 7\frac{3}{8}$
15. $28\frac{1}{2} \times 7\frac{3}{8} \times 1\frac{1}{3}$
16. $1\frac{1}{4} \times \frac{7}{22} \times \frac{1}{121}$
17. $\frac{3}{4} \times 46 \times 1\frac{3}{4} \times \frac{5}{12}$
18. $\frac{5}{8} \times \frac{2}{3} \times \frac{6}{7} \times \frac{3}{10}$
19. $2\frac{1}{3} \times 3\frac{1}{3} \times 4\frac{1}{4}$
20. $5\frac{7}{12} \times 1\frac{1}{2} \times 2\frac{1}{4}$
21. $10\frac{1}{2} \times 2\frac{3}{4} \times 1\frac{5}{12}$
22. $7\frac{1}{2} \times \frac{1}{3} \times 5\frac{1}{2}$
23. $4\frac{3}{4} \times 5\frac{7}{12} \times 3\frac{2}{3}$
24. $84 \times 67 \times 31\frac{1}{2}$
25. $51\frac{1}{2} \times 36 \times 1\frac{3}{4}$
26. $45\frac{1}{2} \times 31\frac{2}{3} \times 1\frac{2}{3}$
27. $16\frac{2}{3} \times 16\frac{2}{3} \times 160$
28. $5\frac{1}{2} \times 5\frac{1}{2} \times 160$
29. $46 \times 37 \times 53 \div 1\frac{7}{8}$
30. $9\frac{1}{3} \times 12\frac{3}{4} \times 2$
31. $2\frac{1}{6} \div 3\frac{1}{4}$
32. $21\frac{1}{2} \times 3\frac{1}{3} \times \frac{1}{5}$
33. $16 \times 15 \times 3 \div 144$
34. $7\frac{3}{5} \times \frac{2}{3} \times 2\frac{1}{4}$
35. $1\frac{3}{4} \div 2\frac{1}{2}$
36. $\frac{1}{2} \times 3\frac{1}{3} \times 2\frac{1}{4}$
37. $\frac{5}{9} \times \frac{3}{7} \times 1\frac{1}{12}$
38. $9\frac{1}{2} \times \frac{3}{4} \times 1\frac{1}{2}$
39. $12\frac{3}{4} \times 7\frac{2}{3}$
40. $5\frac{1}{2} \times 3\frac{1}{2} \times 4\frac{2}{3}$
41. $3\frac{1}{2} \times 4\frac{3}{4} \times 5\frac{3}{8}$
42. $2\frac{1}{3} \times 5\frac{1}{2} \times 7\frac{3}{14}$

172. **Aliquot Parts.** An aliquot part of a number is contained an integral number of times in the number. Thus 2, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are aliquot parts of 10, because $10\div 2=5$, $10\div 2\frac{1}{2}=4$, $10\div 3\frac{1}{3}=3$, and $10\div 5=2$.

173. **Aliquot Parts of 100.** The following are the most important aliquot parts of 100:

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{20}$	$\frac{1}{30}$	$\frac{1}{40}$	$\frac{1}{50}$
50	$33\frac{1}{3}$	25	20	$16\frac{2}{3}$	$12\frac{1}{2}$	10	$8\frac{1}{3}$	$6\frac{2}{3}$	$6\frac{1}{4}$	5	$3\frac{1}{3}$	$2\frac{1}{2}$	2

174. **Multiplication by Aliquot Parts of 100.** Since goods are frequently priced at an aliquot part of $1.00, multiplication by aliquot parts of 100 is important.

Example. At $16\frac{2}{3}$ cents apiece, what is the cost of 276 pieces?
Solution: Since $\frac{1}{6}$ of $100=16\frac{2}{3}$, multiplying by 100 and dividing by 6 gives the same result as multiplying by $16\frac{2}{3}$. Hence the required cost is $27600\div 6=4600$ (cents), or $46.

EXERCISES

1. Give rules for multiplying by $33\frac{1}{3}$, $16\frac{2}{3}$, $12\frac{1}{2}$, $8\frac{1}{3}$, $6\frac{2}{3}$, and $6\frac{1}{4}$.

Write down the products of the following:

2. $33\frac{1}{3}\times 8765$
3. $12\frac{1}{2}\times 8468$
4. $8\frac{1}{3}\times 6754$
5. $6\frac{2}{3}\times 3480$
6. $6\frac{1}{4}\times 3480$
7. $3\frac{1}{3}\times 2910$
8. $2\frac{1}{2}\times 380$
9. $16\frac{2}{3}\times 762$
10. $12\frac{1}{2}\times 492$
11. $33\frac{1}{3}\times 1450$
12. $8\frac{1}{3}\times 3190$
13. 50×590
14. 25×380
15. $16\frac{2}{3}\times 740$
16. $6\frac{2}{3}\times 560$
17. $8\frac{1}{3}\times 1496$
18. $6\frac{1}{4}\times 2370$
19. $3\frac{1}{3}\times 896$
20. $12\frac{1}{2}\times 3460$
21. $2\frac{1}{2}\times 2940$
22. $12\frac{1}{2}\times 864$
23. $16\frac{2}{3}\times 2974$
24. $6\frac{2}{3}\times 89325$
25. $3\frac{1}{3}\times 22470$
26. $33\frac{1}{3}\times 7896$
27. $6\frac{1}{4}\times 798482$
28. $33\frac{1}{3}\times 987$

In extending bills, examples like those given below are of frequent occurrence.

WRITTEN EXERCISES

For each of the following make out a complete bill. Use your own name as buyer, and the name of a firm that you know as the seller. Extend each item and foot each bill. Extend as many items as possible mentally. (See page 74.)

1.

42 yds. at $33\frac{1}{3}$¢
76 yds. at $8\frac{1}{3}$¢
48 yds. at $6\frac{1}{4}$¢
16 yds. at $12\frac{1}{2}$¢
45 yds. at $16\frac{2}{3}$¢

2.

280 lbs. at $33\frac{1}{3}$¢
690 lbs. at 25¢
240 lbs. at 50¢
140 lbs. at $16\frac{2}{3}$¢
380 lbs. at $8\frac{1}{3}$¢

3.

320 yds. at 10¢
700 yds. at 20¢
340 yds. at 25¢
480 yds. at 50¢
2190 yds. at $8\frac{1}{3}$¢

4.

380 yds. at 50¢
160 yds. at $33\frac{1}{3}$¢
280 yds. at 25¢
460 yds. at $12\frac{1}{2}$¢
1240 yds. at $6\frac{2}{3}$¢

5.

750 yds. at 20¢
250 yds. at $12\frac{1}{2}$¢
380 yds. at $16\frac{2}{3}$¢
630 yds. at $33\frac{1}{3}$¢
380 yds. at $6\frac{1}{4}$¢

6.

1780 lbs. at $6\frac{1}{4}$¢
2550 lbs. at 25¢
180 lbs. at 50¢
430 lbs. at $16\frac{2}{3}$¢
600 lbs. at $12\frac{1}{2}$¢

7.

42 ft. at $2\frac{1}{2}$¢
64 ft. at $12\frac{1}{2}$¢
76 ft. at $33\frac{1}{3}$¢
106 ft. at $16\frac{2}{3}$¢
94 ft. at $8\frac{1}{3}$¢

8.

74 lbs. at $6\frac{2}{3}$¢
38 lbs. at $6\frac{1}{4}$¢
92 lbs. at $16\frac{2}{3}$¢
194 lbs. at $12\frac{1}{2}$¢
284 lbs. at $8\frac{1}{3}$¢

9.

218 yds. at 33¢
430 yds. at $8\frac{1}{3}$¢
390 yds. at 25¢
520 yds. at $6\frac{2}{3}$¢
950 yds. at 4¢

10.

816 pt. at $8\frac{1}{3}$¢
34 pt. at $6\frac{1}{4}$¢
240 pt. at $16\frac{2}{3}$¢
390 pt. at $33\frac{1}{3}$¢
294 ft. at $2\frac{1}{2}$¢

11.

57 lbs. at 50¢
89 lbs. at $12\frac{1}{2}$¢
290 lbs. at 25¢
1840 lbs. at 20¢
960 lbs. at $8\frac{1}{3}$¢

12.

290 pcs. at $12\frac{1}{2}$¢
370 pcs. at $16\frac{2}{3}$¢
425 pcs. at 50¢
970 pcs. at $8\frac{1}{3}$¢
1280 pcs. at $6\frac{1}{4}$¢

WRITTEN EXERCISES

Study again pages 88 to 91.

Find the percentage in each of the following:

	Base	Rate		Base	Rate
1.	14.5	3½%	7.	3260	15%
2.	73.80	3¾%	8.	4740	12½%
3.	204.60	3¼%	9.	293.60	25%
4.	43	4½%	10.	764.90	4%
5.	460	5%	11.	4721.80	30%
6.	738.75	6¼%	12.	574.00	3½%

In each of the following find the rate to the nearest hundredth of one per cent:

	Base	Percentage		Base	Percentage
13.	39	24	19.	1¾	⅚
14.	76	108	20.	3¼	2½
15.	2946	165	21.	26.3	21.6
16.	8000	570	22.	⅜	⅚
17.	470	55.1	23.	7/16	3/32
18.	55.1	121	24.	5/16	17/32

In each of the following find the base to the nearest hundredth:

	Rate	Percentage		Rate	Percentage
25.	6¼%	260	30.	6¼%	2460
26.	33⅓%	480	31.	3½%	840
27.	4¾%	5000	32.	3¼%	78
28.	4¼%	18600	33.	16⅔%	49
29.	5¼%	35000	34.	37½%	3690

Study again pages 138, 139.

WRITTEN EXERCISES

	Gross amount	Discount		Gross amount	Discount
1.	$380	20%, 15%	8.	43.70	35%, 10%
2.	1240	25%, 15%	9.	286.60	40%, 5%
3.	260	40%, 20%	10.	38.75	30%, 15%
4.	850	30%, 15%	11.	24.60	20%, 5%
5.	675	25%, 10%	12.	39.50	25%, 10%
6.	86.40	25%, 15%	13.	180.60	30%, 20%
7.	35.65	40%, 10%	14.	260.00	40%, 15%

175. **Single Discount Equal to Discount Series.** It may become important in some cases to decide what single discount is equivalent to a discount series. The following example shows how this is best done:

Example. What single discount is equivalent to a discount series of 35%, 20%, 5%?

Solution: Suppose the gross price to be $100.

$$\begin{array}{r} 5\underline{)65} \\ \underline{13} \\ 20\underline{)52} \\ \underline{2.6} \\ 49.4 \end{array}$$

First deduct 35% of $100, then 20% (1/5) of $65, leaving $52, and then 5% (1/20) of $52, leaving $49.4. Hence the required single rate of discount is $100-49.4=50.6$ (per cent).

Find the single rates equivalent to the following discount series:

15. 30%, 15%
16. 40%, 10%
17. 35%, 20%
18. 25%, 15%
19. 50%, 10%
20. 45%, 15%
21. 40%, 25%
22. 35%, 5%
23. 20%, 15%
24. 20%, 10%
25. 30%, 20%
26. 50%, 20%

176. Method Used in Computing Interest. When the time is a whole number of months the simplest method is to find the interest for one year, and then express the time as a fraction of a year, and multiply by this fraction. This is essentially the cancellation method. (See page 104.)

Thus, to find the interest on \$750 at 7% for 5 months, find 7% of \$750 and multiply the result by 5/12.

That is, 7% of \$750 = \$52.50, and $\frac{5}{12} \times \$52.50 = \$21.87\frac{1}{2}$.

Many examples of this kind may be solved orally.

Thus, to find the interest on \$600 at 5% for 4 months, find 5% of 600, which is \$30, and take ⅓ of \$30, which is \$10.

When the time is given in days, any one of the standard methods may be used. In some respects the bankers' method is the most convenient.

ORAL EXERCISES

In each of the following, find the interest:

	Principal	Rate	Time		Principal	Rate	Time
1.	100	6%	4 yrs.	13.	450	5%	9 mos.
2.	100	6%	3 yrs.	14.	275	6%	1 yr.
3.	100	6%	10 mos.	15.	3000	5½%	1 yr.
4.	100	6%	3 mos.	16.	2400	5½%	1 yr.
5.	100	6%	5 mos.	17.	2000	4%	3 mos.
6.	100	6%	13 mos.	18.	4500	4%	2 mos.
7.	300	7%	4 mos.	19.	500	8%	6 mos.
8.	500	5%	6 mos.	20.	225	5%	1 yr.
9.	800	6%	1 yr. 3 mos.	21.	500	5½%	6 mos.
10.	400	5%	3 mos.	22.	1000	5½%	1 yr.
11.	600	5%	2 mos.	23.	1000	5%	1 yr.
12.	1200	4%	1 mo.	24.	1500	6%	30 days

Study again on pages 104–107 the method selected for finding interest. Notice that whichever method you use, much of the work is done mentally. The form of the work should be such as to make checking easy.

WRITTEN EXERCISES

Find the interest on the following, the rate being 6%.

1. $1240, 43 days	5. $430, 81 days	9. $4160, 63 days
2. 760, 49 days	6. 3420, 41 days	10 3500, 52 days
3. 760, 49 days	7. 4800, 29 days	11. 9250, 39 days
4. 375, 73 days	8. 1200, 87 days	12. 6780, 84 days

Find the interest at 5% on the following:

1. $7400, 17 days	5. $580, 73 days	9. $4950, 39 days
2. 4750, 39 days	6. 42700, 102 days	10. 5970, 67 days
3. 3260, 78 days	7. 5600, 97 days	11. 150, 91 days
4. 890, 94 days	8. 8300, 42 days	12. 280, 84 days

Find the interest at 7% on the following.

1. $580, 71 days	5. $1325, 63 days	9. $9840, 81 days
2. 430, 84 days	6. 12860, 117 days	10. 9100, 92 days
3. 1670, 19 days	7. 3980, 37 days	11. 4380, 47 days
4. 1580, 89 days	8. 870, 48 days	12. 2760, 35 days

Find the interest at 5½% on the following:

1. $3740, 53 days	5. $4990, 43 days	9. $4370, 71 days
2. 6740, 78 days	6. 17400, 64 days	10. 2790, 108 days
3. 9435, 97 days	7. 3980, 37 days	11. 1380, 78 days
4. 12760, 82 days	8. 8560, 42 days	12. 4500, 27 days

GENERAL INTEREST TABLE (5%)

	1 0000	2 0000	3 0000	4 0000	5 0000
60 days	008219	016438	024658	032877	041096
61 days	008356	016712	025068	033425	041781
62 days	008493	016986	025479	033973	042466
63 days	008630	017260	025890	034521	043151
64 days	008767	017534	026301	035068	043836
65 days	008904	017808	026712	035616	044521
66 days	009041	018082	027123	036164	045205
67 days	009178	018365	027534	036712	045890
68 days	009315	018630	027945	037260	046575
69 days	009452	018904	028356	037808	047260

	6 0000	7 0000	8 0000	9 0000
60 days	049315	057534	065753	073973
61 days	050137	058493	066849	075205
62 days	050959	059452	067945	076438
63 days	051781	060411	069041	077671
64 days	052603	061370	070137	078904
65 days	053425	062329	071233	080137
66 days	054247	063288	072329	081370
67 days	055068	064247	073425	082603
68 days	055890	065205	074521	083836
69 days	056712	066164	075616	085068

Above is a portion of a general interest table, such as is used in banks. No decimal points are used in the table, but the computer is always able to locate it properly by noting the amount of the principal. Thus, the interest on $1.00 at 5% for 60 days is $.008219, and not $.08219 or .0008219. The interest on $10 at the same rate and for the same time is $.08219. For $100 the interest is $.8219, for $1000 it is $8.219, and for $10,000 it is $82.19.

Similarly, the interest on $7000 at 5% for 67 days is $64.247, while the interest on $7 for this time and rate is $.064247.

This table gives the exact interest.

Example. Find the interest on $8495 at 5% for 67 days.

Solution:	Interest for $8000 =	$73	425
	Interest for 400 =	3	6712
	Interest for 90 =		82603
	Interest for 5 =		045890
	Total............	$77	968120

If a little care is taken it is not necessary to write down more than two figures to the right of the decimal point.

73	43
3	67
	83
	04
77	97

Thus, instead of .82603, we write .83, and instead of .045890, we may write .05. However, if one number is increased in this manner, the next number should not be. That is, we may write .04 instead of .05, though the latter is a closer approximation to .04589.

In case of a number like .425 we may write 43 or 42, as seems best suited to balance the other approximations.

WRITTEN EXERCISES

Use the table on page 182 in solving the following problems:

	Principal	Rate	Time		Principal	Rate	Time
1.	$8200	5%	67 days	11.	$1282	5%	64 days
2.	450	5%	61 days	12.	3490	5%	69 days
3.	2900	5%	60 days	13.	6520	5%	63 days
4.	6400	5%	62 days	14.	5173	5%	66 days
5.	2560	5%	68 days	15.	8460	5%	61 days
6.	5930	5%	69 days	16.	2940	5%	67 days
7.	10800	5%	65 days	17.	785	5%	69 days
8.	18690	5%	67 days	18.	1374	5%	63 days
9.	34280	5%	64 days	19.	2315	5%	64 days
10.	964	5%	69 days	20.	5246	5%	68 days

177. Promissory Notes. Read again the definitions and explanations on pages 134–137.

ORAL EXERCISES

1. What is meant by a "promissory note"? Name its three essential elements.

2. What is meant by the *maker*, or *drawer*, of a note?

3. What is meant by a negotiable note? A non-negotiable note? Is the following note negotiable? Why?

$1500 August 17, 1917

Ninety days after date I promise to pay

to the order of Clifton R. Holmes

Fifteen Hundred and $\frac{no}{100}$ DOLLARS

At First National Bank, Chicago, Illinois

Value received.

No._____ Due _____

SAMUEL D. BRISTOL

4. If this note is to be sold, how must it be endorsed?

5. If instead of "I promise to pay to the order of," the note read, "I promise to pay to," would the note be negotiable?

6. A note reads, "I promise to pay to bearer." Is it negotiable?

7. What is meant by *date of maturity* of a note?

8. What is a "time note"? A "demand note"?

9. Does the above note bear interest before it is due? Why? Does it bear interest after it is due? At what rate? (See pages 102, 136.)

WRITTEN EXERCISES

1. Make a negotiable promissory note for $250, payable in 60 days to R. L. Moore. Sign your own name to it.

2. Make a note like the preceding one except that it shall be non-negotiable.

3. Make a similar note payable to bearer.

4. Make a note payable to yourself and endorse it in blank. If a note is endorsed in blank, to whom is it payable?

5. Make a note payable to yourself, and endorse it to John Smith, so as to make it negotiable.

6. Make a note like the preceding and endorse it so that it shall be non-negotiable. Is this note negotiable before you endorse it? What is the purpose of making a note non-negotiable?

7. Endorse a note so that you will not be liable in case the maker of the note fails to pay it when due.

8. Write a "Demand Note," and explain how it differs from a "Time Note."

9. The face of a note is $900; time, 90 days; rate of interest, 6%. Find the amount at the date of maturity.

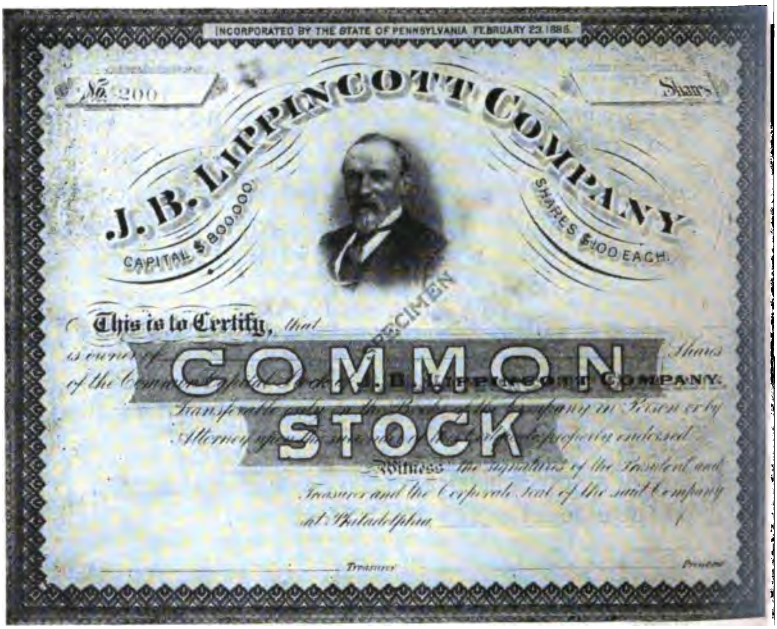
10. A note for $1200 bearing no interest is discounted at 6%, 60 days before it is due. Find the proceeds of the note.

11. A note for $2500, due in 90 days from the time it is made, bears interest at 7%. What will be the amount? If this note is discounted by a bank 30 days before it is due, find the proceeds if the rate of discount is 8%.

12. A note for $3600 made July 20th, 1915, is due November 22d, 1915. The rate of interest is 6%. On September 22d it is discounted at the bank. Find the proceeds if the rate of discount is 7%.

A STOCK COMPANY

178. A Stock Certificate. Below is a blank stock certificate. When filled out such a certificate declares that the person whose name appears on it is the owner of a certain number of shares of the J. B. Lippincott Company's stock, and is therefore entitled to a certain share of its net earnings.



179. Organizing a Stock Company. Some men in a small city wish to build a factory, and decide to organize a *company*. They issue a certain amount of what is called stock, say $100,000. That is, they arrange for 1000 shares, each representing $100, and sell these to whoever wishes to buy them. If one man buys 100 of these shares, that does not mean that the business owes him a certain amount, but simply that he is the owner of one-tenth of the property.

180. **Par Value of Stock.** The shares call for $100 each. This is expressed by saying that their *par value* is $100. If it appears that the business is going to be very prosperous, people may be willing to pay more than $100 a share. In that case the shares are said to be *above par*, or *at a premium*. If, on the other hand, the prospects are not so good, the shares may sell at less than $100 apiece. The shares are then said to be *below par*. If a share is sold for just $100, it is said to sell *at par*.

181. **Importance of Stock Companies.** A very large part of the business done in this country is now carried on by companies of this sort. (The Pennsylvania Railroad system has over $400,000,000 in stock issued, and the United States Steel Corporation has $1,000,000,000). As a concern enlarges its business, additional capital stock is issued, and sold to obtain money with which to carry it on.

182. **Dividends on Stocks.** From time to time the earnings of a company are divided among those who own the stock (the stockholders). Money so paid to the stockholders is called *dividends*. Dividends are always counted as so many per cent of the *par value* of the stock. Thus, if a company whose capital stock is $100,000 divides $6,000 among the stockholders, the persons owning stock get $6 for each share. This is spoken of as a 6% dividend.

183. **Why Stocks are Above or Below Par.** When a company has become well established, and is earning enough to pay all its expenses, and besides to pay a good dividend, say 6% per annum, the stock is likely to sell above par. When interest on money invested on good security is about 5%, people are willing to pay more than $100 for a source of income yielding $6.00 a year. If you watch the papers, you will read accounts of the sales of stocks, and you will notice that the prices are constantly changing.

184. Meaning of Rate in Stock and Bonds. In problems on stocks and bonds, as in all other applications of percentage, we are dealing with three numbers: *base, rate,* and *percentage,* two of which are given and the third is to be found.

An element of confusion in problems on stocks and bonds is that the *rate* is used with two different meanings.

(1) When we say that a stock yields a *6% dividend,* we mean that a dividend of $6.00 is paid on each share of $100 *par value.*

(2) When we say that an investment in a certain stock yields a *6% income,* we mean that the dividend is equal to 6% of the *cost of the stock.*

If we understand this clearly, we shall have little trouble with problems on stocks and bonds.

185. The Stock Exchange. In great cities, there are places called *stock exchanges,* where stocks and bonds are constantly bought and sold. Those who buy and sell on the stock exchange are called *brokers.* Brokers sometimes buy or sell on their own account, but more frequently for others who have requested them to do so. When a broker buys or sells a share for a customer, he gets a commission for so doing, usually ⅛ of one per cent of the par value of the shares bought or sold. Thus, a broker gets $12.50 for buying or selling 100 shares with par value of $100.00 each.

186. Changes in the Price of Stocks. The smallest change possible, in the price of a stock is ⅛ of a dollar. For this reason the only prices between $106 and $107 for which a share can be sold are 106⅛, 106¼, 106⅜, 106½, 106⅝, 106¾, and 106⅞. When the price on a share goes up or down by 1 dollar it is said to change by one whole *point.* The amount of selling and buying of shares may be inferred from the fact that in New York alone the total sales on the stock exchange are several million shares a week.

187. **A Comparative Study.** In all applications of percentage, we are dealing with three numbers: *base, rate* and *percentage*, two of which are given and the third is to be found. A study of the following table will help to identify these elements in the various applications of percentage:

Subject	Base	Rate	Percentage
Interest.........	Principal.........	Rate for one year	Interest for one year
Loan on note....	Face of note	Rate of interest	Interest for one year
Bank discount...	Face of note	Rate of discount	Discount for one year
Trade discount...	Gross price	Rate of discount	Discount
Commission....	Buying price Selling price	Rate of commission	Commission
Stocks and bonds	Par value of stock or bond	Rate of dividend	Dividend
	Cost of stock	Rate of income on investment	Amount of income
Any investment	Amount of investment	Rate of income on investment	Amount of income from investment

It should be encouraging to the pupil to know that a mastery of this table will go far in overcoming the difficulties of the applications of percentage. The pupil must not make the mistake of thinking that committing the words to memory means mastery. The pupil must learn to *understand the meaning and the why* of things.

ORAL EXERCISES

1. If any one of the three numbers, base, rate, or percentage is missing, tell how you would find it.
2. If any one of the three numbers, principal, rate, interest (for one year) is missing, tell how you would find it.
3. Ask and answer similar questions for loan on note, bank discount, commission, trade discount, stocks and bonds, any investment.

188. Stock Quotations. The following is a clipping from a Chicago paper:

NEW YORK STOCK SUMMARY FOR WEEK

		Sales	High	Low	Close	Net chg.
21	Am Sugar	70600	134¾	129	133½	+ ⅝
7	Am Sum Tobacco..	46600	115⅞	110	114½	+ 1
6	do pref	600	96	95	96	..
29	Am Tel & Tel......	14600	108⅛	*105¼	*106	— ⅛
4	Am Tobacco	2700	220	210	219⅝	+ 7⅝
14	do pref new......	400	101½	101¼	101½	+ ½
17	Am Woolen	38500	*117	*103	*116	+ 6½
8	do pref	300	*108	*108	*108	+ ¾
2	Am Writ Pap pref.	3800	47½	44	47½	+ 1
31	Am Zinc & Lead...	13500	23¼	19⅛	23	+ ⅝
21	do pref	1900	60	55	59	..
6	Anaconda	55100	73	69	72⅞	+ 1⅜
2	Assets Realization.	3800	2⅜	1½	2	+ ⅜
6	Asso Dry Goods....	6800	53	47	53	+ 2⅝
19	do 1st pref.......	300	76	74	76	+ 1
8	do 2d pref	300	76	75	75	— 1½
2	Asso Oil	1200	88½	87½	88½	— ¼
3	A T & S F..........	14400	100	98⅝	99½	+ 1⅛
2	do pref	1200	88	87½	87¾	+ ¼
31	A B & A..........	100	9	9	9	— ½
27	Atlantic C L	1400	104	100¼	100¼	— 3¾
8	A G & W I S S...	33800	181	166	179½	— 2½
29	do pref	200	72¼	72¼	72¼	+ ¼
29	Bald Locomotive...	145600	105	93⅛	103⅝	— 2¾

From this we see that during this week there were 70600 shares of American Sugar Co. stock sold on the New York Stock Exchange, that the highest price was $134.75 a share, that the lowest price of the week was $129.00 a share, and that the price at the end of the week was $133.50.

Notice that in the market quotations the price is given without the dollars sign, and also that the fractional parts of a dollar are expressed in common fractions and not in decimals.

1. What information do you find in this stock report about the following shares: American Telephone and Telegraph Co., Baldwin Locomotive Co.? In general, did the price of stocks rise or fall during the week?

2. In a daily paper look up the stock quotations for the past week or for the past day. Discuss whether prices of stocks are going up or down.

189. **Rate of Income on Investments.** The most important practical problem on stocks (and bonds) for persons other than speculators is to find the rate of income on an investment in stocks at a given price when the rate of dividend is known.

Example. What is the rate of income on an investment in stocks yielding 7% dividend if they were bought at $118.75, including brokerage?

Solution: The cost of one share is $118.75, and the income from it is $7. Hence the question is: $7 is how many per cent of $118.75?

The result is: $7 \div 118.75 = .05895 = 5.895\%$.

In problems of this kind, the rate is frequently figured to one-hundredth or even one-thousandth of one per cent.

WRITTEN EXERCISES

Find the rate of income to the nearest hundredth of one per cent on each of the following investments:

	Investment	Income		Investment	Income
1.	$104.625	$6.00	11.	$113.50	$6.00
2.	96.875	5.00	12.	189.75	8.00
3.	127.375	7.00	13.	137.00	7.00
4.	118.375	6.50	14.	87.625	5.00
5.	142.625	7.50	15.	35.75	3.00
6.	83.25	4.50	16.	97.00	6.00
7.	137.50	7.00	17.	130.375	8.00
8.	149.75	7.50	18.	86.50	5.00
9.	75.125	4.00	19.	94.875	4.50
10.	203.625	10.00	20.	70.00	5.00

190. Cost of Stocks. To find the cost of stock bought on the exchange brokerage must be added to the market quotation.

Thus, if American Telephone and Telegraph Co. Stock (see page 190) is bought at $108\frac{1}{8}$ (\$108.12½) its true cost to the buyer is \$108.125+\$.125 (brokerage) = \$108.25. This is more easily found by using common fractions. Thus, $108\frac{1}{8}+\frac{1}{8}=108\frac{1}{4}=108.25$.

Unless otherwise stated, brokerage is ⅛ (\$.125) per share.

ORAL EXERCISES

Find the cost to the buyer of stocks quoted as follows:

1. 96	4. 22⅜	7. 117⅛	10. 113⅞	13. 70
2. 95¼	5. 12⅞	8. 105¾	11. 123½	14. 2½
3. 22¾	6. 105¼	9. 111½	12. 123¾	15. 116

191. Rate of Income on Investment in Stocks. To find the rate of income on an investment in stocks is now easy.

Example. A stock yielding 6½% dividend is bought at 123½. What is the rate of income on the investment?

Solution: The cost of the stock is $123\frac{1}{2}+\frac{1}{8}=123\frac{5}{8}$. Hence the rate of income is $6.5 \div 123.625 \doteq .05258 = 5.258\%$.

WRITTEN EXERCISES

Find the rate of income on investments in stock at the following quotations. Be sure to add brokerage to the cost of the stock.

	Quotation	Dividend		Quotation	Dividend
1.	98	5	6.	189⅝	8
2.	105¼	5½	7.	87.50½	4½
3.	121¾	6½	8.	130¼	7½
4.	54½	4	9.	142	7½
5.	70	6½	10.	205½	12

192. **Bonds and Coupons.** When a city, or state, or school district borrows money, it issues what is called a *bond*. Such bonds are usually issued in denominations of $1000 each. If a $1000 bond bears interest at the rate of 5% this is $25 every six months. If the bond is of the kind called coupon bonds, there are attached to it little slips called coupons. Each coupon calls for $25.00, and one is due every six months. When a coupon is due it may be cut off and presented for payment. In practice, the coupons are deposited in banks for collection, thus saving their owners time and trouble.

There is another kind of bond called a registered bond, which has no coupons attached to it. The owners of registered bonds receive checks for the interest whenever it is due. Liberty Bonds were issued both as coupon bonds and registered bonds, and specimens of both kinds may readily be obtained for inspection.

Business concerns, such as railroads, issue bonds as well as stock. Bonds, differ from stocks in that they bear definite rates of interest, while the income from shares of stock varies with the prosperity of the company. If a company has outstanding both stocks and bonds, the interest on the bonds must be paid before any money is set aside to pay dividends on the stock.

National Governments, states, cities, counties, and private corporations have borrowed tremendous sums of money on bonds. Bonds are constantly sold and bought in all of our larger cities. On the New York Stock Exchange alone bonds to the value of $20,000,000 are frequently sold in one week.

193. **Preferred Stock.** Railroads and other private corporations also issue a kind of stock called *preferred stock*. The preferred stock is like the bonds in that it bears a definite rate of interest, which must be paid before any money is set aside to pay dividends on the other stock, which is called *common stock*. However, interest on bonds must be paid before any dividend is declared even on the preferred stock.

194. The Market Value of Bonds. The price at which a bond sells depends upon the rate of interest it bears, the certainty that it will be paid when due, the length of time before it is payable, and upon the conditions of the money market.

WRITTEN EXERCISES

Find the rates of income in the following, adding brokerage to the given price in each case.

	Price of bond	Rate of Dividend		Price of bond	Rate of dividend
1.	$112.50	4%	7.	$97.00	5%
2.	102.50	5%	8.	129.50	6%
3.	94.75	4½%	9.	124.50	6%
4.	97.00	4%	10.	168.75	7%
5.	96.00	4%	11.	155.50	5%
6.	94.50	4%	12.	225.87½	10%

195. Price of Bonds Influenced by the Time They Have to Run. There is one element entering into the price of bonds, a full consideration of which is too difficult for us at this stage. The general rule is that a long period tends to depress the price of a bond bearing a low rate of interest, and to raise the price of one bearing a high rate of interest.

196. How Corporations Obtain Money. Besides obtaining money from the sale of stocks and bonds, corporations borrow large sums on short term notes. This is rapidly becoming a favorite method for securing funds.

197. Importance of Corporations. According to the census of 1910, 79% of the value of manufactured products was produced by corporations. Establishments each reporting products valued at $1,000,000 or over produced 43.81% of the total. The work of transportation and mining is practically all in the hands of corporations.

198. **Thrift Stamps. Savings Stamps.** United States Thrift Stamps may be bought in denominations of 25 cents, and Savings Stamps in denominations of $5.00. The interest on these stamps is about 4%. One person may not buy more than $1000 worth of Savings Stamps in one year.

199. **Postal Savings Banks.** Small amounts, not more than $100 a month, and not more than $500 for one person, may be deposited in the Postal Savings Banks. This affords the greatest possible security, but the rate of interest is only 2%.

200. **Savings Banks.** Deposits in a savings bank carry from 3 to 4% interest. The security is usually very high, and the money may be had practically at any time.

201. **Stocks and Bonds.** Stocks and bonds may be bought at any time in the open market.

202. **Mortgages.** Money may be loaned on real estate mortgages, and on personal notes. Before investing in a mortgage, one should make sure that there is no other mortgage on the same property. The present value of the property and its probable future value should be examined.

203. **Worthless Stocks.** There is an extensive sale of worthless stocks. Shares are sold in mines which are solid granite mountains; in orange groves which are only pleasant dreams; in irrigated tracts in the West which are as dry as the Sahara. There are hundreds of men in the United States prisons for using the mails to advertise worthless stocks.

Before buying stocks and bonds of a private corporation one should know well its general financial standing.

Any proposition for investment which promises higher returns than the ordinary is probably bad. There is plenty of money seeking investment at the highest safe rate, and those on the "inside" would quickly take advantage of any unusual opportunity.

LIFE INSURANCE

204. A Life Insurance Policy. A contract by which, for a certain consideration, called a *premium*, a company agrees to pay a certain sum in a specified number of years or on the death of the person who is insured, is called a *life insurance policy*. The company which agrees to make the payment is called a *life insurance company*.

205. An Ordinary Life Policy. If the policy is payable on the death of the insured, and if the premiums continue until his death, it is called an *ordinary life policy*.

206. An Endowment Policy. If the policy is payable at the death of the insured or in a certain number of years, in case the insured lives that long, it is called an *endowment policy*.

In case of a 20-year endowment policy the payments continue for 20 years, if the insured lives that long, and at the end of this period the amount of the policy is paid.

207. A Limited Payment Life Policy. If the policy is payable on the death of the insured, but the payments continue only for a fixed number of years, it is called a *limited payment life policy*.

Following is part of a table giving the premiums on three different kinds of policies charged for each $1000 of insurance by a prominent American life insurance company.

	LIFE POLICIES	ENDOWMENT POLICIES	
Age	Payments to continue for life	Payments to continue for twenty years	Payments to continue for twenty years
20	$18.50	$26.10	$48.20
25	20.50	28.10	48.70
30	23.30	31.10	49.60
35	27.10	35.00	50.90
40	32.20	39.80	53.00
45	39.10	46.20	56.40
50	48.50	54.80	62.00

208. Increase in Life Insurance. The amount of life insurance in the United States has increased greatly in the last twenty years, as is shown by the following table:

The insurance in force means all the insurance the companies would have to pay if all the insured were to die at once.

	Insurance in Force	Assets of Companies
1843	$36,500,000	$1,000,000
1867	1,235,000,000	124,534,000
1892	4,898,000,000	907,441,000
1899	6,266,000,000	1,576,000,000
1912	18,003,000,000	4,164,000,000
1915	21,730,000,000	5,190,000,000

Besides this there were in 1915 mutual assessments companies having 8,665,000 members, and carrying $9,491,000,000 insurance.

WRITTEN EXERCISES

1. Find the amount of assets held by the insurance companies for each $1000 insurance for each of the years given in the table.

2. Using the table on the opposite page, find the yearly premium which a man of 25 must pay for a whole life policy for $5,000.

3. A man 25 years of age takes out a $10,000 twenty-year endowment policy. The dividends declared by the company amount to 17% of the total premium. How much more does he get from the company than he paid in in premiums?

4. If in the preceding problem the man had placed the amount of the premium (not deducting the company's dividends) in a savings bank, each year, how much would the premiums amount to at the end of the 20 years, if interest at the rate of 4% is compounded annually by the bank.

Suggestion: By the table on page 132 find the amount of the first premium at the end of 20 years, of the second at the end of 19 years, and so on. Add all these amounts.

209. Property Insurance. There is a large variety of loss of property against which insurance may be bought. Loss of buildings and other property by fire, loss of ships and cargoes by the perils of the sea, loss of crops by hail, floods, draughts, or insects, loss of trade by bad weather on a certain day, loss of property by war—these and a great many other kinds of loss may be covered by insurance.

The most common kinds of property insurance are marine insurance and insurance against loss by fire.

210. Fire Insurance. An ordinary fire insurance policy is a contract to pay all loss up to a certain amount which may arise from the total or partial destruction by fire of a certain specified property.

The policy also covers any loss due to the use of water or any other legitimate means used in extinguishing the fire.

Thus, if a house insured for $10,000, is damaged to the extent of $5800 from fire, and the destruction incidental to the work of extinguishing the fire, then the company must pay the full $5800. If the house is damaged to the extent of $12,000, the company pays $10,000, the full amount of the policy, and no more.

211. Premiums. The amount paid by the insured for the insurance is called the *premium*. The premium varies with the kind of property insured. It is usually computed as a certain per cent of the amount of the policy or as a certain number of dollars per thousand.

212. Open Policy. An open policy specifies the rate at which the policy is issued and the kinds of goods insured, but does not specify the amount of property insured.

Thus, wheat in an elevator may be insured without stating how much wheat is in the elevator. The amount of premium is then computed on the actual amount covered by the insurance from time to time.

Companies usually issue a three-year policy for 2½ times the cost of a one-year policy, and a five-year policy for 4 times the cost of a one-year policy.

YEAR ENDING JANUARY FIRST, 1916

Total premium paid in the United States, $419,361,346.
Total income of insurance companies, $459,361,260.
Fire losses, $221,701,359.
Dividends, $26,509,028.
Other expenses, $157,729,585.
Total insurance carried, $53,000,000,000.
Total assets of all companies, $874,000,000.

TOTALS FOR THE UNITED STATES

Year	Property loss	Insurance loss
1885	$102,818,796	$57,430,789
1890	108,993,792	65,015,465
1895	142,110,233	84,689,030
1900	160,929,805	95,403,650
1905	165,221,650	116,446,324
1910	234,476,650	140,400,000
1916	168,905,100	102,000,000
Total, 33 years	5,359,258,199	3,404,271,685

WRITTEN EXERCISES

1. What per cent of each of the fire losses given in the above table was covered by insurance?

2. At ½% what is the yearly premium on a $3500 policy? What is the premium for 3 years? For 5 years? The customary reductions for long-time insurance are made in both cases.

3. A house was insured for $12,000 at ⅛% premium. After paying four yearly dividends, the house valued at $15,000 is destroyed by fire. What is the loss counting the premiums?

4. A family insured personal property for $2500, at ½%. After paying 9 yearly premiums, they sustained a loss of $460. Did they gain or lose by the insurance, and how much? Do not figure interest on the premiums.

213. What our Taxes are Spent for. Numerous expenditures are made by what are called *public corporations*, such as cities, towns, townships, counties, and states. Public schools are maintained by the town, city or state; roads are built and kept in repair; courthouses are erected for the safe keeping of documents and the transaction of public business; courts of law are maintained; prisons are built and cared for. There are asylums for the insane, homes for the destitute, and sanatoriums for the sick.

In the cities, streets are paved and kept clean. If a heavy snow falls, teams and men are set to work to remove it. Garbage and ashes are removed; public parks are provided and kept in order; there is a police force to protect people against lawlessness.

The Army and Navy are kept up. There are pensions for old soldiers or their widows. Salaries are paid to all United States officials, from the President down. Rivers and harbors are improved, canals are dug, and many other public works are undertaken.

214. Why Taxes are Increasing. As civilization advances, there is an increasing number of things for which the town, city, or state must spend money. In primitive days we built our own roads, fought our own fires, and protected ourselves as best we could against thieves and murderers. All these are now done by the State. That proportion of our expenditures which we may call *collective expenditures* is constantly on the increase. We all chip in, in the form of taxes, and the government spends our money more effecttively for our welfare than we could spend it individually.

EXERCISES

1. Make a list of things for which taxes are spent in your community. Do you benefit from any of these things?
2. Did the Indians pay any taxes? Were they better off than we are for this reason? Why?

215. **Amount of Public Expenditures.** The following tables show the increase in public expenditures. These tables are not brought down to date because of the unusual expenditure on account of the war.

United States Government:

	Population	Expenditure	Per Capita
1870	38,558,371	$293,657,000	
1890	62,947,714	297,736,000	
1910	91,972,266	659,705,391	
1915	100,000,000	731,399,759	

State of Illinois:

1870	2,539,891	$2,066,273.42	
1890	3,826,352	4,907,542.31	
1910	5,638,591	10,508,241.23	
1915	6,069,519	18,528,499	

City of Boston:

1870	250,526	$8,507,544	
1890	448,477	11,903,820	
1910	670,585	21,413,646	
1915	734,747	34,562,665	

WRITTEN EXERCISES

1. Find the *per capita* expenditures in each of the above. Compare these for the years 1870, 1890, 1910, 1917.

Any money collected by the state, county, town, or city, to meet public expenditures, is called a *tax*. With the increase in public expenditures, it is, of course, to be expected that the taxes will increase. At any event, the average total tax *per capita* must increase.

What was the reason for the comparatively large expenditure of the United States in 1870?

There is a large variety of ways for raising money for public expenditure. Some of these we now proceed to study.

216. Real Estate Tax. Land and buildings are called *real estate.* For the purpose of taxation real estate is usually regarded separately from other property.

Men called assessors make an estimate of the value of each piece of land, and the buildings on it. These estimated values are added, giving the total real estate *valuation* of the city. Suppose that in a certain small city this is $1,648,600.00. It is then decided how much tax to raise from real estate. Suppose this is $12,000.

The next step is to make what is called a *tax-table.*

If $12,000 is to be paid on a valuation of $1,648,600, we find by dividing $12,000 by 1,648,600, that each dollar of property must pay $.00728. We make a table showing how much must be paid on any whole number of dollars up to $10; multiples of 10, up to 100; multiples of 100, up to 1000; and so on.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	.00728	$10	.0728	$100	.728	$1000	7.28	$10000	72.80
$2	.01456	$20	.1456	$200	1.456	$2000	14.56	$20000	145.60
$3	.02184	$30	.2184	$300	2.184	$3000	21.84	$30000	218.40
$4	.02912	$40	.2912	$400	2.912	$4000	29.12	$40000	291.20
$5	.03640	$50	.3640	$500	3.640	$5000	36.40	$50000	364.00
$6	.04368	$60	.4368	$600	4.368	$6000	43.68	$60000	436.80
$7	.05096	$70	.5096	$700	5.096	$7000	50.96	$70000	509.60
$8	.05824	$80	.5824	$800	5.824	$8000	58.24	$80000	582.40
$9	.06552	$90	.6552	$900	6.552	$9000	65.52	$90000	655.20

The process of computing the amount of tax to be paid by each individual is called *spreading* the taxes.

Example. Find the tax on real estate assessed at $25,460 from the above table.

Solution:

Tax on	$20,000 =	$145.60
Tax on	5,000 =	36.40
Tax on	400 =	2.91
Tax on	60 =	.44
Tax on	25,460 =	$185.35

1. From this table find the tax on each of the following assessments: $13,340; $8,275; $3,300; $37,400

217. **Personal Property Tax.** Any property whatever, which is not real estate, is called *personal property*. It may consist of horses, carriages, automobiles, furniture, stocks and bonds, merchandise, cash, etc.

It is comparatively easy to decide the value of the real estate owned by a person, but it is difficult to find how much personal property he has.

It is now becoming customary to require each individual to fill out a schedule of all his personal property, and to swear to its substantial accuracy; and in case he fails to do so, the assessors are likely to make a high estimate of the value of his property.

When the valuation of the personal property has been made, the process of *spreading* the tax is the same as on real estate. A tax-table is made, and the tax of each individual computed.

218. **The Tax Rate.** In some localities property is assessed at its full value, while in other localities it is assessed as low as one-fourth its actual value. For this reason (and also for other reasons) the rate used in computing the tax varies very widely.

PROBLEMS

1. In a city, there is an assessed valuation of personal property amounting to $2,864,300. It is desired to raise a tax of $31,800 on this property. Make a tax table for this case, and find the tax on an assessment of $18,870.

2. In Boston the total assessed valuation of all taxable property in the year 1912 was $1,481,779,717. The tax rate was $1.64 per hundred dollars. Make a tax table for Boston for this year, and compute the tax on each of the following valuations: $86,760; $10,300; $8,435; $380,000.

3. Try to find the total assessed valuation of your own town or city, and also the amount to be raised by taxation. Make a tax table and find the tax on certain properties. See if you can actually help spread a tax in your locality.

219. Direct and Indirect Taxes. A tax which is paid directly by each individual to a representative of the government is called a *direct tax.* All other taxes are called *indirect taxes.*

Taxes on real estate and on personal property are direct taxes. We will now consider two kinds of indirect taxes.

220. Internal Revenue Tax. For every box of cigars made and sold in this country, a certain tax is paid to the United States Government. This is called an *internal revenue tax.* A similar tax is paid on a few other articles.

221. Import Duty. Many different kinds of goods bought in foreign countries and brought into the United States are subject to a tax as they enter this country. This is called an *import duty.*

Internal revenue taxes and import duties are *indirect taxes.*

222. Indirect Taxes Paid in the Form of Increased Prices. Indirect taxes are paid by the person who finally buys and uses the goods, not directly to the government, but in the form of increased prices. An example will show just what happens.

The maker of a certain grade of cigars pays an internal revenue of \$3 per thousand, which is regarded by him as an additional element in the cost of production. Suppose the cigarmaker figures on a profit of 25% on the total cost of production. He then adds to the price of the cigars the internal revenue tax plus the 25% of the three dollars, or \$3.75. Hence the manufacturer's price is increased by \$3.75 because of the \$3.00 revenue tax. The wholesaler who buys the cigars from him sells them to a retailer at a profit, say of 20%. This means a profit on the \$3.75 as well as on the rest of the manufacturer's price of the cigars.

So the wholesaler charges \$3.75+20% of \$3.75, or \$4.50 more for the cigars than he would if there had been no tax on them. Suppose the retailer's profit is 30%. Then his profit on the extra \$4.50 is \$1.50. Hence the price to the consumer who buys the

cigars from the retailer will be $4.50+$1.50=$6.00 higher than it would have been if there had been no revenue tax on the cigars. Thus the consumer of the cigars pays not only the three dollars revenue tax, but he pays another three dollars in the shape of profits on the tax.

This is an example of what happens in a greater or less degree in every case of indirect taxation.

It is obvious, of course, that the case cited here is ideal, and that exactly the figures used here would not be the real figures in any practical case. Cigars are sold at certain fairly fixed prices, such as 10 cents apiece, two for a quarter, $4.50, $5.00, and so on per box of 50. Nevertheless the case cited represents the essential elements in the situation.

An indirect tax is very expensive to collect.

PROBLEMS

1. If a revenue tax adds 60 cents to the manufacturer's cost, how much will be added to the consumer's cost if the manufacturer makes a profit of 20%, the wholesaler 15%, and the retailer 40%?

2. The revenue tax on a certain article is $1.10. How much will be added to the consumer's price if there are successive profits of 30%, 20%, and 40% to be paid?

3. If the import duty adds one dollar to the cost of a yard of cloth, how much will be added to the consumer's price if successive profits of 15%, 15%, and 25% are to be paid?

4. If the import duty on a certain automobile is $600, by how much will the retail price of a machine be increased thereby if it is subject to successive profits of 10% and 15%.

5. If the import duty on a certain grade of woollen goods is $.80 per yard, by how much will the retail price per yard be increased if the goods is subject to successive profits of 15%, 18%, and 30%?

223. **Specific and Ad Valorem Duties.** The table below gives the duties on some of the more important articles of import as fixed by the tariff of 1913. Import duty may be *specific*, or *ad valorem*, or both. A specific duty is a fixed amount to be paid on a certain article, regardless of the price at which it was bought. Thus, on a certain grade of gloves the duty is $2.50 *per dozen pairs.*

An *ad valorem* duty is fixed as so many per cent of the price of the goods at the place where they were bought, just before being imported. Thus, imported ready-made woollen clothing is subject to a tax of 35% of the price at which they are bought in the foreign country. When duties are given as per cent, they are *ad valorem* duties. Nearly all duties imposed by the act of 1913 are *ad valorem.*

Ready-made clothing (woollen)	35%
Carpets (velvet)	35%
Ready-made clothing (cotton)	30%
Shirts, collars, cuffs	30%
Tablecloths (cotton)	25%
Oilcloths for floors	20%
Tinplate	35%
Wire	15%
Copper	5%
Sewing silks	15%
Silk handkerchiefs	50%
Silk ribbons	45%
Painted china	55%
Printing paper	12%
Jewelry	60%
Trimmed hats	45%
Automobiles	30%
Cotton thread	22½%
Cotton stockings	40%
Oriental rugs	50%
Views of landscapes,	20¢ per lb.
Men's gloves	$2.50 per doz.
Olive oil	30¢ per gal.

TOPICS FOR DISCUSSION

1. Are any of the articles given in the above list imported and sold in your community?
2. See if you can find out whether any of the duties given above have been changed since the law of 1913 went into effect.
3. What is the purpose of levying import duties?

WRITTEN EXERCISES

Find the import duty in United States money on each of the following:

1. Imported 120 dozen pairs of men's gloves from Canada.

2. Imported 1480 gallons of olive oil from Italy.

3. An invoice from Paris containing:
   Silk yarns for 4790 francs.
   Sewing silks for 2800 francs.
   Silk ribbons for 3000 francs.
   Silk handkerchiefs for 7850 francs.
   (One franc is 19 cents.)

4. An invoice from Hamburg containing:
   Shirts and collars for 12350 marks.
   Cotton tablecloths for 7430 marks.
   (One mark is 24 cents.)

5. Velvet carpets for 2760 pounds sterling.
   Woolen clothing for 10890 pounds sterling.
   (One pound sterling is $4.8665.)

6. Tinplate for 4200 pounds sterling.
   Wire for 1750 pounds sterling.

7. 8490 pounds of landscape views.

8. Cotton thread, 890 pounds sterling.
   Painted china, 1060 pounds sterling.

9. Jewelry for 24800 francs.
   Painted china for 32640 francs.

10. Printing paper for $45,820.

11. Trimmed hats for 15600 francs.
    Oriental rugs for 15600 francs.
    Automobiles for 75350 francs.

PROBLEMS ON TAXATION

1. In a city, with an assessed valuation of $128,500,000, the sum of $1,150,000 is to be raised by direct taxes. Make a tax-table for this levy, and find the tax of each of the men whose assessed valuation are as follows:

George M. Snow.....	$27,400	H. L. Benton........	$78,400
A. S. Stone..........	153,000	D. S. Jordon.........	4,000
Arthur A. Needham..	95,000	E. L. Tait...........	1,900
Samuel E. Caruthers.	140,000	D. E. Pollock........	14,000
H. P. Northern......	45,300	N. J. Lewis..........	24,600
L. S. Andrews.......	3,000	R. T. Boynton.......	12,000
H. S. Severn........	214,000	G. F. Vincent.......	47,700
Carl Angstrom......	57,000	G. P. Mollet........	3,800
A. S. Gordon........	2,150	D. L. Peck..........	19,800
R. T. Sanders.......	15,400	L. C. Parker........	375,000

WRITTEN EXERCISES

2. The import tariff on men's gloves is $2.50 per dozen. By how much per dozen does this increase the retail price of the gloves, if profits of 20%, 25%, and 50% are made by the importer, wholesaler, and retailer, respectively? (See page 204).

3. What is the import duty on the following:
   Silk ribbons invoiced at 4500 francs.
   Silk handkerchiefs invoiced at 3400 francs.
   Sewing silk invoiced at 1650 francs.
   One franc is counted as $.193. (See page 206.)

4. A piece of jewelry is bought by an importer in Holland, the price being equivalent to $100. What is the import duty? (See page 206.)

5. The piece of jewelry in the preceding example is sold by the importer at a gain of 35%. What is the selling price?

224. The Personal Check. The simplest way to pay a bill at a distance is to send a personal check. (See page 123.) All that the payer of the bill needs to do is to write out his check and send it in an ordinary letter. One disadvantage is that when the man who receives the check deposits it in his own bank, that bank will usually not give him credit for it until enough time has elapsed to hear from the bank on which the check is drawn.

225. The Certified Check. Any bank will usually give immediate credit for a check which is "certified" by the bank on which it is drawn. To certify a check, the bank stamps "certified" across the face of it, and deducts the amount of the check from the maker's account. A certified check is good as long as the bank on which it is drawn is solvent.

226. The Banker's Draft. Another method of paying bills at a distance is to buy a *banker's draft.* A banker's draft is simply a check drawn by one bank on another bank.

A BANKER'S DRAFT

Feb. 1, 1917 No. 76

TRADERS' NATIONAL BANK, CHICAGO, ILL.

Pay to the order of Arthur E. Peck $450.00

Four Hundred Fifty and no/100 DOLLARS

The Commercial Exchange National
New York

C. B. Adams,
Cashier.

The banks usually make a small charge for issuing drafts, but this charge varies from time to time, the causes of the variation being too complicated to discuss here.

227. Commercial Drafts. One method of collecting money due at a distance is by means of a *commercial draft*. An example will show how this is done. Suppose Robert E. Hains, of Chicago, owes $800 to Stanley Roberts & Co., of Boston. A draft like the one shown on this page is prepared by Stanley Roberts & Co., and deposited by them in their Boston Bank. This bank gives Stanley Roberts & Co. credit for the face of the draft less whatever charge they may make for collecting. The Boston bank forwards the draft to a Chicago Bank for collection. Robert E. Hains is notified, and if he acknowledges the debt, and wishes to pay it, he at once sends a check to the Chicago bank. The Chicago bank then notifies the Boston bank that the draft has been paid, and that the proceeds are credited to the Boston bank. The draft used in the example is payable at once, and is called a *sight draft.*

A SIGHT DRAFT

$800 Boston, Mass., April 8, 1917

________ At sight ________ pay to

the order of the Newton National Bank, Boston,

Eight Hundred and $\frac{no}{100}$ DOLLARS

Value received and charge to the account of

Robert E. Hains, Stanley Roberts & Co.,
Chicago, Ill. Boston, Mass.

228. Bills of Lading Draft. The following example illustrates another use of the commercial draft. Walter Clark & Co., of Chicago, ship a bill of goods by freight to Gordon Smith & Co., of Omaha. A bill of lading is made out, which makes the goods deliverable in Omaha to the order of Walter Clark & Co.

Walter Clark & Company endorse the bill to their bank, attaching to it a draft on Gordon Smith & Co. for the amount of the shipment. The Chicago bank forwards the draft and bill to a bank in Omaha with which they do business. Gordon Smith & Co. are notified, pay the draft to the Omaha Bank, and receive a bill of lading signed over to them.

If Gordon Smith & Co. do not pay the draft they cannot get the bill of lading from the bank, and hence cannot get the goods from the railway company. This draft is called a *bill of lading draft.*

229. The Time Draft. Another kind of commercial draft is what is called the *time draft.*

Suppose that Samuel Crawford, of Syracuse, buys goods of James Worden, of New York City, on ninety days' credit.

A TIME DRAFT

No. 786	New York City, May 12th, 1918.
Ninety days after date	pay to
the order of the City National Bank, New York,	
Five Thousand Four Hundred and $\frac{no}{100}$	DOLLARS
$5400	Value received, and charge to the account of
To Samuel Crawford, New York	James Worden, New York.

This draft is made out by James Worden, and deposited in his New York Bank, which forwards it to a bank in Syracuse. Samuel Crawford *accepts* the draft by writing across the face of it:

"Accepted, May 20th, 1918.
Samuel Crawford."

This draft now becomes the promissory note of Samuel Crawford, and he must pay it when due.

230. Discounting a Draft. The discount of a time draft is computed on the face of the draft the same as the bank discount of a note (see page 127). The time for which the draft is discounted runs from the day the draft is discounted until it is due. Days of grace are included or not in accordance with local customs.

Example. Find the proceeds of a draft, face value $8600, due Sept. 7th, and discounted July 1st, the rate of discount being 6%.

Solution: The time from July 1 to Sept. 7th is 68 days or $\frac{68}{360}$ years $=\frac{17}{90}$ years. Hence the discount is, $\frac{17}{90}\times\frac{6}{100}\times\$8600=\$97.47$; and the proceeds are, $\$8600-\$97.47=\$8502.53$.

WRITTEN EXERCISES

Find the proceeds of each of the following drafts. Use the exact number of days, and 360 days as one year.

	Face	Date due	Date of discount	Rate of discount
1.	$2400	August 1st	June 7th	6%
2.	$1300	September 19th	July 9th	6%
3.	$540	July 31st	May 7th	6½%
4.	$7800	August 7th	June 9th	5%
5.	$39400	September 14th	June 14th	6%
6.	$490	October 7th	July 18th	4¾%
7.	$200	December 8th	August 13th	5½%
8.	$8300	July 14th	April 24th	5¼%

9. An automobile company in Detroit sells machines to a western dealer for $35800. The bill is payable in 90 days. In payment the company receives a time draft and discounts it 75 days before it is due. What are the proceeds of the draft if the rate of discount is 5½%?

231. Postal Money Orders. Payments of small bills, such as subscriptions for magazines and payments for mail orders are often made by *postal money orders.* The charges made by the Post Office for these orders are:

Amounts less than or equal to \$2.50	3 cents
Amounts greater than \$2.50 to \$5.00	5 cents
Amounts greater than \$5.00 to \$10	8 cents
Amounts greater than \$10 up to \$20	10 cents
Amounts greater than \$20 up to \$30	12 cents
Amounts greater than \$30 up to \$40	15 cents
Amounts greater than \$40 up to \$50	18 cents
Amounts greater than \$50 up to \$60	20 cents
Amounts greater than \$60 up to \$75	25 cents
Amounts greater than \$75 up to \$100	30 cents

A money order is made payable at a definite post office, and to a definite person. No other post office will pay the order, and the payee must be identified. A money order can be signed over only *once* to another person or institution. This enables one to deposit money orders in a bank. A money order is not issued for more than \$100. Large amounts are never sent by money order because the cost is high, .3%, and also because it would be inconvenient to handle \$100 money orders to pay one large sum.

232. Express Money Orders. Express companies sell *express money orders.* These have the advantage that they are payable at *any* office of the express company to a properly identified owner of the order.

233. Sending Money by Telegram or Cable. When instant payment is of great importance, payment is sometimes ordered by *telegram*, or by *cable* in case the payment is to be made overseas.

The rates for sending money by telegram may be learned from any telegraph office.

234. Travelers' Checks. The express companies and some bankers' associations issue what are called travelers' checks.

These checks are in denominations of $10, $20, $50, $100, and $200. The buyer signs his name on each check as he buys it; and when he wishes to use it, he identifies himself by duplicating this signature. The rapidity with which such a signature is always written, and the comparatively small amount of each check, makes forgery practically unknown. Hence, if such checks are lost, no one else can get the money on them, and the owner will eventually recover their value.

Travelers' checks are accepted all over the civilized world by steamship companies, railroads, hotels, and the larger business concerns. A charge of 1% of the value of the checks is usually made by the bank or company issuing them. They are always payable at par.

WRITTEN EXERCISES

A man buys travelers' checks amounting to $4500. After traveling six months, he turns in checks to the amount of $2000, which he has not used. If he had left the $2000 in the bank, the money would have drawn interest at the rate of 2% per year. How much did he lose by taking out the $2000 more than he needed on his journey, counting the charge for the checks?

235. **Service of the Banks in Paying Bills at a Distance.** The full reason why payments at a distance are made through banks is rather difficult to understand. We can acquire a partial comprehension of it, however, by studying a special case.

Throughout the year breadstuffs and meats are being shipped east from the North Central States. At the same time clothing, shoes, and other manufactured goods are being shipped into the North Central States from New England and the Middle Atlantic States. Each shipment is made on a separate order, and payment must be made for it. If cash were to be used to make each payment an immense amount of cash would be shipped in both directions. Indeed, it would be perfectly possible to have millions in actual money shipped from New York to Chicago, the same day that an equal number of millions were shipped from Chicago to New York.

This waste of work and expense is saved by the banks. If merchants in Chicago owe one million to merchants in New York, and at the same time other merchants in New York owe a million to merchants in Chicago, the banks, by means of their drafts, enable both millions to be paid without transferring a single cent in cash. The Chicago bank sells drafts on New York, and the New York banks sell drafts on Chicago, and these indebtednesses are cancelled off.

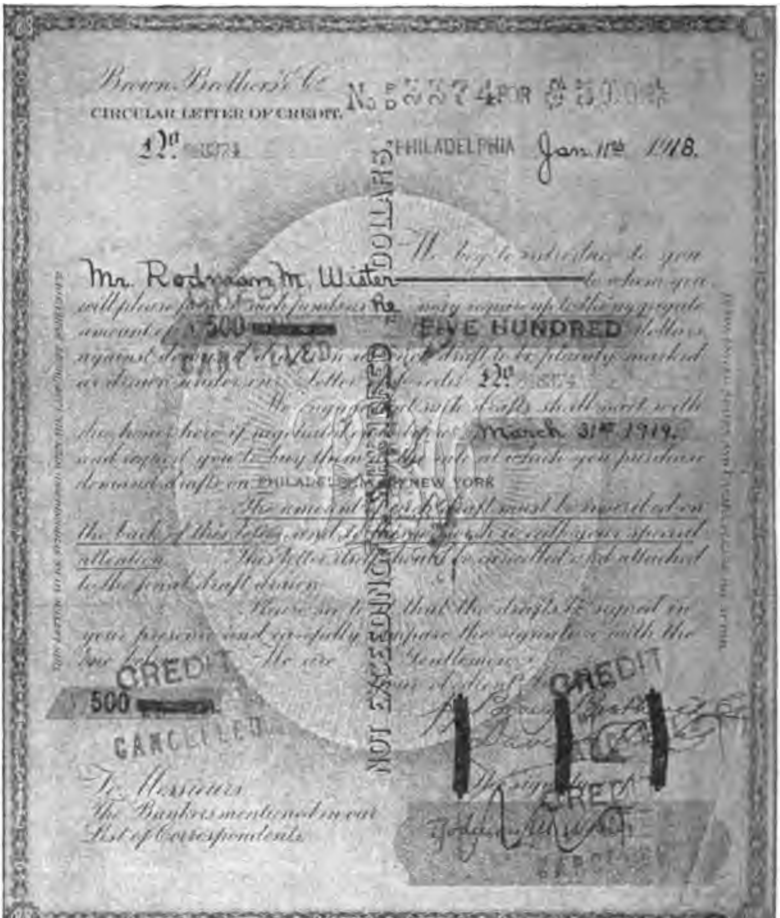
If during any comparatively short period, say one or two months, the payments to be made by Chicago people in New York is about the same in total amount as the payments to be made by New York people in Chicago, then no cash is sent.

If for any length of time the total of the payments to be made by Chicago in New York exceed by a considerable amount the total of the payments to be made by New York in Chicago, then, and only then, is money actually shipped from Chicago to New York.

TOPIC FOR DISCUSSION

Why are there many bills to be paid at a distance?

LETTER OF CREDIT



Brown Brothers & Co.

CIRCULAR LETTER OF CREDIT.

PHILADELPHIA Jan. 11th 1918.

Mr. Rodman M. Wister

500 FIVE HUNDRED

March 31st 1919.

PHILADELPHIA NEW YORK

NOT EXCEEDING DOLLARS

CREDIT 500 CANCELLED

To Messieurs

The Bankers mentioned in our List of Correspondents

The above is a copy of a real letter of credit issued to Mr. Rodman M. Wister and used by him.

236. Letters of Credit. People traveling either in this country or abroad and thus needing to obtain cash in many different places often carry what is called a *letter of credit.*

A letter of credit is a statement issued by a bank to the purchaser requesting bankers in various parts of the world to pay him any sum he wishes, up to a specified amount. The buyer of the letter of credit signs his name on the face of it.

When applying to a banker for money on this letter, Rodman M. Wister is required to make out a draft on the Brown Brothers & Co. for the amount. His signature is carefully compared with that on the letter of credit, and if found to be genuine, the money is paid over to him, and the amount noted on the back of the letter. In this way any banker who is requested to advance money on the letter will know just how much has been paid on it by other bankers.

ENDORSEMENTS ONCE MADE HEREON, OF PAYMENTS, MUST BE ALLOWED TO REMAIN WITHOUT ALTERATION OR ERASURE. CARE SHOULD THEREFORE BE TAKEN TO UNDERSTAND EXACTLY ABOUT PAYMENTS DESIRED, BEFORE INSCRIBING AMOUNTS BELOW.
ON THE PAYMENT OF ANY SUM EXHAUSTING THIS CREDIT, THIS LETTER MUST BE SURRENDERED BY THE HOLDER, AND ATTACHED BY THE BANKER NEGOTIATING THE DRAFT TO THE SAID DRAFT.

DATE WHEN PAID	BY WHOM PAID	NAME OF TOWN	AMOUNT PAID, EXPRESSED IN WORDS	AMOUNT IN FIGURES	
Mar. 8/18	Lehigh Valley Trust Co	Allentown Pa	Thirty Dollars	$30	00
Apr 30/18	Lehigh Valley Trust Co	Allentown Pa	Twenty-five Dollars	25	00
May [illegible]/18	LEHIGH VALLEY TRUST COMPANY	Allentown Pa	Thirty Dollars	30	00

The above is a copy of part of the endorsements made on the back of Mr. Wister's letter of credit by bankers who advanced money on it.

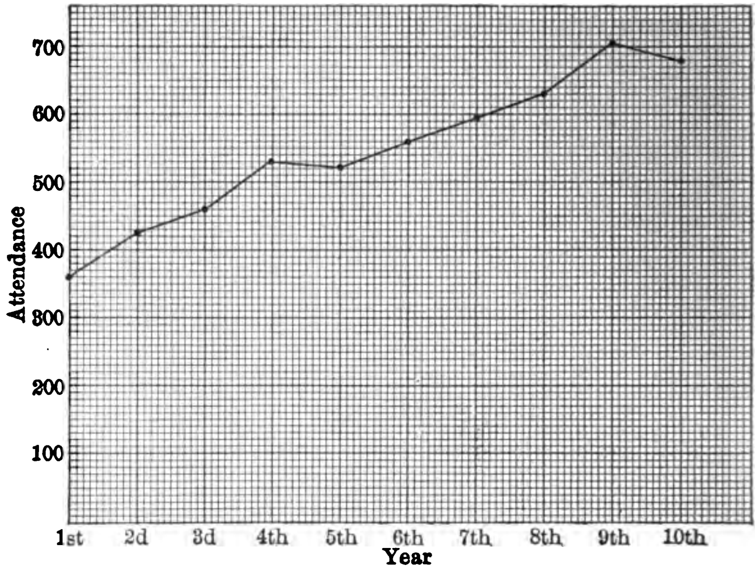
The banker who sells a letter of credit usually charges 1% on the face of it as his commission.

TOPICS FOR DISCUSSION

1. If a letter of credit is lost or stolen can anyone beside the owner get money on it?

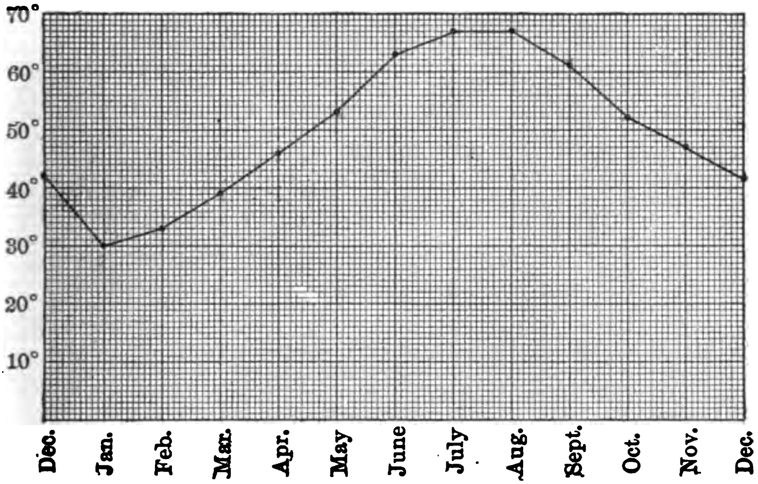
2. Compare the convenience of travelers' checks and letters of credit.

GRAPHIC REPRESENTATION



A certain grammar school building was finished just ten years ago. The total attendance the first year was 358; the second year, 424; and for the years following, 458, 531, 520, 560, 595, 630, 705, 680. This attendance is represented approximately in the diagram or *graph* shows on this page. The height of the curve shows the number of pupils, and the distance to the right from the left margin of the graph to any point on the curve shows the year represented by that point. Each small vertical space represents 10 pupils. The dots are placed as nearly in the right places as possible, and then connected as shown in the figure.

Find the total attendance in your own school for each of the last ten years. Get a large sheet of ruled paper, and make a graph like this to represent the growth of your school. Hang it up to show visitors how your school has been growing.

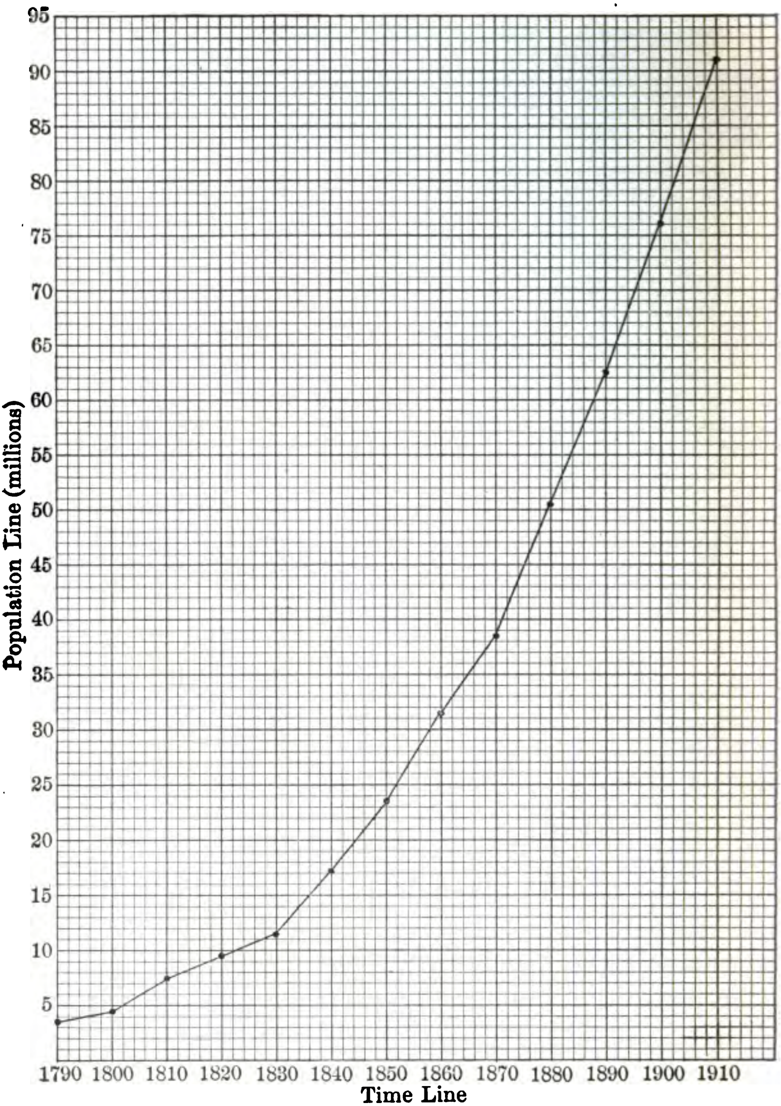


The average monthly temperatures for New York City beginning with January are: 29°, 33°, 39°, 46°, 53°, 63°, 67°, 67°, 61°, 52°, 47°, 41°. These temperatures are represented in the above graph.

1. The average monthly temperatures at St. Vincent, Minnesota, are: 5° below zero; 15°, 35°, 55°, 60°, 66°, 63°, 55°, 40°, 22°, 5°. Make a graph like the above to show the yearly change at St. Vincent. The teacher will help you in representing temperatures below zero.

2. Find the average monthly temperatures throughout the year at your home, and make a graph representing them. This graph will tell a good deal about the climate in your place. A really fine graph of this kind should be made and hung up either in your room or in the principal's office.

3. Find the hourly temperatures for one day in your vicinity, and make a graph to represent them. You can get these temperatures from a newspaper.


95
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Population Line (millions)
1790 1800 1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910
Time Line

The graph on the opposite page represents the population in the United States from 1790 to 1910. The dots represent the actual population as counted by each census, and the line drawn through these points represents to a fair degree of approximation the population for any year during this period.

From this graph answer the following questions:

1. In what year was the populations of the United States 10 millions? 12 millions? 15 millions? 20 millions? 30 millions? 40 millions? 60 millions? 80 millions?

2. What was the population of the United States in each of the following years: 1812, 1825, 1836, 1848, 1865, 1875, 1885, 1895, 1905?

3. During which decade did the population increase most rapidly? During which decade did it increase most slowly?

4. Ask and answer other questions which can be answered from this graph.

5. The population of the State of New York for each census since 1790 was: 340,000; 589,000; 959,000; 1,373,000; 1,919,000; 2,429,000; 3,097,000; 3,881,000; 4,383,000; 5,083,000; 6,003,-000; 7,289,000; 9,114,000.

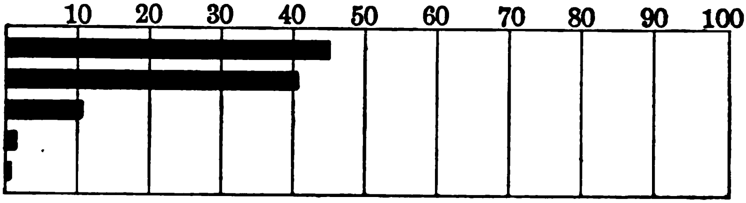
(These populations are given to the nearest thousand.)

Make a graph representing the increase in population in that state. Make each large vertical space represent a half million.

6. Find the population of your own state for each census since its admission to the Union. Make a graph to represent its increase in population during tbat time. You should make a large fine graph to hang up in your room.

7. Find the population of your own city for each census as far back as you can. Make a graph to represent the increase of population in your city. Select a convenient scale. If the city is small one large vertical space may represent 1000 population.

1. In every 100 persons in the world 45 are yellow, 41 are white, 11 are black, 2 are brown, and 1 is red.



2. Using horizontal bars as above, represent the world's commerce as given here.

1850	4 billions	1860	7.2 billions
1870	10.5 billions	1880	14.5 billions
1890	16.8 billions	1900	21.5 billions
1910	32.4 billions	1913	40.4 billions

3. The average incomes of 153 graduates of Princeton University in the class of 1901 were as follows: Represent these data bv a graph like that on page 220.

1902	$750	1907	$2400
1903	950	1908	2400
1904	1250	1909	2750
1905	1700	1910	3250
1906	2100	1911	3750

4. The average retail prices in the United States of 15 principal articles of food from 1890 to 1912. (Since the beginning of the war these prices have practically doubled). Represent these graphically.

1890	102.	1898	99.4	1906	122.4
1891	103.6	1899	100.5	1907	122.4
1892	101.7	1900	102.9	1908	132.5
1893	104.6	1901	109.2	1909	140.3
1894	99.5	1902	116.8	1910	148.5
1895	97.2	1903	116.9	1911	146.9
1896	94.9	1904	118.3	1912	157.9
1897	96.4	1905	118.3		

MISCELLANEOUS PROBLEMS

1. The following represents the average load of freight per train on one of the great American railways:

1879	195 tons	1888	360 tons	1897	483 tons	1906	1147 tons
1880	193 tons	1889	468 tons	1898	497 tons	1907	1132 tons
1881	214 tons	1890	444 tons	1899	644 tons	1908	1058 tons
1882	233 tons	1891	425 tons	1900	751 tons	1909	1192 tons
1883	259 tons	1892	493 tons	1901	758 tons	1910	1207 tons
1884	266 tons	1893	465 tons	1902	791 tons	1911	1159 tons
1885	299 tons	1894	469 tons	1903	951 tons	1912	1215 tons
1886	303 tons	1895	458 tons	1904	1012 tons	1913	1242 tons
1887	313 tons	1896	453 tons	1905	1078 tons	1914	1250 tons

Represent these data by a curve as on page 220.

In 1910 the total exports and imports of the United States were $1,744,984,720 and $1,557,819,988, respectively. In this year the exports exceed the imports by $187,164,732, or 12.0% of the imports.

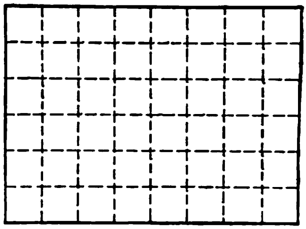
Year ending June 30	Total exports	Total imports	Per cent excess of exports over imports
1900......	1,394,483,082	849,941,184	....
1902......	1,381,719,401	903,320,948	....
1904......	1,460,827,271	991,087,371	....
1906......	1,704,864,500	1,226,563,843	....
1908......	1,860,773,346	1,194,341,792	....
1910......	1,744,984,720	1,557,819,988	....
1912......	2,204,322,409	1,653,264,934	....
1914......	2,364,579,000	1,893,926,000	....
1916......	4,333,659,000	2,197,884,000	....

2. In the above table find the per cent excess of exports over imports for each of the years given, and make a graph representing these per cents.

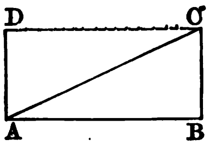
CHAPTER IV

Mensuration

237. Area of a Rectangle. The area of a rectangle is the number of square units contained in it, and is equal to the product of the length and width.

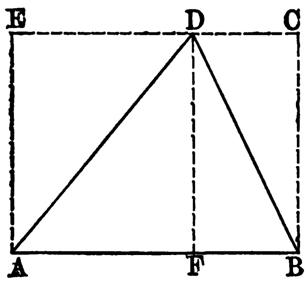


238. Area of a Right Triangle. A triangle is a right triangle if one of its angles is a right angle. The area of the right triangle ABC is one-half the area of the rectangle ABCD.



That is, the area of the triangle equals $\frac{AB \times BC}{2}$.

239. Base and Altitude of a Triangle. A line drawn from one vertex of a triangle perpendicular to the opposite side, is called an *altitude* of the triangle, and the side to which it is perpendicular is called a *base* of the triangle.



Thus, in the triangle ABC above, AB may be considered as a base and BC the altitude.

In the triangle ABD to the left, AB is a base and DF the altitude on it.

240. Area of Any Triangle. It is easy to see that the triangle ABD is one-half the area of the rectangle ABCE. Hence, the area of this triangle equals $\frac{AB \times DF}{2}$.

From this conclude that the following rule holds:

The area of any triangle is equal to one-half the product of its base and altitude.

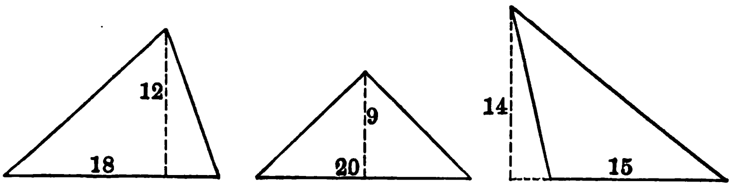
ORAL EXERCISES

Find the areas of the triangles having the following dimensions:

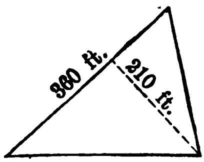
	Base	Altitude		Base	Altitude
1.	10	8	6.	30	10
2.	16	10	7.	8	20
3.	12	12	8.	24	12
4.	20	16	9.	100	50
5.	40	20	10.	12½	8

WRITTEN EXERCISES

1. Find the area of a triangle having a base of 15 inches and an altitude of 12½ inches.



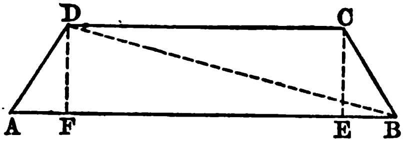
2. Find the area of each of the triangles represented in the above figures.



3. A triangular city block represented in the figure is sold for 45¢ a square foot. What is the selling price?

4. The base and altitude of a certain triangle are 24 and 16 respectively. Another side of this triangle is 18. Find the altitude on this last side.

241. The Trapezoid. In the figure ABCD, the lines AB and CD are parallel. Such a figure is called a *trapezoid.*



The parallel sides are called the *bases* of the trapezoid, and the perpendicular distance between them is called the *altitude.*

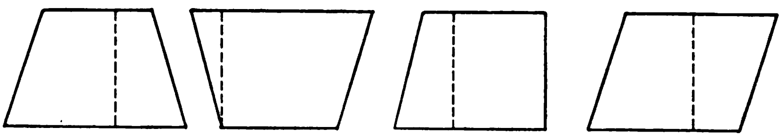
A trapezoid may be divided into two triangles as shown in the figure. The altitudes CE and DF of the triangles are equal to the altitude of the trapezoid. The area of the trapezoid is equal to the sum of the areas of the triangles, but the areas of the triangles are $\frac{AB\times CE}{2}$ and $\frac{CD\times CE}{2}$.

Hence the area of the whole trapezoid is

$$\frac{AB\times CE+CD\times CE}{2}=\frac{(AB+DC)CE}{2}.$$

The rule for finding the area of a trapezoid may now be stated as follows:

The area of a trapezoid is equal to one-half the sum of the bases multiplied by the altitude.



These figures show different shaped trapezoids.

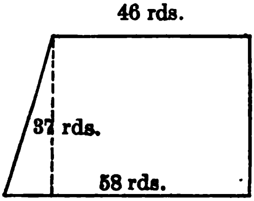
PROBLEMS

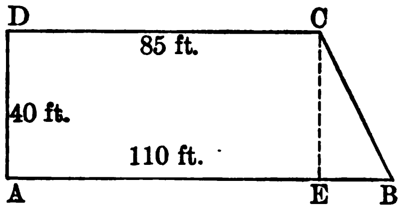
Solve as many as you can orally:

1. In the first figure, above, AB = 16, CD = 10, and CE = 8, find the area of the trapezoid, using the general rule stated above.

2. In the same figure find the areas of the triangles ABD and CDB and add them. Compare with the area found in Example 1.

3. Using the dimensions given in the first figure, below, find the area in acres of the trapezoid.





4. Find the area in square feet of the second trapezoid.

5. In the second figure find the length CE and EB. What is the area of the triangle BCE?

6. Find the area of the rectangle AECD and add to the area of the triangle BCE. Compare with the area found in Problem 4.

7. Find the area of a trapezoid whose bases are 24 feet and 18 feet respectively, and whose altitude is 12 feet.

8. The bases of a trapezoid are 12 feet and 8 feet respectively. What is its altitude if the area is 70 square feet?

9. The area of a trapezoid is 840 square inches, and its altitude is 14 inches. Find the sum of its bases.

10. If in the first figure on page 226, DC = 42, FD = 18, AF = 8, EB = 6, find the area of the trapezoid ABCD in two ways, and compare the results.

11. If in the first figure of page 226, AB = 60, AF = 6, EB = 4, EC = 21, find the area of the trapezoid in two distinct ways, and compare results.

12. Draw a trapezoid like the second figure on this page, and divide it into a rectangle and a triangle. Find the area of the trapezoid in two ways, if its bases are 36 and 48, and its altitude 16, and compare.

SQUARE ROOT

242. The Square of a Number. The result obtained by multiplying a number by itself is called the *square* of the number.

Thus, 4 is the square of 2, 9 is the square of 3, and $\frac{49}{16}$ is the square of $\frac{7}{4}$.

The square of a number is indicated by writing a small figure 2 a little above and to the right of it.

Thus, $2^2=4$, $3^2=9$, and $(\frac{7}{4})^2=\frac{49}{16}$.

243. A Perfect Square. A number which is the square of an integer or of a fraction is called a perfect square.

Thus, 4, 9, $\frac{49}{16}$ are perfect squares.

244. The Radical Sign. The square root of a number is indicated by placing the symbol $\sqrt{\ }$, called the *radical sign,* over it.

Thus, $\sqrt{4}=2$, $\sqrt{9}=3$, and $\sqrt{49/16}=\frac{7}{4}$.

245. Approximating $\sqrt{2}$. The number 2 is not a perfect square, and its root cannot be found exactly. We may, however, approximate it as nearly as we please.

Thus, $(1.4)^2=1.96$ and $(1.5)^2=2.25$. Hence $\sqrt{2}$ lies between 1.4 and 1.5.

Again, $(1.41)^2=1.9881$ and $(1.42)^2=2.0164$. Hence $\sqrt{2}$ lies between 1.41 and 1.42.

$(1.414)^2=1.999396$ and $(1.415)^2=2.02225$. Hence $\sqrt{2}$ lies between 1.414 and 1.425.

1.414 is the nearest approximation to $\sqrt{2}$ to three decimal places.

246. Approximating the Square Root of any Number. The first step in approximating the square root of a number is to find two numbers between which its square root lies.

In the case of small numbers we can easily find two consecutive integers between which the root lies.

Thus, $\sqrt{15}$ lies between 3 and 4; since $3^2=9$ and $4^2=16$.

Similarly, $\sqrt{34}$ lies between 5 and 6 because $5^2=25$ and $6^2=36$.

Example. Find two consecutive integers between which lies the square root of 78; also two such integers between which lies the square root of 246.

Solution: $\sqrt{78}$ lies between 8 and 9, because 78 is greater than $8^2=64$, and less than $9^2=81$.

Similarly, $\sqrt{246}$ lies between 15 and 16, because 246 is greater than $15^2=225$, and less than $16^2=256$.

ORAL EXERCISES

Find consecutive integers between which each of the following lies:

1. $\sqrt{45}$
2. $\sqrt{56}$
3. $\sqrt{66}$
4. $\sqrt{84}$
5. $\sqrt{91}$
6. $\sqrt{34}$
7. $\sqrt{7}$
8. $\sqrt{73}$
9. $\sqrt{4}$
10. $\sqrt{52}$
11. $\sqrt{94}$
12. $\sqrt{63}$

Since $10^2=100$, and $20^2=400$, the square root of any number between 100 and 400 must lie between 10 and 20.

Since $20^2=400$, and $30^2=900$, the square root of any number between 400 and 900 must lie between 20 and 30.

In each of the following, state whether the square root lies between 0 and 10, between 10 and 20, between 20 and 30, between 30 and 40, and so on.

13. $\sqrt{1240}$
14. $\sqrt{4216}$
15. $\sqrt{3290}$
16. $\sqrt{6060}$
17. $\sqrt{6845}$
18. $\sqrt{7491}$
19. $\sqrt{8265}$
20. $\sqrt{9180}$
21. $\sqrt{5628}$

Since $700^2=490{,}000$, and $800^2=640{,}000$, it follows that the square root of a number between 490,000 and 640,000 lies between 700 and 800.

In each of the following, state whether the square root lies between 0 and 100, between 100 and 200, and so on.

22. $\sqrt{18264}$
23. $\sqrt{236280}$
24. $\sqrt{459261}$
25. $\sqrt{583900}$
26. $\sqrt{678000}$
27. $\sqrt{892000}$

247. Method for Finding First Approximation to a Square Root. The method for finding a first approximation to the square root of a number is illustrated by the following examples:

To find a first approximation to $\sqrt{4297}$, divide the number into groups of two digits in each, beginning at the right. Thus, 42, 97. The square root of the first group to the left, or 42, lies between 6 and 7. Hence the square root of 4297 lies between 60 and 70. But 4297 is about midway between $60^2 = 3600$ and $70^2 = 4900$. Hence, 65 is taken as a first approximation.

To find a first approximation to $\sqrt{75982}$, divide the number into groups as in the first case, 7, 59, 82. The square root of the first group to the left, or 7, lies between 2 and 3. Hence $\sqrt{75982}$ lies between 200 and 300. But 75982 is considerably nearer $300^2 = 90,000$ than $200^2 = 40,000$. Hence 270 may be taken as an approximation.

To find an approximation to $\sqrt{.0798}$ divide the number into groups of two, beginning at the decimal point. Thus, .07, 98. Then .28 may be taken as a first approximation. Explain.

To find a first approximation to the square root of a number, use the following rule:

(1) *Divide the number into groups of two digits each, beginning at the right, or at the decimal point in case there is one.*

(2) *Find two consecutive integers between which lies the root of the left group, and annex to these numbers as many zeros as there are groups remaining.*

(3) *Estimate the root between these two numbers.*

ORAL EXERCISES

Find the first approximation to each of the following:

1. $\sqrt{3946}$	5. $\sqrt{60541}$	9. $\sqrt{91472}$
2. $\sqrt{5986}$	6. $\sqrt{14506}$	10. $\sqrt{242690}$
3. $\sqrt{13462}$	7. $\sqrt{450610}$	11. $\sqrt{5089}$
4. $\sqrt{84746}$	8. $\sqrt{5940642}$	12. $\sqrt{639486}$

Example. Find the square root of 5382.

$$74\overline{)5382}^{\,72.73} \qquad 73.37\overline{)5382}^{\,73.354} \qquad 73.362\overline{)5382}^{\,73.3622}$$

Solution: We estimate the root to be 74. Dividing 5382 by 74, we get 72.73+. Hence the root must lie between 74 and 72.73. We take half the sum of these numbers, or 73.37, as a second approximation. Dividing 5382 by this number, we get a quotient 73.354. Hence the root lies between 73.37 and 73.354. Half the sum of these numbers, 73.362, is the third approximation. Dividing again we get 73.3622. Hence the root lies between 73.362 and 73.3622. We take 73.3621 as the final approximation, which is certainly accurate to four decimal places.*

248. Rule for Approximating Square Root. To find the square root of a number use the following rule:

(1) *Find a first approximation.*

(2) *Divide the number by the first approximation.*

(3) *Take half the sum of the first approximation, and the first quotient as a second approximation and divide the number by it.*

(4) *Take half the sum of the second approximation and the second quotient, as a third approximation, and divide the number by it.*

(5) *Continue this process as far as may be necessary to attain the required closeness of approximation.*

The root always lies between the last divisor and the last quotient.

The approximation is accurate to at least as many decimal places as the last divisor and quotient are alike.

WRITTEN EXERCISES

Find correct to two places of decimals each of the square roots indicated on the opposite page.

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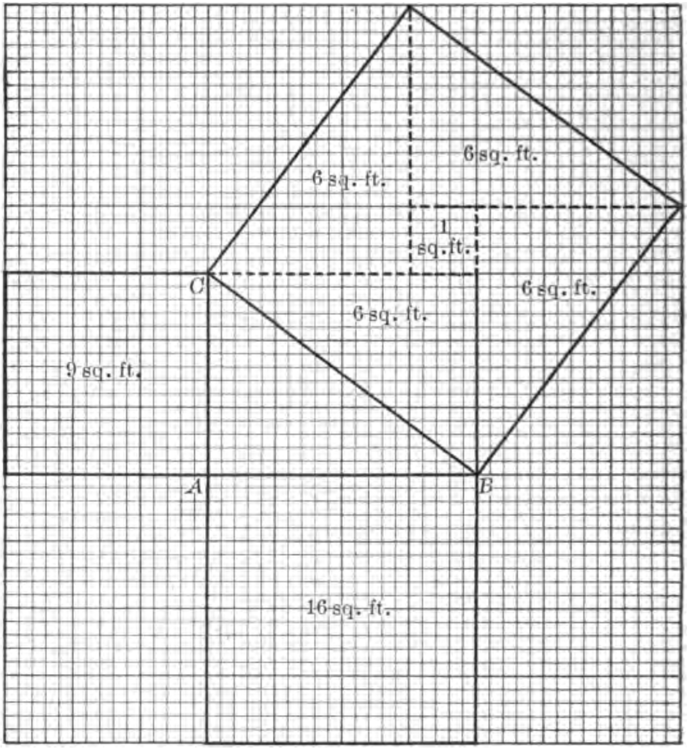
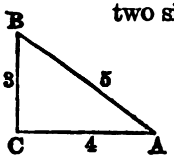
*Very few numbers whose square roots are needed in practice are exact squares. Hence their roots cannot be found exactly, and an approximate root is the only possible result to be found. The number of decimal places needed in the root depends upon the problem in which the finding of the root arises.

THE PYTHAGOREAN PROPOSITION

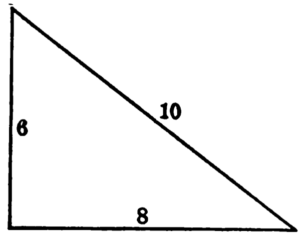


249. The Right Triangle. Squares are constructed on the three sides of the right triangle ABC. The sides AB and AC are 4 and 3 units long, and the squares on these sides contain 16 and 9 square units respectively. That is, the sum of these squares is 25 square units. By studying the figure we find that the square on the side BC (the hypothenuse) also contains 25 square units. Hence we find that in this triangle the square on the hypothenuse is equal to the sum of the squares on the two sides.

250. The Pythagorean Proposition. We have just found that if two sides of a right triangle are 3 and 4, then the hypothenuse is 5.



If two sides of a right-angled triangle are 6 and 8 respectively, then the hypothenuse is 10. Again, we notice that $6^2+8^2=10^2$.

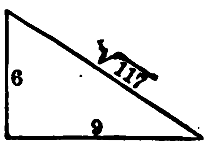


The following holds for all right-angled triangles:

The square on the hypothenuse of a right triangle is equal to the sum of the squares on the other two sides.

This was discovered by the Greek mathematician, Pythagoras, about 580 B.C. Hence, it is called the *Pythagorean proposition.* This is one of the most important, if not *the* most important, propositions in mathematics.

By this proposition we can find the length of the hypothenuse of a right triangle when the other two sides are known.

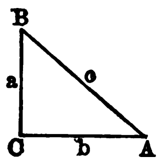


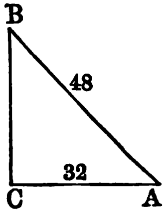
Thus, if the two sides are 6 and 9, then the square on the hypothenuse equals $6^2+9^2=117$. Hence to find the hypothenuse, we need only find $\sqrt{117}$.

WRITTEN EXERCISES

In the following, find each correct to two places of decimals:

1. If in the triangle ABC $a=10$, $b=9$, find c.
2. If $a=57$, $b=31$, find c.
3. If $a=142$, $b=178$, find c.
4. If $a=194$, $b=87$, find c.
5. If $a=638$, $b=76$, find c.

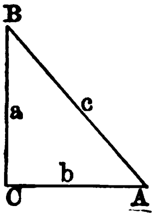




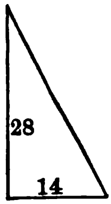
If in the triangle ABC, AB=48, and AC=32, we can find the square of BC by squaring 48, and from this square subtracting the square of 32. That is, $BC^2 = 48^2 - 32^2 = 2304 - 1024 = 1280$. Hence, $BC = \sqrt{1280} = 35.78$.

WRITTEN EXERCISES

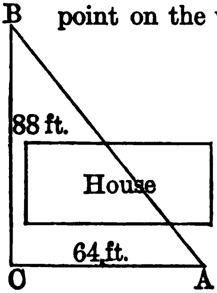
In the following find results to two places of decimals:



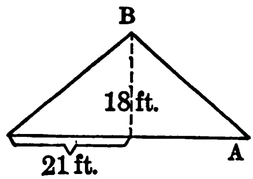
1. If c=74, b=39, find a.
2. If c=18.36, a=12.47, find b.
3. If c=259, a=187, find b.
4. If c=95.24, b=37.95, find a.
5. If c=39.43, b=14.65, find a.
6. If A=49, b=36, find c.



7. How long is a ladder which, when placed at a point 14 feet from the side of a house, reaches to a point on the wall 28 feet from the ground?

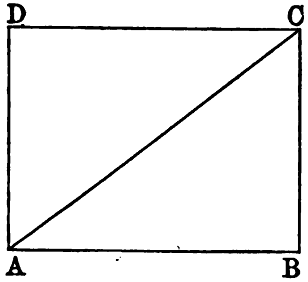


8. It is desired to find the distance from a point A to a point B. Between the points is a house, as indicated in the figure. The lines AC and CB are measured, the point C being so placed that the angle at C is a right angle. From the lengths given in the figure, find the distance from A to B.



7. A house is 42 feet wide; the peak of the roof is 18 feet above the side walls. How long must the rafter AB be if it extends $1\frac{1}{2}$ feet beyond the wall?

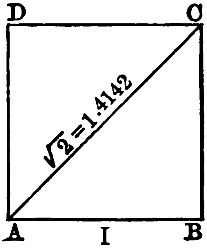
(*Suggestion:* Find the hypothenuse of the triangle, and add $1\frac{1}{2}$.)



251. **Diagonals.** A line joining opposite vertices of a four-sided figure is called a *diagonal* of that figure. In the figure ABCD, the line AC is a diagonal.

WRITTEN EXERCISES

1. Find the length of a diagonal of a unit square correct to four places of decimals.



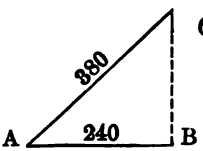
2. Find correct to three places of decimals the length of a diagonal of a rectangle 7 feet long and 4 feet wide.

3. Find the length of the longest cane which can be placed in the bottom of a trunk 42 inches long and 28 inches wide.

(*Suggestion:* $42^2+28^2=2548$. Find $\sqrt{2548}$.)

4. Find the length of a diagonal of a room 18 feet wide, and 28 feet long.

5. An athletic field is 600 feet wide and 800 feet long. What is the length of the longest straight-way running track which can be laid out on it.



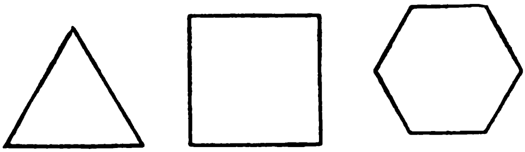
6. A kite string is 380 feet long. A boy standing at B directly under the kite is 240 feet from a boy at A holding the end of the string. How high above the ground is the kite?

7. A guy rope is fastened to a smoke stack 75 feet above the ground. How long must the rope be, if it is fastened to the ground 75 feet from the smoke stack?

8. An athletic field is 480 feet wide. How long must it be in order to permit a diagonal running track 800 feet long.

252. Regular Polygons. Figures like triangles, quadrilaterals, and figures having a larger number of sides are called *polygons*.

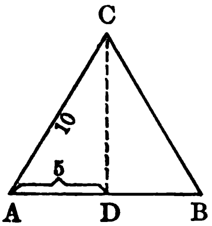
If all the sides of a polygon are equal to each other, and all its angles equal to each other, the polygon is said to be *regular*.



A regular triangle — A regular quadrilateral (square) — A regular hexagon (6-sided figure)

253. Area of Equilateral Triangles. A regular triangle is frequently described by merely calling it an equilateral triangle.

Example. Find the area of an equilateral triangle whose sides are 10 inches.



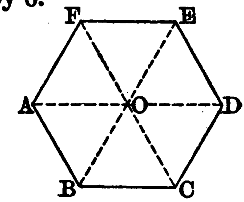
Solution: To find the area of the triangle we need to find its altitude. Draw CD perpendicular to AB. Then in the right triangle, ADC, AC = 10, AD = 5. Then $CD = \sqrt{10^2 - 5^2} = \sqrt{75} = 8.66$.

Hence, the area is $\frac{10 \times 8.66}{2} = 43.3$.

WRITTEN EXERCISES

1. Find the altitude of an equilateral triangle whose sides are 18 inches.
2. Find the altitude of an equilateral triangle whose sides are 24 inches.
3. Find the altitude of an equilateral triangle whose sides are 64 rods.
4. Find the area of an equilateral triangle whose sides are 36 rods.
5. Find the area of an equilateral triangle whose sides are 16 feet.

254. **Area of a Regular Hexagon.** A regular hexagon may be divided into six equilateral triangles as shown in the figure. Then its area may be found by finding the area of one of these triangles and multiplying this by 6.



255. **Center, Perimeter, Apothem of Regular Polygons.** Any regular polygon may be divided into equal triangles, as shown in the figure. The point O, where all the triangles meet, is called the centre of the polygon.

The total distance around the polygon is called the perimeter.

A line from the center, drawn perpendicular to one side of the polygon, is called its apothem.

256. **Area of any Regular Polygon.** To find the area of the polygon shown in the above figure, we may find the area of each triangle separately, and then multiply by the number of triangles.

Another method is to first add all the bases of the triangles, and multiply the sum by the common altitude, or the apothem. Half of this product is the area of the polygon.

Since the sum of the bases is the perimeter of the polygon, we have the following rule:

The area of a regular polygon is equal to one-half the product of the perimeter and the apothem.

The chief use of this rule is in connection with the area of the circle. This will be shown on the next page.

WRITTEN EXERCISES

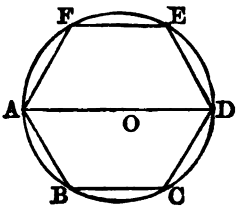
1. A side of a regular hexagon is 8 inches. Find its area.

(*Suggestion:* Find the area of an equilateral triangle whose sides are 8, and multiply by 6.)

2. Find the area of a regular hexagon whose sides are 15 inches.

257. The Circle. A figure formed by a curved line, every point of which is equally distant from a point within is called a *circle*.

The point within is called the *center*.



A straight line passing through the center and having both ends in the circumference is called a *diameter*.

258. Length of the Circumference. Suppose a regular hexagon is inscribed in a circle as shown in the figure. Evidently the circumference of the circle is longer than the perimeter of the polygon. Since the perimeter of the regular hexagon is just three times the diameter, it follows that the circumference is more than three times the diameter. $3\frac{1}{7}$ times the diameter is a pretty good approximation to the length of the circumference. When greater accuracy is required, the diameter may be multiplied by 3.1416 to find the circumference.

Measure the circumference of a circular object, and also its diameter (measure the distance around by using a string and then measure the string). Multiply the diameter by $3\frac{1}{7}$ to see how nearly the product equals the circumference.

WRITTEN EXERCISES

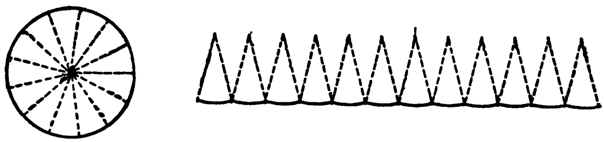
1. The diameter of the driving wheels of a locomotive is 78 inches. Find the circumference.

2. The diameter of a great fly-wheel is 18 feet. How far does a point on its rim go when the wheel makes one complete revolution?

3. The circumference of a wheel is 14 feet 8 inches. What is its diameter?

4. Find the circumference of an automobile tire whose diameter is 35 inches. How far will such a tire roll in making one revolution?

259. **Area of a Circle.** To find the area of a circle draw radii as shown in the figure, dividing it into pieces very nearly like triangles. If these are regarded as triangles with bases equal to the length of the arc, and altitudes equal to the radius of the circle we shall get an area which is very nearly the area of the circle. To get the area of all these triangles, we multiply the sum of their bases by half their altitude.



The sum of the bases is the circumference of the circle. Hence, we have the rule:

The area of a circle equals the circumference multiplied by half the radius.

Compare this with the rule for finding the area of a regular polygon.

If we denote the radius by r, then the circumference is $3.1416\times 2r$, and the area is $\frac{1}{2}\times r\times 3.1416\times 2\,r = 3.1416\,r^2$.

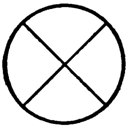
The number 3.1416 is usually denoted by the Greek letter π.

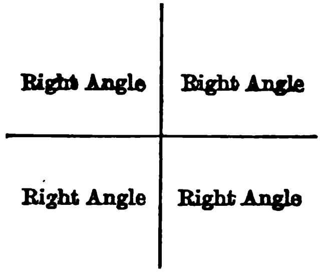
Hence, the area of a circle $= \pi\times r^2 = \pi r^2$

WRITTEN EXERCISES

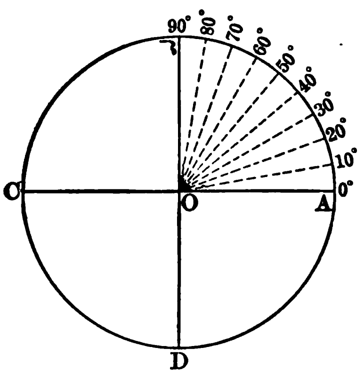
1. Find the area of a circle whose radius is 5 inches.
2. Find the area of a circle whose radius is 16 inches.
3. Find the area of a circle whose circumference is 36 inches. (*Suggestion:* First find the diameter, and then the radius.)
4. A cylindrical tank has a radius of 4 feet. Find the area of one end of this tank.
5. Which has the greater area, and how much—a circle with a diameter 12 inches, or a square whose sides are 11 inches?

260. Right Angles. If two lines meet in a point, they are said to *intersect* at that point. Two intersecting lines make four different angles. If the four angles made by two inter-





secting lines are all equal, the angles are called *right angles.* The lines are said to be at *right angles* to each other, or to be *perpendicular* to each other.



261. A Degree of Arc. If two diameters of a circle are at right angles, they divide the circle into four equal parts. In the figure, the four arcs AB, BC, CD, DA are equal. If one of these arcs, as AB, is divided into 90 equal parts, each part is called one degree. In the figure, the arc AB is divided into 9 equal parts. Hence each of these parts is 10 degrees.

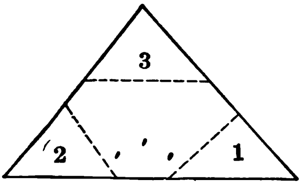
262. A Degree of Angle. The angle at the center of the circle is also measured in degrees. Thus, the angle AOB = 90 degrees. One degree of an angle is $\frac{1}{90}$ of a right angle.

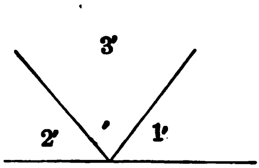
The degree is used as a unit of measurement of both arcs and angles. The symbol ° is used to indicate degrees of both arcs and angles. Thus 15° means 15 degrees.

263. Circular Measure. In measuring angles or arcs, one degree is divided into 60 minutes, and one minute into sixty seconds. The symbols for minutes and seconds are ′ and ″ respectively.

TABLE OF CIRCULAR MEASURE

$60'' = 1'$
$60' = 1°$
$360° = 1$ circumference.





One of the most interesting facts discovered in geometry is that the sum of the three angles in any triangle is equal to two right angles, or 180°. You can easily show this by cutting off the three corners of a triangle and putting them together, as shown in the figure.

WRITTEN EXERCISES

Add the following angles:

1.	74°	18′	25″	2.	104°	34′	50″
	59°	36′	45″		16°	45′	38″

3. Find the difference between 115° 36′ 29″ and 84° 17′ 24″.

4. Find the difference between 84° 47′ 18″ and 61° 32′ 52″.

5. Find the difference between 90° 30′ 25″ and 54° 45′ 50″.

6. Find the difference between 180° and 147° 40′ 36″.

7. If one angle of a triangle is 90° (a right angle), what is the sum of the other two angles?

264. Ratio. *The ratio of one number to another is the quotient obtained by dividing the first by the second.*

Thus, the ratio of 6 to $3 = 6 \div 3 = 2$, and the ratio of 3 to 6 is $3 \div 6 = \frac{1}{2}$.

265. Antecedent, Consequent. In the ratio 2 to 5, 2 is called the *antecedent* and 5 the *consequent*. Similarly, in any ratio, the number given first is the antecedent and the one given second is the consequent.

266. Ratio as a Fraction. Since division may be indicated by a fraction, a ratio may always be written as a fraction.

Thus, the ratio of 2 to 5 is $\frac{2}{5}$, and the ratio of 7 to 3 is $\frac{7}{3}$.

The sign : is used to indicate ratio. Thus, $2:5$ is the "ratio 2 to 5," or $\frac{2}{5}$.

Thus, we see that $2:5 = 2 \div 5 = \frac{2}{5}$, or, in general, antecedent: consequent = dividend ÷ divisor = $\frac{\text{numerator}}{\text{denominator}}$.

We have frequently dealt with ratios, though we have not called them by that name. When a ball team plays 17 games, and wins 8, the ratio of the games won to games played is $8:17$, or $\frac{8}{17}$.

267. Reduction of Ratio to Simplest Form. Since a ratio may be expressed as a fraction, it follows that the *antecedent and consequent of a ratio may both be divided or both multiplied by the same number without changing its value.*

Thus, $3:18 = 1:6$ (dividing by 3), and $\frac{3}{4}:\frac{5}{6} = 9:10$.

EXERCISES

Reduce each of the following ratios to simplest form.

1. $7:21$	5. $6:15$	9. $\frac{3}{4}:\frac{1}{2}$	13. $5:\frac{5}{6}$
2. $\frac{1}{2}:8$	6. $8:12$	10. $\frac{2}{3}:\frac{3}{4}$	14. $\frac{4}{5}:\frac{5}{8}$
3. $\frac{2}{3}:6$	7. $3:\frac{1}{2}$	11. $\frac{3}{8}:\frac{3}{4}$	15. $1\frac{1}{2}:3\frac{1}{3}$
4. $4:\frac{5}{6}$	8. $6:\frac{2}{3}$	12. $\frac{5}{8}:\frac{7}{12}$	16. $7\frac{2}{3}:9\frac{3}{4}$

268. Approximate Ratios. Ratios of numbers are frequently expressed approximately.

Thus, the ratio $\frac{25}{6}$ when expressed to the nearest integer is 4, and when expressed to the nearest tenth is 4.2.

To obtain an approximate ratio in the form of an integer or a decimal fraction, divide the antecedent by the consequent. The process consists simply in reducing a common fraction to a decimal.

Example 1. In 1900 there were 208,903 boot- and shoe-makers in the United States. For how many persons did each shoemaker make shoes if the total population was 75,945,000?

Solution: $\frac{75995000}{208,903}=364$ (approximately, which is the result correct to the nearest integer).

Hence 1 person in 364 was a shoemaker. That is, the ratio of the number of shoemakers to the whole population was 1 to 364.

Approximate ratios as shown in these examples are in more common use than any other form of ratio, and should be clearly understood.

EXERCISES.

In the year 1910 the population of the city of New York was 4,766,883. The numbers of people engaged in certain occupations are given below. Find the ratio to the nearest unit of each of these numbers to the total population.

Actors,	7,966	Nurses	17,213
Bakers,	13,312	Plasterers	6,388
Blacksmiths,	7,922	Plumbers,	19,564
Brick- and Stone-masons,	15,804	Policemen,	10,689
Carpenters,	41,442	Retail Dealers,	115,128
Chauffeurs	9255	Salesmen (Stores)	94,206
Delivery-men,	22,127	Servants,	139,987
Doctors,	8,241	Stenographers,	40,111
Lawyers,	10,563	Teachers,	27,324
Mail-carriers,	4,267	Waiters,	29,617

269. Proportion. A *proportion* is an equality between two ratios.

Thus, $\frac{3}{6}$ $\frac{4}{8}$ or 3 : 6 4 : 8 is a proportion.

270. Means and Extremes. The first and the last number in a proportion, when written in the form $3:6=4:8$ are called the *extremes,* and the other two numbers are called the *means.*

Thus, in the proportion $3:6=4:8$, 3 and 8 are the extremes and 6 and 4 are the means.

In every proportion, the product of the extremes is equal to the product of the means.

Thus, in $3:6=4:8$, $3\times8=6\times4$.

271. Solving a Proportion. If three of the four numbers of a proportion are given, the fourth number may always be found. Let the unknown number be represented by x.

Example. Find the value of x (the unknown number) in $x:3=7:9$ or $\frac{x}{3}=\frac{7}{9}$.

Solution: Since $\frac{x}{3}$ or $\frac{1}{3}$ of $x=\frac{7}{9}$, $x=3\times\frac{7}{9}=\frac{21}{9}=2\frac{1}{3}$.

In this proportion x (one of the extremes) is equal to the product of the means $(3\times7=21)$ divided by the other extreme (9).

The finding of one number of a proportion when the other three are given is called *solving the proportion.*

In a proportion either extreme is equal to the product of the means divided by the other extreme, and either mean is equal to the product of the extremes divided by the other mean.

$$\text{Thus, in } 3:6=4:8,\ 3=\frac{6\times4}{8}$$

$$\text{and}\quad 6=\frac{3\times8}{4},\ 4=\frac{3\times8}{6},\ 8=\frac{6\times4}{3}.$$

By means of this rule any one of the four numbers of a proportion may be found when the other three are given. It is the most important rule in proportion.

Example. Find x in $3:x=7:12$.

Solution: $x=\frac{3\times12}{7}=5\frac{1}{7}$.

WRITTEN EXERCISES

Find the value of x in each of the following:

1. $x : 5 = 7 : 9$
2. $x : 2 = 3 : 5$
3. $x : 3 = 5 : 9$
4. $3 : 7 = x : 2$
5. $11 : 15 = x : 8$
6. $5 : 14 = x : 7$
7. $3 : x = 5 : 9$
8. $5 : x = 4 : 11$
9. $7 : x = 9 : 13$
10. $2 : 5 = 7 : x$
11. $3 : 7 = 5 : x$
12. $4 : 9 = 6 : x$

Example. A field containing 35 acres yields 1240 bushels of corn. At this rate, what will be the yield of a field containing 64 acres?

Solution: The ratio of the acreages of the two fields must be equal to the ratio of the yields. Let x = the number of bushels in the unknown yield.

Thus, $x : 1240 = 64 : 35$ and $x = \frac{1240 \times 64}{35} = \frac{15972}{7} = 2281\frac{5}{7}$ (bushels).

13. If coal is sold at $7.50 per ton, what is the value of a load containing 3560 pounds of coal?

Suggestion: If x = the cost in cents, then $x : 750 = 3560 : 2000$.

14. If an automobile uses $5\frac{1}{2}$ gallons of gasoline going 65 miles, how many miles does the car go on 45 gallons?

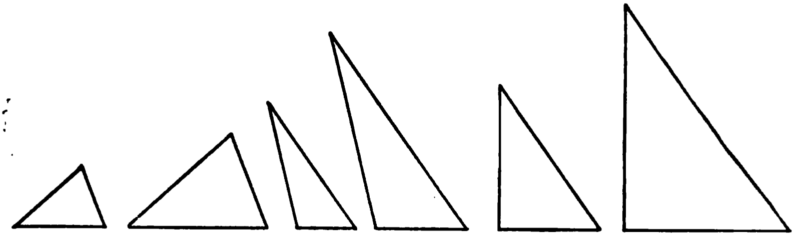
15. In a city, a lot 30 feet wide, and 90 feet deep sold for $3250. At this rate, what is the price of a lot 25 feet wide, and 150 feet deep?

Suggestion: $x : 3250 = 25 \times 150 : 30 \times 90$.

16. If a Turkish rug 7 feet by $8\frac{1}{2}$ feet costs $85, what should be the price of a rug of the same quality 10 feet wide and 12 feet long?

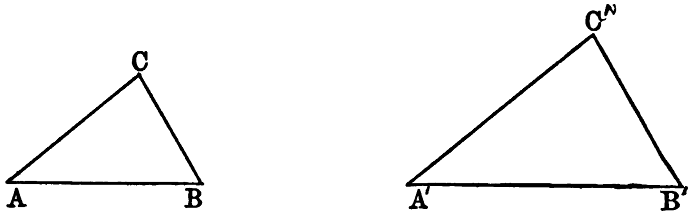
17. If 57 tons of hay are required to feed 36 head of cattle for $4\frac{1}{2}$ months, how much hay will be needed to feed 19 head $2\frac{1}{4}$ months?

272. Similar Figures. Two figures which have the same shape are *similar*.



In the figure three pairs of similar triangles are shown.

273. Equal Angles in Similar Triangles. The triangles ABC and A′B′C′ are similar. The angles of one are equal to the angles



of the other. That is, the angle A is equal to the angle A′, angle B is equal to angle B′, and angle C is equal to angle C′.

The following is an important property of triangles:

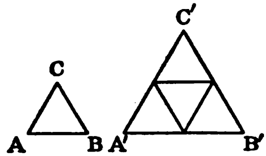
If the angles of one triangle are equal to the angles of another, the two triangles are similar.

274. Corresponding Sides of Similar Triangles. The sides opposite equal angles of similar triangles are corresponding sides.

Thus, in the triangles ABC and A′B′C′ above, the sides BC and B′C′ are corresponding sides because they are opposite the equal angles A and A′. Similarly, AB and A′B′ are corresponding sides, because they are opposite the equal angles C and C′, and AC and A′C′ are corresponding sides because they are opposite the equal angles B and B′.

275. Corresponding Sides of Similar Triangles Form a Proportion.

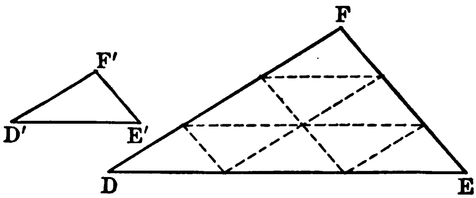
Each of the small triangles into which the triangle A′B′C′ is divided is equal to the triangle ABC. (Notice that triangles ABC and A′B′C′ are similar.)



ORAL EXERCISES

How does the side AB compare in length with the side A′B′?
How does the side AC compare in length with the side A′C′?
How does the side BC compare in length with the side B′C′?
From the exercise we see that, AB : A′B′ = 1 : 2
BC : B′C′ = 1 : 2
CA : C′A′ = 1 : 2

Hence, AB : A′B′ = BC : B′C′ = CA = C′A′. (1)



In the triangles DEF and D′E′F′ how does the side D′E′ compare in length with the side DE?

How does the side E′F′ compare in length with the side EF?

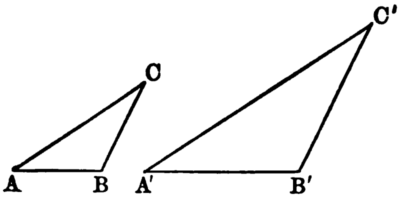
How does the side FD compare in length with the side F′D′?
Hence we see that DE : D′E′ = 1 : 3
EF : E′F′ = 1 : 3
FD : F′D′ = 1 : 3

Therefore DE : D′E′ = EF : E′F′ = FD : F′D′. (2)

In equations (1) and (2) the ratios are ratios of corresponding sides. These equations are examples of an important property of similar triangles.

In two similar triangles the ratio of any pair of corresponding sides is the same as the ratio of any other pair of corresponding sides.

This property of similar triangles makes them very useful in finding heights and distances which we cannot measure directly.



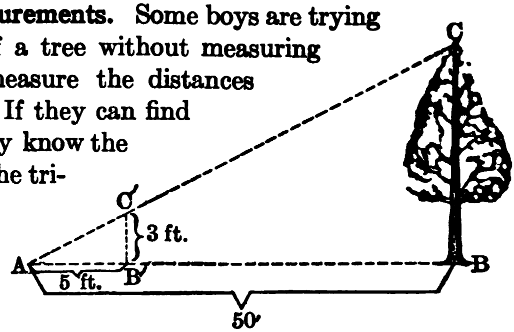
The two triangles in the figures are similar. Which angles are equal?

What ratios of sides are equal to the ratio $\frac{AB}{A'B'}$?

Sometimes the last property of similar triangles given on the preceding page is stated:

In similar triangles the corresponding sides form a proportion.

276. Indirect Measurements. Some boys are trying to find the height of a tree without measuring it directly. They measure the distances AB, AB′, and B′C′. If they can find the length of BC they know the height of the tree. The triangles AB′C′ and ABC are similar. Therefore, BC : B′C′ = AB : AB′. Hence, $BC = B'C' \times \frac{AB}{AB'}$. (See page 244.)



Suppose they find AB = 50 feet, AB′ = 5 feet, B′C′ = 3 feet; then $BC = 3 \times \frac{50}{5} = 30$.

That is, the point C is 30 feet above the point B, and the tree is therefore 30 feet high.

In making these measurements, care must be taken to make the triangle ABC and A′B′C′ similar. The triangles will be similar if the lines B′C′ and BC are parallel. If the tree stands perpendicularly, all that is needed is therefore to hold the rod B′C′ perpendicular. Note carefully that if B′C′ and BC are parallel the triangles will be similar. Similar triangles are used, directly or indirectly, in nearly all indirect measurements.

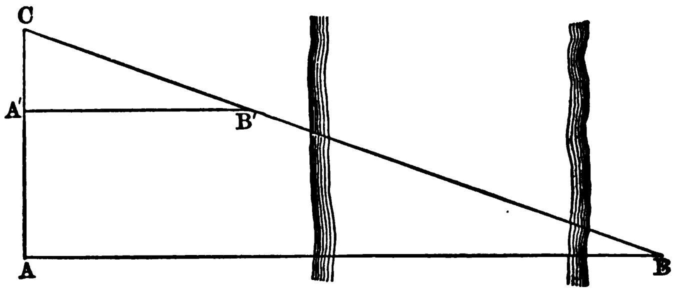
WRITTEN EXERCISES

In the second figure on the opposite page, find BC for each of the following values of the other lines.

1.	AB = 100 feet	AB′ = 8 feet	B′C′ = 6 feet
2.	AB = 85 feet	AB′ = 10 feet	B′C′ = 4 feet
3.	AB = 15 feet	AB′ = 9 feet	B′C′ = $5\frac{3}{4}$ feet
4.	AB = 158 feet	AB′ = 9 feet	B′C′ = $4\frac{1}{2}$ feet

By the methods used in these problems you can evidently find the height of any tree, building, flagstaff, or any other object which it is difficult to measure directly.

Try to measure the height of some such object. Make several attempts, and see how nearly alike your results are.

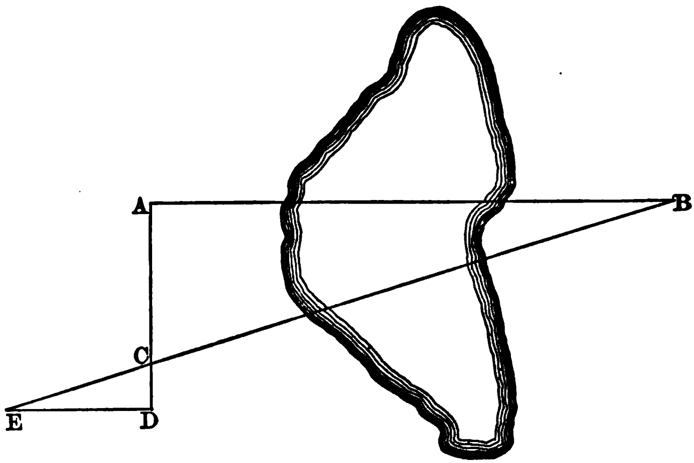


To measure the distance across a river, two boys arrange two triangles, ABC and A′B′C so that the line A′B′ is parallel to AB. This makes the two triangles similar.

Hence, $AB : A'B' = AC : A'C$, or $AB = \frac{A'B' \times AC}{A'C}$.

If the lines AC, A′C, and A′B′ are measured, we can now find the length of AB. Suppose A′B′ = 40 feet, A′C = 25 feet, and AC = 90 feet.

Then $AB = \frac{40 \times 90}{25} = 144$ (feet).



This figure shows how some young people measured the distance across a lake. They wanted to find the distance from A to B. They selected point D so as to make the angle A a right angle. Then they sighted from E to B and marked a point C on the line AD. The triangles EDC and CAB are similar.

Hence $AB : ED = AC : CD$ or $AB = \frac{ED \times AC}{CD}$.

The lines ED, DC, and CA are measured, and are found to be 50, 24, 4, respectively, as shown in the figure.

Then, $AB = \frac{50 \times 24}{4} = 300$ (feet).

1. Draw a figure to show how you could measure the distance across a wide river.

2. Draw a figure to show how you could measure the height of a tree.

3. Try to measure the distance between two points by the method shown on page 249.

4. Try to measure the height of a tall building or a flagstaff by means of the method shown on page 248.

277. **Indirect Measurements in Geography.** Finding distances without measuring them directly is called indirect measurement. By means of indirect measurements a great many facts are learned which could never be obtained in any other way. The height of Mount Everest is given as 29002 feet, though no one has ever reached the top of it, and still the height is known more accurately than it could ever be measured by any direct means.

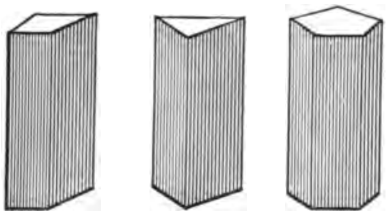
278. **Indirect Measurements in Astronomy.** We know the distance from the earth to the moon, to the sun, to the various planets, and even to many of the fixed stars, though the nearest of these is over two hundred thousand times as far away as the sun. We know the size of the moon, of the sun, and of all the planets. It is perfectly clear that none of these distances can ever be measured directly, and that our knowledge of them must forever be derived from indirect measurements.

279. **Indirect Measurements in Surveying Land.** Even in surveying large tracts of land, indirect measurements are used. One side of a triangle is measured at the outset, and after that only the angles are measured, and the lengths of the sides are computed. This saves a tremendous amount of trouble, since it would be very difficult, for example, to measure directly the horizontal distance between two points on opposite sides of a mountain.

You see from all this that indirect measurements are very important. A large amount of mathematics have been built up, and very fine instruments have been made, just for the purpose of making such measurements.

The size of the earth and the distances between remote points on it are known much more accurately than they could be measured directly except by the most laborious processes. The shape of the earth is also learned by indirect measurements.

280. Prisms, Bases, Altitude, Lateral Area. We have already found the number of bushels contained in bins and boxes, the number of cubic yards in excavations, the number of gallons in tanks, the number of cubic feet in a room, and so forth. We will now study this subject more in detail.



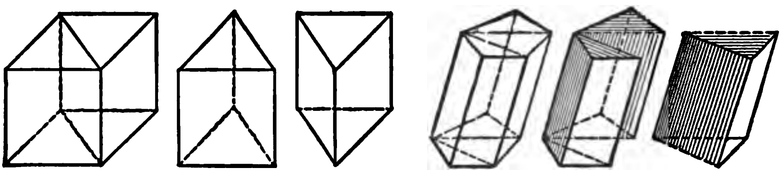
Solids like those shown in the figure are called *prisms*.

A prism has two parallel and equal faces which are called its *bases*.

In the figure, each prism stands on one of its bases, and the other base is the white surface on top. The shaded portion constitutes the *lateral area* of the prisms.

The distance between the bases is called the *altitude*.

281. Right Prisms. A prism is called a *right prism* when its sides are perpendicular to the bases. Only right prisms are considered here.



Rule: To find the volume of a prism, multiply the area of the base by the altitude.

We have already used this rule in finding the contents of rectangular boxes. In the figure above, a rectangular prism is divided into two triangular prisms, and we see at once that the rule for finding the volume holds for triangular prisms. But any prism can be divided into triangular prisms, as in the next figure; and hence the rule holds for any prism whatever.

WRITTEN EXERCISES

1. Find the volume of a prism whose altitude is 10 inches, and whose base is a square 6 inches on each side.

2. Find the area of an equilateral triangle whose sides are 10 inches. (See page 236.)

3. Find the volume of a prism whose altitude is 12 inches and whose base is an equilateral triangle with sides 12 inches.

Suggestion: First find the area of an equilateral triangle whose sides are 12 inches.

4. Find the area of an equilateral triangle whose sides are 8 inches.

5. Find the area of a regular hexagon whose sides are 8 inches.
Suggestion: See Example 1, page 237.

6. The base of a right prism is a regular hexagon whose sides are 8 inches. Find the volume of this prism if its altitude is 8 inches.

7. The base of a right prism is a right triangle whose sides about the right angle are 3 and 13 inches. Find the volume of this prism if its altitude is 12 inches.

8. The volume of a prism is 640 cubic inches. Find its altitude if the base is a rectangle 10 by 8 inches.

9. The volume of a prism having a square base, is 384 cubic inches, and its altitude is 8 inches. Find a side of the base.

10. Find the area of a trapezoid whose parallel sides are 11 and 16 inches, and whose altitude is 18 inches.

11. The hypothenuse of a right triangle is 17.4, and one side is 13.7. Find the third side.

12. Find the circumference of a circle whose radius is $5\frac{1}{2}$ inches.

13. Find the area of a circle whose diameter is $8\frac{1}{4}$ inches.

282. The Lateral Area of a Prism. Since the lateral area of a right prism consists of rectangles, it can be found by adding the areas of these rectangles.

In a right prism the altitude of each rectangle is the same as the altitude of the prism. Hence the lateral area equals the sum of the bases multiplied by the altitude of the prism.

Since the sum of these bases is equal to the perimeter of the base of the prism, we have the rule:

To find the lateral area of a right prism, multiply the perimeter of the base by the altitude of the prism.

To find the total area of a prism, add the areas of the two bases to the lateral area.

WRITTEN EXERCISES

1. Find the total area of a cube whose edges are 12 inches.

2. Find the lateral area of a prism whose altitude is 18 inches and whose base is a rectangle 10 by 14 inches.

3. Find the total area of the prism in Example 2.

4. Find the lateral area of a prism whose altitude is 16 inches and the perimeter of whose base is 24 inches. Does the shape of the base make any difference in this problem?

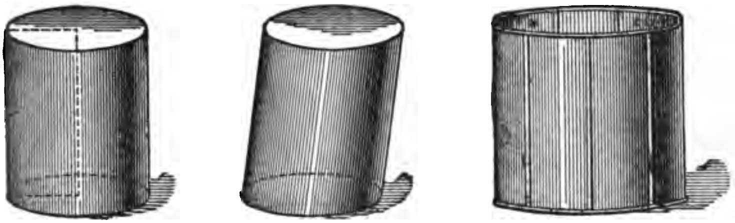
5. Find the total area of a prism whose altitude is 10 inches, and whose base is an equilateral triangle with sides 10 inches.

Suggestion: Find the area of an equilateral triangle whose sides are 10 inches.

6. Find the total area of a prism whose altitude is 14 and whose base is a regular hexagon with sides 4.

Suggestion: Find the area of a regular hexagon by using the method on page 237.

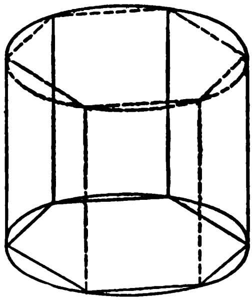
7. Find the total area and also the volume of a right prism whose altitude is 16 inches, and whose base is an equilateral triangle whose sides are 8 inches.



283. **Cylinders.** Figures like the above are called cylinders. Iron smokestacks, stovepipes, rolls of paper, the lower parts of milk bottles, milk cans, etc., are cylinders.

The two flat sides of a cylinder are its bases.

The curved surface is its lateral area. The cylinder is called a right cylinder if the lateral surface is at right angles with the bases.



284. **Area and Volume of Cylinders.** From the figure we see that a prism can be constructed which shall differ as little as we may wish both in area and in volume from those of a cylinder. Indeed, for the purpose of finding the rules for surface and volume, a cylinder may be regarded as a prism with a large number of very small sides.

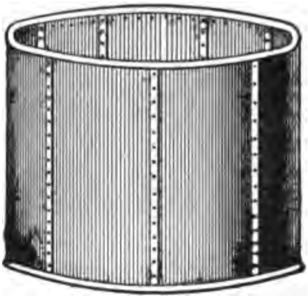
Hence we have the two following rules for finding the lateral area and the volume of a right cylinder:

To find the lateral area of a right cylinder, multiply the perimeter of the base by the altitude.

To find the volume of a cylinder, multiply the area of the base by the altitude.

The rule for finding the lateral area of a cylinder may also be obtained by wrapping a sheet of paper around it, and then laying the paper flat and measuring it.

WRITTEN EXERCISES



1. A roll of wrapping paper is 2 feet wide and 12 inches in diameter. Find the area of the outside layer of the paper.

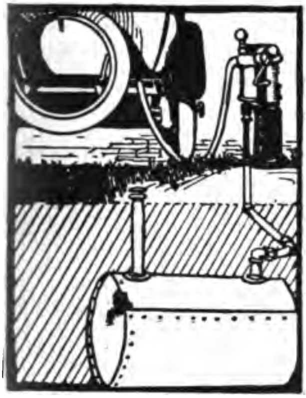
2. A piece of stovepipe 18 inches long is 8 inches in diameter. Find the area of the iron sheet from which it is made, if 2 inches of it is allowed for making the seam.

3. A water tank, open at the top, is 8 feet in diameter, and 6 feet high. Find its total surface area. Find the number of square feet of sheet steel used in making it by adding $\frac{1}{8}$ for wastage and seams.

4. Iron plate $\frac{1}{8}$ inch thick weighs 5 pounds to the square foot. Find the weight of 200 feet of iron pipe made of $\frac{1}{8}$ inch material, if the diameter of the pipe is 4 inches.

5. One of the largest gas tanks ever constructed was 293 feet in diameter and 180 feet in height. At 4 cents a square yard, find the cost of painting this tank. (The lateral area and the top are painted.)

6. In arid regions over-ground water mains are built of inch-board held in place by iron rings. How many feet (square feet) are required to build one mile of 18-inch main, adding $\frac{1}{5}$ for waste and matching the lumber.



1. A gasoline tank is 24 inches in diameter, and 30 inches long. What is its capacity in gallons? One gallon contains 231 cubic inches.

2. The following dimensions are taken from a price list of underground gasoline tanks. Fill in the third column of the first table, giving the capacity:

Diameter inches	Length inches	Capacity gallons
24	36	
34	30	
34	36	
34	42	
40	42	

3. Fill in the third column of the second table, giving the capacity.

(One barrel equals 4.21 cubic feet.)

Diameter	Height	Capacity barrels
6 ft.	6 ft.	
8 ft.	5 ft.	
8 ft.	8 ft.	
10 ft.	8 ft.	
10 ft.	10 ft.	

4. How many cubic feet of gas does it require to fill the tank described in Example 5 on the opposite page? At 80 cents per thousand cubic feet, what is the value of this gas?

5. If the dimensions of the water tank described in Example 3 on theopposite page are inside measures, how many gallons will the tank hold?

6. Kerosene oil is hauled in "tank cars." How many barrels of oil will a tank hold if it is 8 feet in diameter and 30 feet long?

285. **Practical Uses of Arithmetic.** The use of arithmetic in our daily lives is of greater importance than is generally understood. No individual can regulate his expenditures intelligently unless he keeps an account showing his income and the purposes for which he expends it. The expenditures of a family can never be wisely made unless a complete household account is kept, running over a series of years. No woman can keep house as economically as possible unless she figures the value of the food she buys at varying prices. A farmer, no matter how intelligent and successful, will be more successful if he keeps accounts showing which hen, which cow, or which field pays him the best. A few minutes spent each day in intelligent application of the arithmetic learned in the grade schools will pay a higher income per hour than any any other work we are likely to do.

286. **The Fundamental Operations of Arithmetic.** Any work whatever involving arithmetic consists of a series of steps each of which is an addition, a subtraction, a multiplication, or a division. Experience has shown that people will avoid doing whatever is difficult and laborious. Only those persons who can perform the operations of arithmetic with ease, accuracy, and comfort will use them freely. If you can add or multiply only with difficulty and with a feeling of dislike and fatigue, the chances are that you will do no more of it than seems absolutely necessary. The result will be that you will neglect to make that use of arithmetic which is necessary for the highest success. Hence it is of the very greatest importance that you should practice on the fundamental operations until you can perform them rapidly and with practically absolute accuracy.

287. **Business Usages.** Through long and varied experience those who are doing the world's business have learned a great deal about the most convenient forms for doing

the number work that is needed in practical life. We should learn as much as possible about such forms in order that we may be more likely, as a matter of daily practice, to do that work with numbers which is required for our highest welfare.

EXERCISES

1. Add the columns on page 6 of this book. Do not copy the numbers. Add each column both ways to make sure your sums are right. Work as rapidly as you can, and time yourself to see how long it takes to get these sums.

2. Add the columns on page 7, following the directions given under Example 1, above.

3. Find the sum required on page 8, adding horizontally.

4. Find the totals in Example 1 on page 9.

5. Find the sum in each line and the amount saved each month in the account given on pages 12, 13. Also find the sum of each column and the sum of these sums. Add the sums obtained by adding each line and see that your work checks.

6. Copy and add the numbers given in Examples 1–7 on page 16. Remember that in practice accurate copying is just as important as accurate adding.

7. Copy examples at the bottom of page 16 and subtract.

8. Copy and find products in the examples at the top of page 20, timing yourself as suggested under Example 1, above.

9. Estimate rapidly quotients in the examples at the top of page 25. After you have estimated all the quotients, find whether your estimates are correct.

10. Find the quotients to the nearest integer in ten examples on page 25, timing yourself as suggested above.

11. In your opinion, which of the four fundamental operations occur most frequently in practice? Give reasons for your answers.

THE FUNDAMENTAL OPERATIONS ON FRACTIONS

ORAL EXERCISES

1. Describe a convenient method for reducing fractions to a common denominator.
2. Give the rule for finding the product of two fractions.
3. Give the rule for dividing a number by a fraction.
4. Give a rule for changing the form of a fraction without changing its value.
5. Give the sums required at the bottom of page 56.
6. Give the products in the exercises on page 58.
7. Give the products in all the exercises on page 61.
8. Give the quotients required on page 62.
9. Give the sums required on page 172.
10. Give the differences required on page 172.
11. Give the products required on page 174.
12. Give the quotients required on page 174.

WRITTEN EXERCISES

1. Solve the written exercises on page 55.
2. Solve the miscellaneous exercises in multiplication of fractions given on page 61. Use the four-step method when that is more convenient.
3. Find the sums in the first column of exercises on page 173.
4. Find the sums in the second column of exercises on page 173.
5. Find the results in the first column of exercises on page 175.
6. Find the results in the second column of exercises on page 175.

ORAL EXERCISES

1. State the principle of place value in the decimal notation. Does this principle apply to decimal fractions as well as to whole numbers?

2. If a common fraction is regarded as an indicated division, how may it be reduced to a decimal?

3. How can you decide, without dividing, whether or not a common fraction may be reduced to an exact decimal equivalent?

4. Give a rule for placing the decimal point in the result in addition and subtraction.

5. Give a rule for placing the decimal point in a product.

6. If the divisor is a whole number and the dividend a decimal, how is the decimal point placed in the quotient?

7. Describe the process of finding the quotient when the divisor is a decimal.

WRITTEN EXERCISES

1. Carry out the work required in exercise 3 on page 67, arranging the numbers in the order of their magnitude.

2. Reduce the common fractions in the exercises on page 68 to 5-place decimals.

3. Reduce the decimal fractions given on page 68 to common fractions.

4. Rule paper and make out the cash accounts required on page 77. Find the balance in each case and close the account.

5. Study the short cuts on pages 70, 71. Solve the exercises on page 71.

6. Study the common fraction equivalents of certain per cents required at the bottom of page 87.

THE PRINCIPLE OF PRODUCT AND FACTORS

1. If the product of two numbers and one of the numbers are given how is the other number found?

2. Give an example which illustrates the equation:

$$\textit{numbers of articles} \times \textit{price} = \textit{cost} \qquad \text{(c)}$$

If two of the three numbers involved in this equation are given, state how the third number may be found.

3. What three kinds of problems may be solved by means of equation (c)?

4. For each of the equations given below give one problem illustrating it.

$$\begin{array}{rll}
\textit{speed} \times \textit{time} &= \textit{distance} & \text{(s)} \\
\textit{length} \times \textit{width} &= \textit{area} & \text{(a)} \\
\textit{length} \times \textit{width} \times \textit{depth} &= \textit{volume} & \text{(v)} \\
\textit{base} \times \textit{rate} &= \textit{percentage} & \text{(p)} \\
\textit{marking price} \times \textit{rate discount} &= \textit{discount} & \text{(d)} \\
\left.\begin{array}{r}\textit{buying price} \times \textit{rate gain} = \textit{gain} \\ \textit{buying price} \times \textit{rate loss} = \textit{loss}\end{array}\right\} & & \text{(p \& l)} \\
\textit{selling price} \times \textit{rate commission} &= \textit{commission} & \text{(c)} \\
\textit{investmemt} \times \textit{rate of income} &= \textit{income} & \text{(i)} \\
\textit{assessed valuation} \times \textit{rate of taxation} &= \textit{tax} & \text{(t)} \\
\textit{principal} \times \textit{rate} \times \textit{time} &= \textit{interest} & \text{(int)}
\end{array}$$

5. Give three different kinds of problems which may be solved by each of the equations, (s), (a), (p), (d), (p & l), (c), (i), (t), in Example 4, and state how each problem may be solved.

6. Give four different kinds of problems which may be solved by each of the equations (v), (int), in Example 4, and state how each problem may be solved.

7. Give an equation by means of which problems in loss or gain may be solved when profit or loss is computed as a certain rate on the selling price.

1. Give reasons for reducing fractions to decimals. Why are baseball scores given in decimals?

2. Give reasons for reducing numbers to per cents. Thus, why is the fat content of milk given in per cents?

3. In computing discount on goods sold, what number is used as the base? Give reasons for commercial discounts.

4. When several discounts are given, how are they computed?

5. In computing loss or gain, what number is usually used as the base?

6. Give reasons why as a general rule there must be gain in commercial transactions.

7. What is the general rule for computing commission on any transaction?

8. Enumerate different kinds of business that is done on a commission basis. Why is business done on commission?

9. In computing interest what number is used as the base? How is time usually counted in computing interest?

10. What are the main business usages governing bank discount?

11. Describe the process of levying a general property tax. What are the steps involved in spreading a tax?

12. Give the rule for computing a broker's commission for selling stocks and bonds.

13. What is meant by par value of a stock or a bond?

14. Give general directions which should be followed in making an investment.

PROBLEMS WITHOUT NUMBERS

1. Tell how you would find a quotient to the nearest integer.

2. Tell how you would find a quotient correct to four places of decimals.

3. Tell how you would keep your own cash account. Tell which items you would put on the debit side, and which on the credit side.

4. Tell how you would keep account with a field on a farm.

5. A troop of boy scouts raised vegetables and general garden truck. Tell how you would keep an account to find just how much money they made.

6. Tell what are the essential items of a bill. What are the differences between a bill and an account?

7. How would you find the average of the grades you receive one term in school?

8. If you know the weight of a cubic foot of solid wood and also the weight of one cord of this wood, how can you tell how many per cent of a pile of this wood is solid wood?

9. If you know how many per cent of wheat is ground into flour, how can you tell how many bushels of wheat are required to make a barrel of flour?

10. If you know the buying price and the rate of gain, how do you find the gain and the selling price if the gain is computed on the buying price?

11. If you know the buying price and the rate of gain, how do you find the gain and the selling price if the gain is computed on the selling price?

12. If you know the rate of interest, the yearly cost of upkeep and original cost per mile of a road, how can you find the total cost per mile per year?

1. If you know how much a man paid for a certain property and his yearly net income from it, how can you find the rate per cent income on the investment?
2. If you know the rate of commission which the agent charged for selling a house and also how much the owner received for it, how can you tell the selling price?
3. If you know the rate of dividend on a stock and the price at which it was bought, how do you find the rate of income on the investment?
4. If you know the total assessed valuation of a city and also the amount to be raised as a tax on this property, how do you find the rate of taxation?
5. How do you find a person's property tax if you know the tax rate and his assessed valuation?
6. If you know the diameter of an automobile wheel how can you tell the number of revolutions it makes in going one mile?
7. If you know the length and width of a rectangle, how do you find the length of a diagonal?
8. If you know the length and the diameter of a cylindrical tank, how do you find how many gallons it will hold?
9. Describe a method for measuring the distance across a lake without crossing it.
10. What items would you need to know to find the total yearly cost of keeping an automobile?
11. Tell why several discounts are sometimes given. Does it make any difference which discount is deducted first?
12. How can you find a single discount equivalent to a series of three given discounts? What is the most convenient method for determining which of two discount series is the most advantageous?

MISCELLANEOUS PROBLEMS

1. Find the number of bushels in a bin $9\frac{1}{2}$ feet long and $6\frac{3}{4}$ feet wide if the grain is $5\frac{1}{2}$ feet deep.

2. Sirloin steak contains about 957 calories in food value per pound, and halibut steak contains about 457 calories in food value per pound. Find the cost per 1000 calories at each of the following prices: Sirloin 25¢, 30¢, 35¢, 40¢, 45¢ per pound. Halibut 15¢, 20¢, 25¢, 30¢, 35¢ per pound. Find the cost per pound of sirloin and halibut in your town, and compare the cost of 1000 calories at these prices.

3. A farmer raised 870 tons of sugar beets. How many hundred pounds of sugar were made from these beets if 13.7% of the beets were sugar?

4. Make out a bill using the following data: J. V. Haugan, of Buffalo, N. Y., sold to Mrs. Edward A. Lavell, of Niagara Falls, N. Y., one dining table, \$85; eight dining-room chairs at \$18.50; one set of dishes, \$65.00; one sideboard, \$110.00; one sewing table, \$22.00. Discount 15%. Extend and foot bill and deduct the discount.

5. A business house has a capital stock of \$800,000, and does a yearly business of \$4,800,000. If the average net profit on the business is 4.8%, what rate per cent of the capital stock is this?

6. Last year a farmer fed 65 head of cattle through the winter, using 97 tons of hay and 43 tons of straw. This winter he proposes to feed 93 head of cattle. How much hay and straw should he have?

7. A company which had a capital of \$250,000 declared a dividend of 8%, and carried \$10,500 to surplus. If its total business for the year amounted to \$1,650,000 what rate net profit did the company make on this business?

1. A cubic foot of water weighs 62.5 pounds. The weight of one cubic foot of solid oak is 77% that of water. What is the weight of one cord of oak wood if 56% of the cord is solid oak?

2. A cow is tethered to a rope 25 feet long. Find the circumference of the largest circle around which it can travel. Also find the area of this circle.

3. A farmer wishes to have 80 bushels of seed corn in the spring. How many bushels should he select in the fall if corn shrinks 14.5 from October till May?

4. A retail dealer bought goods at 25% and 10% off. What was the list price of an article for which he paid $40.50?

5. A steel casting shrinks $\frac{1}{48}$ of its length when cooling from the red hot temperature at which it is cast to normal temperature. How long must the red hot casting be made if it is to be 5 feet 6 inches long when cooled?

6. Flour of a certain grade is equal in weight to 75% of the weight of the wheat from which it is made. If the weight of this flour required to make bread is about 67% of the weight of the bread, find the number of pounds of bread which can be made from one bushel of wheat.

7. At 925 shingles for a square of roofing (100 square feet) how many shingles are needed for a roof each side of which is 48 feet long and 22 feet wide?

8. Find to the nearest hundredth the length of the diagonal of a square whose sides are 37.

9. If one ton of soft coal occupies 38 cu. ft., find the number of tons in a bin 12 feet long and $8\frac{1}{2}$ feet wide if the coal is $5\frac{1}{4}$ feet deep.

1. A farmer is offered \$.75 a bushel for his potatoes in the fall. He decides to keep them and sells them in the spring for \$.95 a bushel. If 22% of the potatoes spoiled during the winter, find his loss or gain per cent.

2. A silo in the form of a cylinder is 24 feet high and 16 feet in diameter, inside measurements. At 40 cubic feet a ton, how many tons of silage does it hold?

3. One pound of California walnuts contains about 706 calories in fats, one pound of butter contains 3490 calories in fats, and cottolene contains 4080 calories in fats. At the prices in your community, which of these is the cheapest source of fats? Which is the most expensive?

4. By careful selection of seed corn a farmer increased the yield of a field containing 124 acres by 14%. How much did he gain by this selection if corn was worth 84 cents a bushel and if the yield after the increase was 68 bushels to the acre?

5. It takes 350 watts to run a washing machine. What is the cost per hour of running this machine if electric current is sold for 8 cents per kilowatt hour? (1 kilowatt = 1000 watts).

6. A business man discounts a note bearing no interest 84 days before it is due. Find the proceeds if the rate of discount is $6\frac{1}{2}$%, and if the face of the note is \$45,000.

7. If it costs \$17,500 per mile to build a concrete road, and if the upkeep is \$35 per mile a year, what is the cost per year of this road, the interest being $5\frac{1}{2}$%?

8. A retail dealer sells a fountain pen for \$4.00, thereby making a gross profit of approximately 23%. At what price did he buy the pen if it was sold at a multiple of 25 cents.

9. A pattern of wood is made for an iron casting. If the pattern weighs $\frac{1}{17}$ as much as the casting, what will be the weight of the casting if the pattern weighs 5 lbs. 7 oz.?

1. An agent sold a used automobile, and remitted \$675 to his principal. What was the selling price if the agent charged a commission of 10%?

2. A farmer delivered milk to the creamery as follows: Sunday, 350 lbs; Monday, 364 lbs; Tuesday, 348 lbs; Wednesday, 378 lbs; Thursday, 318 lbs; Friday, 392 lbs; Saturday, 374 lbs. The milk averaged 28 cents a gallon. During the week he took from the creamery 190 gallons of skimmed milk for feed at 6 cents a gallon, and 7 pounds of butter at 47 cents a pound. How much did the creamery owe him on the week's business? (8.6 pounds is regarded as one gallon.)

3. The first cost of an automobile was \$1850. Tires cost 3 cents per mile, gasoline and oil $2\frac{3}{4}$ cents per mile, repairs for the season \$39.50. During the first year the car was driven 8400 miles. Find the total cost per mile of running this car if it depreciated 30% in one year, the rate of interest being 7%.

4. In a city with an assessed valuation of \$8,850,000, find the tax of a man whose property is assessed at \$14,800 if \$55,000 is raised from a property tax.

5. A fenced field is 112 rods long and 80 rods wide when measured clear up to the fence. There is a strip 3 feet all around the field next the fence which is not cultivated. Find the number of square feet not cultivated. (One rod is $16\frac{1}{2}$ feet.) How many acres is this if one acre contains 43,560 square feet? Give result to the nearest hundredth of an acre.

6. How many pounds of sugar beets containing $14\frac{3}{4}$% of sugar are required to make a hundred-pound bag of sugar?

7. An ordinary gas burner consumes $6\frac{1}{2}$ cu. ft. of gas per hour. In canning 12 quarts of corn one burner is used 8 hours. At \$1.10 per thousand cubic feet, how much does it cost for gas to can this corn?

1. Find the amount of \$250 compounded semi-annually at 4% for 8 years.

2. Make out a note in favor of A. C. Walker for \$480 dated May 7, 1920, due in 90 days, bearing interest at 7%. Sign your own name to the note.

3. Endorse the note in Example 2, for A. C. Walker, so as not to make him liable in case the note is not paid when due. What is this endorsement called?

4. A family expect to use 20% of their total income for rent, 25% for food, 20% for clothing, 20% for other expenses. They expect to save the rest. What should be the income to justify \$45 a month for rent? On this basis how much should they save?

5. One year a high grade cow gave 11,780 lbs. of milk containing an average of 4.6% butter fat. How many pounds of butter can be made from this milk if 85% of the butter is butter fat?

6. A dairy man's sales for one week were: 208 qts. 1 pt; 197 qts; 187 qts. 1 pt; 203 qts; 212 qts. 1 pt; 201 qts. 1 pt; 194 qts. How many quarts and pints did he sell? At $12\frac{1}{2}$ cents a quart, how much was this milk worth?

7. How high is a tree which casts a shadow of 168 feet if at the same time a vertical stick 5 feet long casts a shadow 8 feet 9 inches long?

8. A man rented a farm containing 260 acres at \$4.50 per acre. One year he sold 18 head of cattle averaging \$85.00 per head, 78 hogs averaging \$37.50 per head, 57 tons of hay at \$14.25 a ton, 1280 bu. of corn at 87 cents, and other farm products for \$560. He paid \$985 for labor, and \$670 for machinery and other expenses. At the end of the year his stock, implements, and so on, were worth \$350 more than at the beginning of the year. How much did he get for his own labor and as profits on his investment?

1. A man bought New York Central stock at a market quotation of $81\frac{3}{4}$. What was the rate of income on this investment if the stock pays a 6% dividend? Find the result to the nearest hundredth of one per cent.

2. A man bought $4\frac{1}{4}$% bonds at a market quotation of $96\frac{1}{4}$. Find the rate of income on this investment to the nearest hundredth of one per cent.

3. At $1.10 a thousand cubic feet, find the amount of a gas bill which gives the meter reading this month as 47,850, and last month as 42,460.

Suggestion: The gas used is $47{,}850-42{,}460=5{,}390$ cu. ft.

4. A dealer in second-hand automobiles sold two machines at $900 each. On one he gained 50%, and on the other he lost 10%. Find the total loss or gain.

5. A coal dealer bought coal at the mine at $4.80 per long ton, (2240 lbs.). What price per short ton (2000 lbs.) was this? If the consumer bought the coal at $5.75 per short ton, how many per cent more than the dealer did he pay for the coal?

6. A grocer bought potatoes at $1.00 a bushel and sold them at $2\frac{1}{2}$ cents a pound. What was his gain per cent if the potatoes shrank 10%. (One bushel of potatoes weighs 60 lbs.)

7. A boy brought to school checks for coal delivered as follows: 4980 lbs; 5760 lbs; 6130 lbs; 5930 lbs; 6170 lbs; 5380 lbs. At $7.75 a ton (2000 lbs.) what is the value of this coal?

8. A family paying $50.00 a month rent for a heated apartment decided to build a house of their own. The lot cost $600, the building of the house $2800, fuel $85 per year, taxes $30, repairs $60. The rate of interest was 6%. Did they gain or lose by building, and how much?

9. What is the tax on a property valued at $16,500 if it is assessed at $\frac{2}{3}$ its value, and if the rate is 1.2%?

1. A man 35 years old takes out a twenty payment life policy for \$5000. If he dies at the age of 50, find the amount he would have in the bank if he had deposited his yearly premiums at 4% interest compounded annually.

2. A man owns a house valued at \$12,500 which he rents for \$720 a year. Repairs are \$125 a year, taxes \$160 a year, insurance \$48 a year, and depreciations \$150 a year. What rate of income does he get on his investment?

3. A traveling man received a salary of \$3000 a year plus 8% on all sales above \$50,000 a year. What was his yearly income if he sold \$115,000 of goods?

4. The total surface of a cube is 600 square inches. Find its volume.

5. A street 3800 feet long and 42 feet wide is paved at a cost of \$2.45 per square yard. The curbing costs 55 cents per lineal foot of curbing. Find total cost of this improvement.

6. What is the cost of an article sold for \$75.00 with discounts 30%, 10%, 2%?

7. A book listed at \$1.50 is bought by a dealer at list price with discounts of 25% and 10%. How much does he pay for the book? What is the rate of gain on this purchase price if he sells the book at list price?

8. A train goes 125 miles in 3 hrs. 18 minutes. What is the average speed in miles per hour? Find result to the nearest tenth of a mile.

9. A family bought a lot for \$1500 and built a house on it costing \$9800. Taxes are \$115 a year, insurance \$55, repairs on the house \$160, and depreciation \$200 a year. If interest is figured at 7% on the investment how much is the cost per year of living in this house? What monthly rent does this amount to?

1. One pound of chocolates contains 2770 calories; one pound of wheat flour contains 1600 calories; one pound of oatmeal contains 1810 calories. At the prices prevailing in your neighborhood, which of these foods is the least expensive per thousand calories? Which is the most expensive?

2. One pound of milk contains 60 calories of protein; 160 calories of fat, and 90 calories of carbohydrates. What per cent of the total calories of milk is in each of these three forms?

3. Wheat sold at $1.35 a bushel is raised at a cost of $.95 a bushel. Find the gain per cent, using cost as the base, also using selling price as the base.

4. A $1000 bond carrying 5% interest is bought for $985, no brokerage. What is the rate of income on this investment? Find result to the nearest hundredth of one per cent.

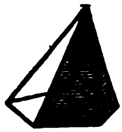
5. The rooms in a house are of the following dimensions: $16\frac{1}{2}' \times 14'$; $18' \times 15'$; $14' \times 12'$; $15\frac{1}{2}' \times 12'$; $16' \times 12\frac{3}{4}'$; $13' \times 10'$; $12' \times 11'$. How many board feet of flooring are required for these rooms, allowing $\frac{1}{6}$ of the lumber for matching and waste in cutting?

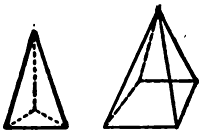
6. Mrs. Jones bought $3\frac{3}{4}$ yards of velvet at $3.75 a yard; $8\frac{1}{2}$ yards of dress goods at $2.25 a yard; $4\frac{1}{4}$ yards of dress goods at $1.75 a yard; trimming and other items for $9.60. What was the amount of her bill?

7. At 95 cents per cubic yard what is the cost of making an excavation 27 feet wide, 50 feet long, and $5\frac{1}{2}$ feet deep?

8. A stock yielding 7% dividend is bought at a cost of $112\frac{3}{8}$, including brokerage. What is the rate of income on this investment?

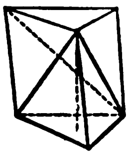
9. A note for $3600 bearing no interest is discounted 75 days before it is due. Find the proceeds if the rate of discount is 7%.

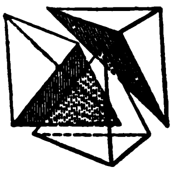


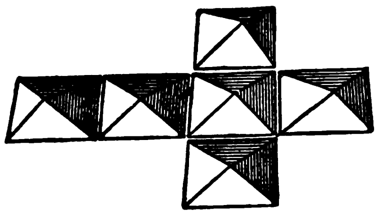


288. Pyramids. Solids of the general shape shown in these figures are called *pyramids.* A pyramid has a polygonal base, and its sides are triangles, which have their vertices in the same point, called the *apex* of the pyramid.

If the base of a pyramid is a triangle, the pyramid is called *triangular.* If the base is a quadrilateral, the pyramid is *quadrangular,* and so on. The perpendicular distance from the apex to the base is the *altitude.*







289. Volume of Pyramid. In the figure to the left a triangular prism is cut into three pyramids, which have equal volumes. In the next figure a cube has been cut into six equal pyramids, each of which has a base equal to a side of the cube, and an altitude equal to half the height of the cube.

From these figures we see that:

The volume of a pyramid equals one-third the volume of a prism having the same base and altitude.

Hence we have the rule:

The volume of a pyramid equals one-third the product of its base and altitude.

290. **Regular Pyramid.** If the sides of a pyramid are all equal triangles, the pyramid is said to be *regular*. The altitude of each of these triangles is called the *slant height* of the pyramid.

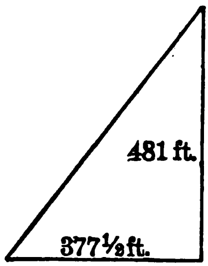
291. **Lateral Area of Pyramid.** The sum of the areas of these triangles is therefore the perimeter of the base multiplied by half the slant height.

WRITTEN EXERCISES

1. The base of the great pyramid of Gizeh in Egypt (see figure page 274) was 755 feet square, and its altitude was 481 feet. Find the volume of this pyramid.

2. Find the slant height of the great pyramid of Gizeh.

Suggestion: Find the hypothenuse of the triangle shown in the figure.



3. Find the lateral area of this pyramid.

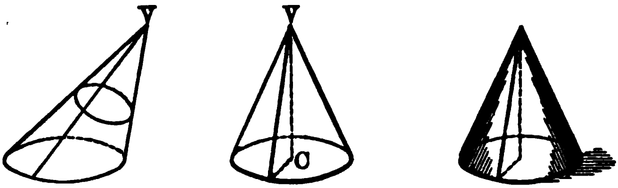
4. A triangular pyramid has a regular base, each side of which is 10 inches. The altitide is 8 inches. Find its volume.

5. Find the slant height of a square pyramid the sides of whose bases are 6 inches, and whose altitude is 10 inches. Find the lateral area and the volume.

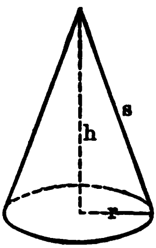
6. Find the volume of a triangular pyramid, the sides of the base being 15 inches each, the altitude of the pyramid being 7 inches.

7. Find the value of a pyramid whose base is a regular hexagon with sides 6 inches, and whose altitude is 12 inches.

8. Find the lateral area of the pyramid in Example 7 if its slant height is 13.08. Show how the slant height of this pyramid may be computed.



292. Cones. If we fasten a string at a point marked V, and move the other end along a curve such as the circle C, the string will trace out a *conical surface*. A *cone* has one flat surface called its base, on which it stands. The point V at the summit of the cone is called its *vertex* or *apex*.



293. Right Circular Cone. A cone whose base is circular, and whose vertex is located vertically above the center of the base, is called a *right circular cone*. In this book we deal only with right circular cones.

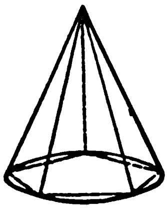
294. Slant Height, Altitude. A line drawn from the vertex to the edge of the base is called the *slant height*, s, of the cone. The perpendicular distance from the vertex to the base is called the *altitude*, h, of the cone. r is the radius of the base. The lines r, h, and s, form a right-angled triangle.

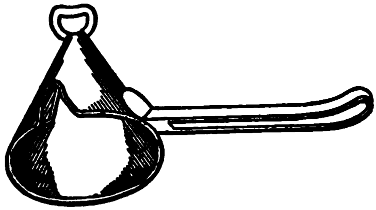
WRITTEN EXERCISES

1. The radius of the base of a cone is 10 inches, and the altitude is 8 inches. Find the slant height.

2. The altitude of a cone is 13 inches, and the slant height is 16 inches. Find the radius of the base.

3. The radius of the base of a cone is 7 inches, and the slant height is 14 inches. Find the altitude.





295. Volume and Area of Cone. From the figure to the left, we see that a pyramid can be constructed which shall differ as little as we please, both in volume and area, from those of a cone. Hence, we conclude that the lateral area and the volume of a cone are found exactly as are those of a pyramid. That is:

The volume of a cone equals one-third the product of the base and altitude.

The lateral area of a cone equals the perimeter of the base multiplied by one-half the slant height.

WRITTEN EXERCISES

1. Find the volume of a cone if its altitude is 8 feet, and the radius of the base is 5 feet.

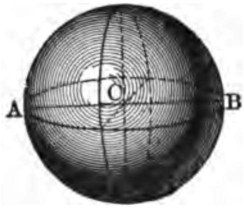
2. How many square feet of material are required to make a conical tent 9 feet high and 10 feet across at the base?

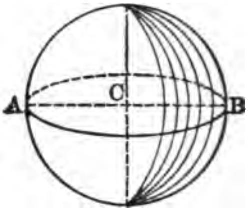
3. How many cubic feet of air will the tent in Example 2 hold?

4. Find the lateral areas of the cones described in Examples 1, 2, and 3 on the opposite page.

5. Find the volumes of these same cones.

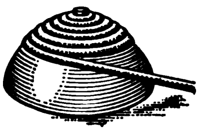
296. Sphere. A *sphere* is a perfectly round ball, such that all points on its surface are the same distance from a point within it which is called its *center*.





297. Radius, Diameter. A line connecting the center and the surface of a sphere is called a *radius*, and a line through the center of the sphere and terminating at both ends in the surface is a *diameter* of the sphere. In the figure CA is a radius and AB is a diameter.

298. The Area of a Sphere. This figure indicates a method of finding the area of a sphere. The length of the rope that is used to cover one hemisphere, when compared with the length used to cover a circular disk of the same diameter, gives approximately the surface of the sphere as compared with the area of the circle. In geometry, very much more accurate methods than this are used. The result is:



The surface of a sphere is four times the area of a circle of the same radius.

Since the area of a circle is πr^2, it follows that the surface of a sphere of radius r is $4\pi r^2$.

Cut an apple into two equal parts, and then cut one of these parts into two equal parts. The curved surface of one of the small parts is equal to the area of the flat surface of the half apple.

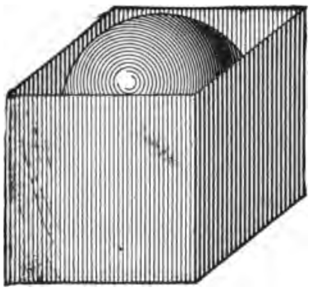
One way to compare the area of the sphere is to weigh a hemispherical shell and a circular disk of the same thickness.

WRITTEN EXERCISES

1. Find the area of a sphere whose radius is 4 inches.

2. Find the area of a sphere whose radius is $7\frac{1}{2}$ inches.

3. The radius of the moon is 1080 miles. Find the area of its surface. How does this area compare with that of the North American Continent, which is 8,038,000 square miles. Is it greater or less than that of the continent of Asia, which is 17,058,000 square miles?



4. A sphere fits exactly into a cubical box. Find the ratio of the area of the sphere to the total surface of the cube.

Solution: Let r = the radius of the sphere. Then its area is $4\pi r^2$. The edge of the cube is $2r$, and its total surface is:

$$6\,(2r)^2 = 24r^2.$$

Hence, the ratio of the areas is $4\pi r^2 : 24r^2 = \pi : 6$.

5. Find the radius of a sphere whose area is 8 feet.

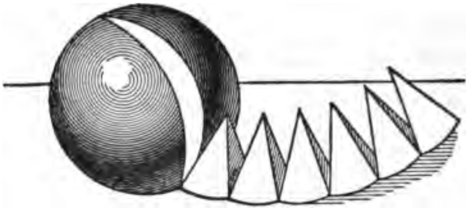
Solution: Let r = radius of the sphere.

Then $8 = 4\pi r^2$. Hence $r^2 = \dfrac{8}{4\pi} = \dfrac{2}{\pi}$ and $r = \sqrt{\frac{2}{\pi}}$

Before finding the square root, reduce $\frac{2}{\pi}$ to a decimal, carrying it to four places.

6. The smaller of the two moons of Mars is supposed to be about 10 miles in diameter. If this is correct, what is the area of its surface? How does this area compare with that of the county in which you live?

7. The dome of a great building is in the shape of a half sphere. Find its area if its radius is 30 feet.



299. The Volume of a Sphere. The figure illustrates how a sphere may be cut into pieces each of which looks like a pyramid. The vertex of each pyramid is at the center of the sphere. The bases are curved surfaces, but if each piece is cut very small, the curvature of the base is so small that it may be regarded as flat.

The surface of the sphere will form the bases of all these pyramids, and the altitude of each pyramid is the radius of the sphere. Hence, the sum of their volumes is the area of the sphere multiplied by one-third the radius.

Hence we have the rule:

The volume of a sphere equals the surface times one-third the radius.

Example. Find the volume of a sphere whose radius is 6 inches.

Solution: The surface $= 4\pi r^2 = 4 \times 3\frac{1}{7} \times 6^2 = 4 \times \frac{22}{7} \times 36 = 452\frac{4}{7}$.

Hence the volume $= \frac{6}{3} \times 452\frac{4}{7} = 905\frac{1}{7}$.

WRITTEN EXERCISES

1. Find the volume of a sphere whose radius is 9 inches.
2. Find the volume of a sphere whose radius is 12 inches.
3. Find the volume of a sphere whose radius is $\frac{3}{4}$ inch.
4. Find the volume of a sphere whose diameter is 10 inches.
5. One cubic foot of iron weighs 480 pounds. Find the weight of a solid iron ball which is 6 inches in diameter.

300. The Cube of a Number. The product obtained by using a number as a factor three times is called the cube of the number.

Thus, $2\times2\times2=8$ is the cube of 2, $3\times3\times3=27$ is the cube of 3, and $\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=\frac{8}{27}$ is the cube of $\frac{2}{3}$.

To indicate that the cube of a number is to be taken, the number 3 is written to the right of and a little above the number.

Thus, $2^3=8$, $3^3=27$, and, $(\frac{2}{3})^3=\frac{8}{27}$

State how you find the cube of a number.

ORAL EXERCISES

Find the values of the following:

1. 1^3	4. 4^3	7. $(\frac{1}{2})^3$	10. $(\frac{3}{4})^3$	13. $(\frac{2}{5})^3$
2. 2^3	5. 5^3	8. $(\frac{1}{3})^3$	11. $(\frac{2}{3})^3$	14. $(\frac{3}{5})^3$
3. 3^3	6. 10^3	9. $(\frac{1}{4})^3$	12. $(\frac{1}{5})^3$	15. $(\frac{4}{5})^3$

301. Short Rule for Volume of Sphere. Since the surface of the sphere is $4\pi r^2$, and since the volume equals the surface multiplied by $\frac{1}{3}r$ we have that the volume of the sphere equals $\frac{1}{3}r\times4\pi r^2=\frac{4}{3}\pi r^3$.

The volume of a sphere equals the cube of the radius multiplied by $\frac{4}{3}$ times π.

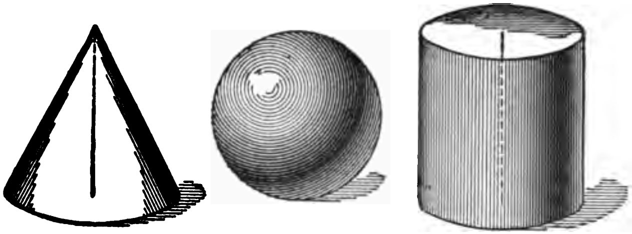
Example. Find the volume of a sphere whose radius is $\frac{3}{4}$.

Solution: Volume $=\frac{4}{3}\pi(\frac{3}{4})^3=\frac{4}{3}\cdot\frac{22}{7}\cdot\frac{27}{64}=\frac{11\cdot9}{7\cdot8}=1.768$.

EXERCISES

Find the volumes of the spheres whose radii are given here:

1. 8	5. 9	9. 12	13. 1.2
2. 3	6. 4	10. $\frac{3}{5}$	14. 2.3
3. 6	7. $\frac{2}{3}$	11. 3	15. 4.5
4. 10	8. $\frac{4}{5}$	12. .7	16. 3.5



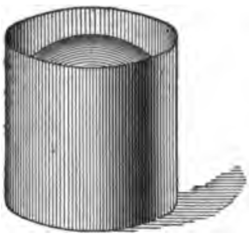
WRITTEN EXERCISES

1. Find the volume of a cone 2 feet in diameter and 2 feet high.

2. Find the volume of a cylinder 2 feet in diameter and 2 feet high.

3. Find the volume of a sphere 2 feet in diameter.

4. Add the volume of the sphere and the cone in Examples 3 and 1, and compare with the volume of the cylinders in Example 2.



The volume of the sphere is exactly $\frac{2}{3}$ the volume of the cylinder, and the cone is just half of the sphere. When we come to study geometry, we shall be able to prove this statement fully. This was discovered by the Greeks about 2300 years ago, and was regarded by them as a most wonderful fact.

5. The largest balls used for bowling are 27 inches in circumference. Find the radius of such a ball.

6. Find the volume in cubic inches of the ball described in the preceding problem.

7. Balls used in bowling are made of wood called *lignum vitæ*, which weighs on an average about 62 pounds per cubic foot. Find the weight of the ball whose volume is required in the preceding problem.

8. What is the approximate diameter of a standard baseball? How many cubic inches are there in such a ball?

PROBLEMS

1. An iron ball used in the governor of an engine is 4 inches in diameter. What is the weight of this ball if iron weighs 480 pounds per cubic foot?

2. The diameter of a standard billiard ball is $2\frac{1}{16}$ inches. What does it weigh if made of ivory, which weighs 114 pounds to the cubic foot?

3. Find the volume of a cube whose edges are all 6 inches.

(Notice that a sphere of radius 3 inches will just fit into the cube in Problem 3.)

4. Find the volume of a regular hexagonal prism the edges of whose base are all 3 inches and whose altitude is 6 inches.

5. Find the volume of a pyramid having a square base, 8 inches on a side, and an altitude 6 inches.

6. Find the volume of a cone 8 inches in diameter and having an altitude 6 inches. By how much do the volumes in problems 5 and 6 differ?

Explain the relation of the pyramid in Problem 5 to the cone in Problem 6.

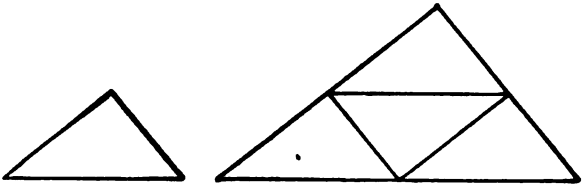
7. Find the volume of a regular hexagonal prism if the sides of the base are 4 inches, and the altitude 6. Compare this volume with the volumes found in Problems 5 and 6.

8. Find the volume of a cylinder with radius 3 inches and altitude 10.

9. Find the volume of a prism with a square base, whose sides are 6 inches and whose altitude is 10 inches.

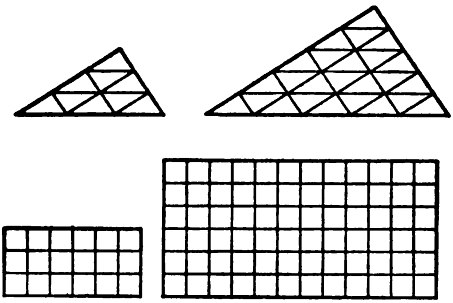
10. Find the volume of a regular hexagonal prism, the sides of whose base are 3 inches and whose altitude is 10 inches.

Explain the relation of the prisms in Problems 9 and 10 to the cylinder in Problem 8.



302. **Areas of Similar Figures.** In these triangles the sides of the larger are twice as long as those of the smaller. The area of the larger triangle is 4 times the area of the smaller. The ratio of the sides is $1:2$, and the ratio of the areas is $1:4$. That is, the ratio of the areas is equal to the ratio of the squares of the sides. This turns out to be true of all similar figures. That is:

The area of any two similar figures are in the same ratio as the squares of their corresponding straight sides.



Verify this rule by counting the small triangles in the two similar triangles in the figure, and the squares in the two similar rectangles.

Example 1. In two similar triangles, two corresponding sides are 3 and 10. Find the ratio of their areas.

Solution: The required ratio $= 3^2 : 10^2 = 9 : 100$.

Example 2. In two similar polygons, two corresponding sides are 2 and 15. Find the ratio of their areas.

Solution: The required ratio $= 2^2 : 15^2 = 4 : 225$.

PROBLEMS

1. In two similar triangles, the ratio of two corresponding sides is 3 : 7. Find the ratio of their areas.

2. In two similar rectangles, the ratio of two corresponding sides is 7 : 15. Find the ratio of their areas.

3. Find the ratio of the areas of two circles whose radii are 2 and 7. (Note that circles are similar figures.)

4. In a garden is a circular flower bed with a walk around it. The width of the walk is equal to the radius of the flower bed. Find the ratio of the areas of the walk and the flower bed.

Suggestion: The area of the circle consisting of the walk and the flower bed is 4 times the area of the flower bed. Why?

5. Two circular disks are made of the same thickness, and the same kind of material. The smaller is 7 inches in diameter, and weighs 18 ounces. The larger is 13 inches in diameter. What is its weight?

6. A card held in front of a light casts a shadow on the wall, similar in shape to the card. The width of the card is 8 inches, while the width of the shadow is 4 feet, 8 inches. How many times the area of the card is the area of the shadow?

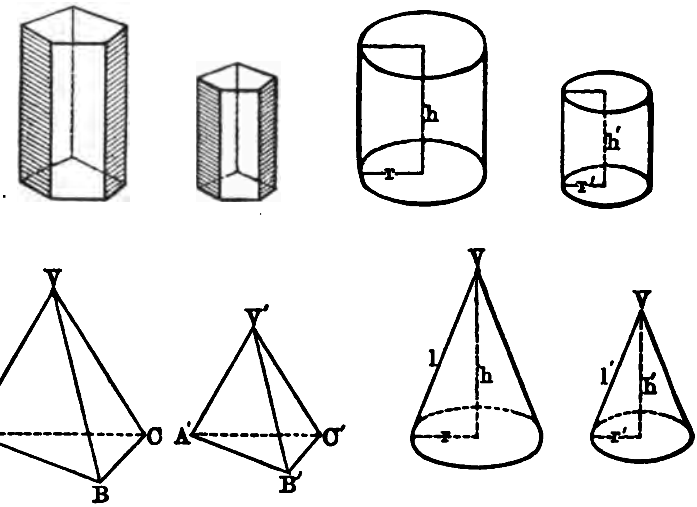
7. A set of drawings for a house are made to the scale $1'' : 4'$. What area of floor space is represented by one square inch in the drawings?

8. A map is made to the scale 1 inch : 10 miles. How many square miles of territory are represented by one square inch of map?

9. A set of drawings for a house are made to the scale $1'' : 3'$. What fraction of the floor of a room will be occupied by the drawing of that floor?

303. Surfaces of Similar Solids. The figures below show four pairs of similar solids. It is easy to see that the surfaces of similar solids are similar figures. Hence we have:

The ratio of the surfaces of similar solids equals the ratio of the squares of their corresponding straight lines.



Example 1. The ratio of the diameters of two spheres is 2 : 5. Find the ratio of their surfaces.

Solution: The required ratio $= 2^2 : 5^2 = 4 : 25$.

Example 2. Two stoves are exactly the same shape. One is 21 inches high, and the other is 29 inches high. What is the ratio of their surfaces?

Solution: The required ratio $= 21^2 : 28^2 = 441 : 841$.

Example 3. Two gasoline tanks are of the same shape, and are made of the same kind and thickness of material. One is twice as long as the other. If the smaller weighs 72 pounds, what does the larger weigh?

Solution: Let x = weight in pounds of the larger tank.

Then, $x : 72 = 2^2 : 1^2 = 4 : 1$, and $x = 4 \times 72 = 288$ (lbs.).

304. Volumes of Similar Solids. The following examples lead naturally to the rule giving the ratio of the volumes of similar solids.

WRITTEN EXERCISES

1. The radius and altitude of a cylinder are both 1. Find its volume.

2. The radius and altitude of a cylinder are both 2. Find its volume. Compare the volume with that of Example 1.

3. The base of a right prism is a square whose sides are 1, and its altitude is 2. Find its volume.

4. The base of a right prism is a square whose sides are 2, and its altitude is 4. Find the volume and compare with that of Example 3.

5. The radius of a right circular cone is 1, and its altitude is 1. Compare the volume of this cone with that of a cone whose radius and altitude are both 2.

The cylinders of Problems 1 and 2 are of the same shape, that is, they are similar. The ratio of their radii and also of their altitudes is 1 : 2. The ratio of their volumes was found to be 1 : 8.

The prisms of Problems 3 and 4 are similar, and the cones of Problem 5 are similar. In both cases their dimensions are in the same ratio, 1 : 2, and their volumes are in the ratio 1 : 8.

Following is the rule giving the ratio of the volumes of similar solids:

The ratio of the volumes of two similar solids is equal to the ratio of the cubes of their corresponding straight lines.

Example 1. The diameters of two bowling balls are 5 inches, and 7 inches. Find the ratio of their weights.

Solution: The required ratio is $5^3 : 7^3 = 125 : 343$.

Example 2. If shells for a 12-inch and a 15-inch gun are of the same shape and material, find the ratio of their weights.

Solution: The required ratio $= 12^3 : 15^3 = 4^3 : 5^3 = 64 : 125$.

PROBLEMS

1. Two Oriental rugs are similar in shape. One is $7\frac{1}{2}$ feet wide, and the other, $5\frac{1}{3}$ feet wide. Find the ratio of their areas. If the smaller rug costs \$65, what should be the price of the larger one?

2. Two fields are similar in shape. One is $2\frac{1}{3}$ times as long as the other. Find the ratio of their areas.

3. Two milk cans are similar in shape. One is 18 inches tall, and the other is 24 inches tall. Find the ratio of their volumes.

4. If the cans in the preceding problem are of the same thickness, and made of the same kind of material, what is the ratio of their weights when empty? (What is the ratio of the surfaces?)

5. Find the surface of a sphere whose radius is 5 inches.

6. Using the result of Example 5, how can you most easily find the surface of a sphere whose radius is 15 inches?

7. Find the volume of a sphere whose radius is 4 inches.

8. Using the result of Example 7, find the volume of a sphere whose radius is 7 inches.

9. Two boxes are exactly of the same shape. The ratio of their lengths is $\frac{2}{3}$. Find the ratio of their cubical contents. Also find the ratio of their weights if they are made of the same kind of material.

10. Find the lateral area of a cylinder whose diameter and altitude are both 10 inches.

PROBLEMS

1. Find the lateral area of a cylinder whose diameter and altitude are both 6 inches.

2. Find the surface of a sphere whose radius is 3.

3. Find the lateral area of a cylinder whose diameter and altitude are both 10 inches. Also find the surface of a sphere whose radius is 5 inches. Compare these areas.

Note that the sphere of Example 2 will just fit inside the cylinder of Example 1, and that the sphere of Example 3 will just fit inside the cylinder of that example. In each case the lateral area of the cylinder is exactly equal to the surface of the sphere.

4. Find the volume of the cylinder in Example 1, and of the sphere in Example 2, and compare them.

5. Find the volumes of the cylinder and the sphere in Example 3, and compare them.

6. Two rifle bullets are of the same shape. The ratio of their diameters is 25 : 32. Find the ratio of their weights.

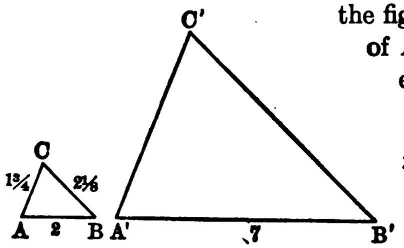
7. Assuming that the projectiles fired from 11-inch cannon and from 15-inch cannon are similar in shape, find the ratio of their weights.

8. If a man 5 feet 9 inches tall weighs 160 pounds, what should be the weight of a man 6 feet tall, supposing them to be similar in shape?

9. Two ocean steamships are 560 feet and 900 feet long respectively. If these steamers are similar in shape, what is the ratio of their volumes? What is the ratio of their surfaces?

10. If all the dimensions of a tree be multiplied by 2, by how much will its weight be multiplied? By how much will its surface be multiplied?

MISCELLANEOUS PROBLEMS IN MENSURATION



1. If the triangles ABC and A′B′C′ in the figure are similar, find the lengths of A′C′ and B′C′, using the dimensions given in the figure for the other sides.

2. Draw a figure showing how to find the height of a building by means of similar triangles.

3. Draw a figure showing how to find the distance across a river by means of similar triangles.

4. Find the area of a triangle with base 46 feet, and altitude 32 feet.

5. One side of a triangle is 24 inches, and the altitude upon this side is 18 inches. Another side of this triangle is 32 inches. Find the altitude upon the second side.

6. Find the area of a trapezoid whose bases are 14 rods and 25 rods, and whose altitude is 17 rods.

7. Find the side of a square whose area is equal to that of a rectangle 64 rods by 144 rods.

Suggestion: Find the area in square rods and approximate the square root.

8. Find a side of a square field which contains 45 acres.

9. Find the dimensions in feet of a square lot containing just one acre of ground.

10. Find the length of the hypothenuse of a right triangle if its two other sides are 10 inches and 8 inches respectively. Find result to the nearest hundredth.

1. The hypothenuse of a right triangle is 14 inches, and one side is 12 inches. Find the length of the remaining side.

2. Find the volume of a rectangular bin 10 feet long, 8 feet wide, and 7 feet deep. If 35 cubic feet hold one ton of coal, how many tons will this bin hold?

3. A room is 14 feet wide, and 18 feet long. What is the length of a diagonal of the room?

4. A diagonal of a room is 21 feet, and the length of the room is 18 feet. Find its width.

5. The sides of an equilateral triangle are 48 feet. Find its area.

6. Find the area of a regular hexagon whose sides are 4 inches.

7. Find the area of a circle whose radius is 10 inches.

8. Find the radius of a circle whose circumference is 17 feet.

9. The diameter of the wheel on a railway passenger-coach is 36 inches. How far does this wheel go in turning around once? How many times will this wheel turn around in going one mile? How many times will this wheel go around in going from Chicago to Boston, a distance of 1120 miles?

10. Find the side of a square whose area is equal to that of a triangle, with a base 12 feet and altitude 8 feet.

11. Find the area in acres of a rectangular plot 287 feet wide and 490 feet long.

Surveyors usually measure lengths in feet, and hence obtain the area in square feet. To reduce to acres, they divide the number of square feet by 43560.

12. The hypothenuse of a right triangle is 24, and one side is 18. Find the length of the remaining side.

13. Find the altitude of an equilateral triangle whose sides are 25 inches.

MISCELLANEOUS PROBLEMS IN MENSURATION

1. Find the volume of a prism whose base is a rectangle 4 by 6 inches, and whose altitude is 10 inches.

2. Find the volume of a prism whose base is an equilateral triangle, with sides 10 inches, and whose altitude is 12 inches.

3. Find the volume of a prism whose base is a regular hexagon, with sides 8 inches long, and altitude 10 inches.

4. Find the total area of the prism in Example 2.

5. Find the total area of the prism in Example 3.

6. Find the total area of a cylinder whose radius is 12 inches and altitude 10 inches.

7. Find the volume of the cylinder in Example 6.

8. The ratio of two corresponding sides of similar triangles is 3 : 11. Find the ratio of their areas.

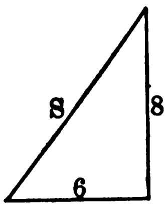
9. Find the volume of a pyramid whose altitude is 8, and whose base is a square with sides 7.

10. The base of a cone is a circle with radius 6 inches. What is the altitude of the cone if its volume is 150 cubic inches?

11. Find the volume of a sphere whose radius is 4 inches.

12. Of two spheres, one has $3\frac{1}{2}$ times the radius of the other. Find the ratio of their volumes.

13. If projectiles for 8-inch and 11-inch guns are of the same shape, find the ratio of their weights.



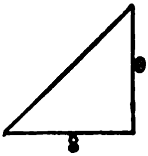
14. Find the volume of a circular right cone with a radius of 6 inches and altitude of 8 inches.

15. Find the lateral area of the cone in Example 14.

Suggestion: Find S in the right triangle shown in the figure. This will be the slant height of the cone.

Find the Areas of the following figures

1.



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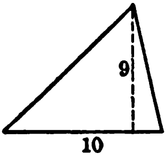

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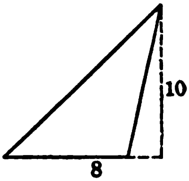

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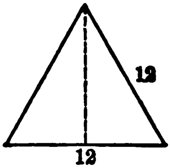

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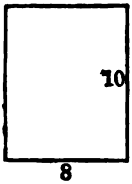

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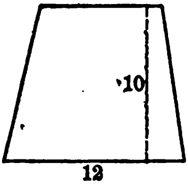

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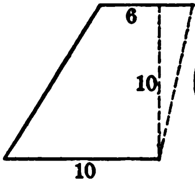

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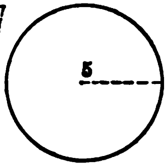
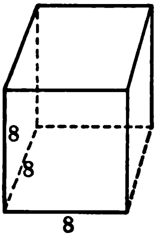


Find the Volume of the following

9.



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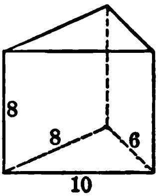

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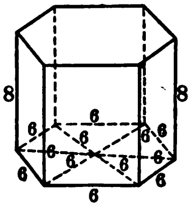

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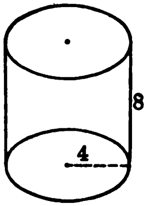

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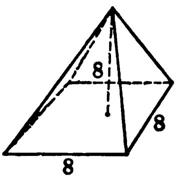

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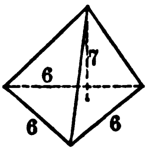

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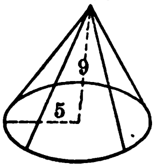
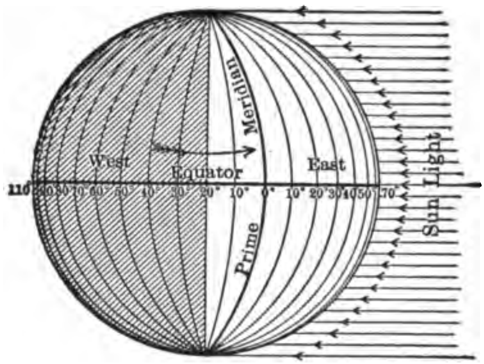


16.

LONGITUDE AND TIME

305. **The Earth's Equator.** The *equator* is a great circle around the earth exactly midway between the two poles.

The *meridians* are half circles running from pole to pole.



306. **The Prime Meridian.** The meridian passing through the observatory at Greenwich, near London, England, is called the *prime meridian.*

307. **Longitude.** Longitude is measured east and west from the prime meridian. In the figure, the distance from the prime meridian is indicated by degrees marked on the equator. The distance of a place in degrees east or west of the prime meridian is called the *longitude of that place.*

We know that the earth turns around eastward, so that a point on the equator moves clear around the circle, or 360°, in 24 hours. Such a point moves $\frac{1}{24}$ of 360°, or 15°, in one hour.

ORAL EXERCISES

1. In the figure above, the point marked 70° east is directly under the sun. That is, there is noon at all places 70° east of Greenwich. How long before there will be noon at a place 45° east of Greenwich?

To answer this question, find out how long it will take the point marked 45° east to move 25° eastward.

2. When it is noon 70° east of Greenwich, how long before it will be noon at 25° east? At 10° east? At 5° west? At 65° west?

Since 15° of longitude corresponds to 1 hour of time, it follows that $\frac{1}{60}$ of 15°, or 15′ of longitude, corresponds to 1 minute of time, and $\frac{1}{60}$ of 15′, or 15″, of longitude corresponds to one second of time.

This gives us the following table:

360° longitude corresponds to 24 hours of time.
15° longitude corresponds to 1 hour of time.
1° longitude corresponds to 4 minutes of time.
15′ longitude corresponds to 1 minute of time.
1′ longitude corresponds to 4 seconds of time.
15″ longitude corresponds to 1 second of time.

WRITTEN EXERCISES

1. Draw a figure like the one on the opposite page, placing the prime meridian so that it will be noon at a place 30° east longitude.

2. From this figure answer the following questions: When it is noon at a place 30° east longitude, how long before it will be noon at 15° east longitude? On the prime meridian? At 30° west longitude? At 45° west longitude? At 60° west longitude? At 90° west longitude?

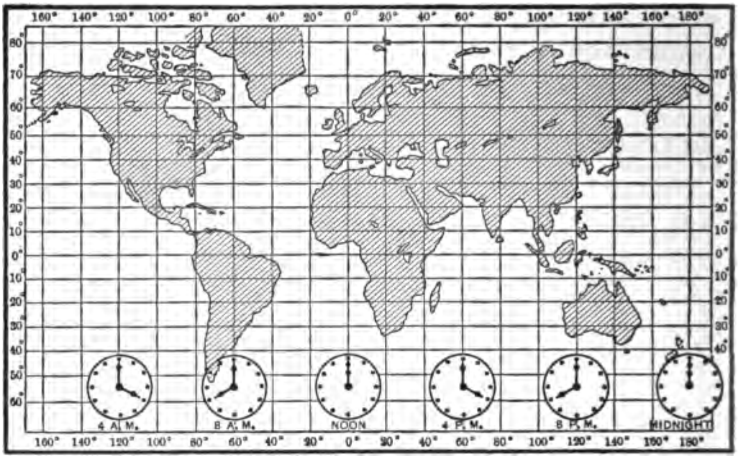
3. If two places differ by 75° in longitude, by how much do they differ in time?

4. If two places differ by 38° in longitude, by how much do they differ in time?

5. If two places differ by 18° 35′ in longitude, by how much do they differ in time?

6. If two places differ by 2 hours, 30 minutes in time, by how much do they differ in longitude?

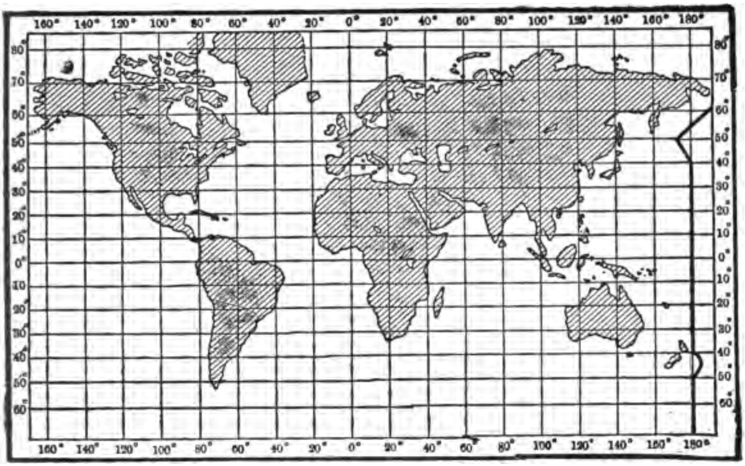
7. If two places differ by 3 hours, 17 minutes, and 30 seconds in time, by how much do they differ in longitude?



The above map shows noon at the prime meridian. It is then 4 P. M. at a point 60° east, 8 P. M. at a point 120° east, and midnight at a point 180° east. It is 8 A. M. at a point 60° west, and 4 A. M. at a point 120° west.

If a person goes around the earth, eastward, it is clear that he will lose one day. That is, he will follow the sun so that each of his days will be a little longer by the watch. When he has completed his journey, he will have seen the sun rise and set just once less than the people who lived in one place during this time. If, on the other hand, he journeyed around the earth westward his days would be a little shorter by the watch, and in going clear around he would see the sun rise and set once oftener than the people who lived in one place during this time. To avoid getting his dates wrong, it would be necessary to throw out one day at some point in the journey in case he goes westward, and to add a day at some point in case he goes eastward.

A person crossing the Atlantic Ocean on a steamer finds that he is obliged to change his watch every day, sometimes by as much as nearly an hour.



308. The International Date Line. By international agreement, an *international date line*, shown in the above map, has been fixed.

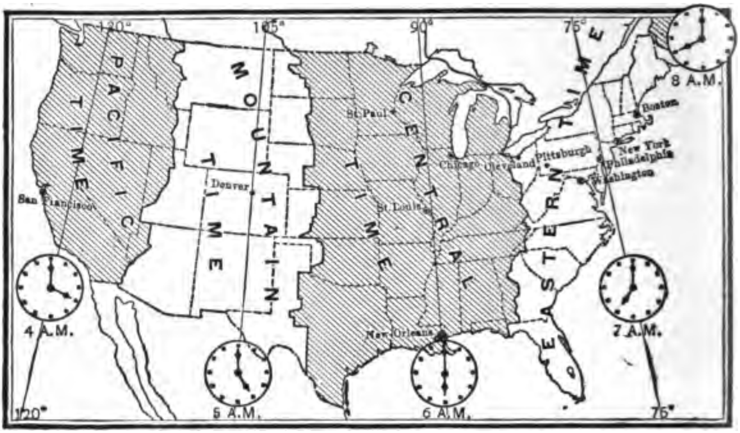
If a traveler going westward, crosses this line April 4th, he will call the next day April 4th also, and if he crosses it going eastward on that date, he will call the next day April 6th.

WRITTEN EXERCISES

1. What is the difference in time between two places, one of which is 65° west, and the other 90° west of the prime meridian?
Suggestion: First find the difference in longitude.

2. Washington, D. C., is 77° 5′ west. When it is noon at Washington, what is the solar time at each of the following places:

Boston	71° 10′ West	Paris	2° 30′ East
New York	74° 0′ West	Berlin	13° 20′ East
Baltimore	76° 40′ West	Rome	12° 29′ East
Chicago	87° 35′ West	St. Petersburg	30° 20′ East
Minneapolis	93° 20′ West	Bombay	72° 48′ East
Seattle	122° 15′ West	Alexandria (Egypt)	29° 53′ East



309. Standard Time. If a person goes one degree eastward or westward, his watch will be wrong by 4 minutes. Hence to keep his watch correct, such a person would need to keep changing it constantly. To avoid this inconvenience, the railways have adopted what is called *standard time*. The North American continent is divided into five belts running north and south. In the whole eastern belt, the time used is correct solar time, for 60° west. This is called *Atlantic time*. In the next belt, the time used is correct solar time for 75° west. This is called *Eastern time*. In the remaining belts, the time used is the correct solar time for 90°, 105° and 120°, respectively. These are called *Central, Mountain* and *Pacific time* respectively.

Each railway changes its time at some convenient city on its line. Hence, the boundaries between these time-belts are jagged.

The clocks on the map show standard time in each belt when it is noon at the prime meridian.

EXERCISES

In what time belt is each of the places in Example 2, page 299?

MISCELLANEOUS PROBLEMS

1. What is the difference between standard and solar time in Chicago, where the longitude is 87° 35′?

Suggestion: What is the difference in longitude between Chicago and a point 90° west?

2. Is standard time too fast or too slow, and how much, at each of the following cities:

New York City,	longitude 74°	Jacksonville, Florida,	81° 39′
Boston,	longitude 71° 10′	Oklahoma City,	97° 30′
Minneapolis,	longitude 93° 20′	Helena, Montana,	112° 3′

3. On a steamer going eastward across the Atlantic, the clock is set ahead 34 minutes in one day. How many degrees and minutes has the steamer traveled in the 24 hours?

4. On a cloudy day, the captain of a steamer judged the location on the ship by knowing the direction and speed of the ship. If he decides that the ship had moved eastward 9° 20′, how much is the ship's clock set forward?

5. A man leaves New York City for Europe with his watch keeping eastern standard time (75° west). After being out several days his watch is 3 hours 53 minutes slow. In what longitude is he?

6. A man whose watch is keeping standard Central time, sails for the Mediterranean. One day he finds that his watch is 7 hours 17 minutes slow. In what longitude is he?

7. A telegram must reach London by 3 P. M. At what time (standard time) must it be sent from Denver, if one hour is allowed for sending the telegram?

8. If the voting places close at the same hour (standard time), in Boston and in San Francisco, how much earlier do they close in Boston than in San Francisco?

FEDERAL FARM LOANS

310. Farm Loans Through Loan Associations. Under the provisions of the Federal Farm Loan Act of 1916, ten or more persons engaged in farming or about to purchase farms may form a farm loan association, for the purpose of obtaining loans from their federal farm loan bank. The principal regulations about such loans are:

(*a*) The loan must be secured by a first mortgage on farm land and must not be for more than 50% of the value of this land.

(*b*) The loan cannot be for less than $100 nor for more than $10,000.

(*c*) The money must be used only for (1) the purchase of farm land; (2) for the payment of a debt on the land contracted before the organization of the first national farm loan association in the county in which the land is located; (3) for the purchase of stock or machinery or for the improvement of the farm; (4) or for the payment of any debt on the farm if the money borrowed was used as stated under (1) and (3).

(*d*) Under no circumstances will the rate of interest be more than 6%. There shall be no charges of any kind to raise the expense to the borrower beyond this rate.

(*e*) The loan must be repaid in equal annual or semi-annual instalments. After 5 years the whole loan or any part of it may be repaid at any time.

311. Loans Through Joint Stock Land Banks. Under this same act, Joint Stock Land Banks may be organized for the handling of farm loans from the federal land banks. Loans obtained through the joint stock land banks are not limited to $10,000, need not be used for any specific purpose, and the borrower is not obliged to be actually engaged in farming. All that is required is that he owns the farm land and can give a first mortgage on it.

312. Amortization of Federal Farm Loans. The borrower agrees to make a certain number of equal annual or semi-annual payments sufficient to cover the interest on the loan, and to repay the loan in a period not less than five years or more than forty years. He may, however, repay the whole loan or any part of it after five years. The payment of equal instalments to cover interest and principal in a given time is called *amortization* (the word really means "killing") of the debt. Below is given part of an amortization table used by a joint stock land bank. The period of the loan in this case is 33 years, the rate of interest 6%, the payments are made semi-annually, and the loan is $1000.

Payment No.	Installment	Interest	Applied on Principal	Principal Still Unpaid
1.........	$35.00	$30.00	$5.00	$995.00
2.........	35.00	29.85	5.15	989.85
3.........	35.00	29.70	5.30	984.55
4.........	35.00	29.54	5.46	979.09
5.........	35.00	29.37	5.63	973.46
35.........	35.00	21.34	13.66	697.72
36.........	35.00	20.93	14.07	683.65
37.........	35.00	20.51	14.49	669.16
38.........	35.00	20.07	14.93	654.23
39.........	35.00	19.62	15.38	638.85
62.........	35.00	4.66	30.34	124.97
63.........	35.00	3.75	31.25	93.72
64.........	35.00	2.81	32.19	61.53
65.........	35.00	1.85	33.15	28.38
66.........	29.23	.85	28.38	....

EXERCISES

1. Using the table given here, find the semi-annual payment on a 33 year loan of $7500.

2. Make and solve other problems on federal farm loans.

313. **Building and Loan Associations.** Building and loan associations originated in England in 1795, and were first established in the United States in 1831. They are now popular both in the United States and in Europe. The purpose of these Associations is "to encourage thrift and saving and to facilitate the owning of houses by its members." Shares ranging in par value from \$25 to \$500 (but generally \$200) are issued by these Associations and sold to subscribers on weekly or monthly payments, 25 cents per week or \$1.00 per month for each share. These payments continue until their sum, together with the apportioned profits, amount to the par value of the shares. When the shares are paid up the member is entitled to withdraw the full value of his shares.

314. **Sources of Profit of a Building and Loan Association.** The sources of profit of building and loan associations are the interest paid on loans, fines and premiums paid by members, and profits from withdrawals.

Members borrowing from an association pay interest just the same as if they borrowed from any other source. Fines are usually imposed on members for failure to pay instalments promptly as they fall due. Sometimes the fine is so much per share on which payment has been delayed. Ten cents a share is a usual fine. Again, the fine may be a certain per cent on the unpaid dues.

315. **Method of Making Loans.** Money is usually loaned only to members of the association. If a person not a member wishes to borrow he becomes a member by subscribing for stock at least to the par value of the amount he wishes to borrow.

If there is demand for more loans than there is money available, loans are made to those offering the highest rate of interest, or to those offering the largest cash premium, or to those offering to make the largest number of payments in advance on the stock for which they have subscribed.

316. **Rate of Interest Paid.** The actual rate of interest paid is often high, though it may be paid partly in the form of premiums or advance payment on shares. Building and loan associations are usually exempt from usury laws, because the profits are shared by the members.

317. **Book Value of Shares.** From time to time the earnings of the association are apportioned among the various shares. For the method of making this apportionment see page 304. The book value of a share consists of all payments made on it together with the profits apportioned to it.

318. **Rate of Interest on Investments in Building and Loan Associations.** The rate obtained by those who subscribe for shares and continue their payments until the shares are fully paid for is usually rather high. However, those who withdraw the cash value of their shares before they are paid up get the actual amount they have paid in plus a very low rate of interest. The withdrawal value is nearly always lower than the book value, the exact value being fixed from time to time by the directors of the Association.

In general it may be said that investment in a building and loan association is profitable for those who always make their payments promptly and who continue to do so until the shares are fully paid up. There are financial penalties of many kinds for those who do not make all payments on time. Indeed this is one of the reasons for the high returns of those who meet all their obligations promptly.

319. **The Several Series Plan.** There are several different kinds of associations; but the most common is the so-called several series plan. Suppose an association is started January 1, 1919. A series of 600 shares may be issued. As soon as these are fully subscribed for, another series is issued, and so on. In many cases new series are issued only quarterly or semi-annually or annually.

320. Finding Book Values of Shares. The method of finding the book value of shares is best shown by an example:

Shares were issued and subscribed for as follows: January 1, 1919, 500 shares; April 1, 600 shares; July 1, 400 shares; October 1, 800 shares. Profits, January 1, 1919, to July 1, 1919, \$180; July 1, 1919, to January 1, 1920, \$540. Find the book value of the shares January 1, 1920, their value being computed semi-annually.

We first find the book value July 1, 1919. To find the investment for one month for one share of each series we notice that the investment for the first month is \$1.00, for the second month \$2.00, and so on. Hence, in the case of the first series we need to find the sum $1 + 2 + 3 + 4 + 5 + 6 = 21$, and for the second series, $1 + 2 + 3 = 6$.

Solution:

Investment for one mo. on a share of 1st series = \$21
Investment for one mo. on a share of 2nd series = 6
Total investment for one mo. 1st series = 21×500 = \$10500
Total investment for one mo. 2nd series = 6×600 = 3600
Total investment for one mo. . . . . = \$14100

\$180 ÷ 14100 = \$.012766 = profit on one dollar for one month.
Hence, $21 \times .012766 = .26781$, profit on one share of 1st series $6 \times .012766 = .076596$, profit on one share of 2nd series.
Hence the book values are:
6. + .26781 = 6.27, for 1st series
3. + .076596 = 3.08, for 2nd series

Check: $500 \times .26781$ = \$133.91
$600 \times .076596$ = 45.96
Total profit \$179.87

The difference of \$.13 is due to omission of decimals. The error is not sufficient to warrant the use of further decimal places.

To find the book value January 1, 1920, each share of the first series is credited with an investment of \$6.27 for the whole period and each share of the second series with \$3.08 for the whole period.

Investment for one mo. of 1st series = $6 \times 6.27 + 21 = 58.62$
Investment for one mo. of 2nd series = $6 \times 3.08 + 21 = 39.48$
Investment for one mo. of 3rd series = 21.
Investment for one mo. of 4th series = 6.

We now proceed as above to find the required value.

PROBLEMS

1. In an association just starting, shares were issued and subscribed for as follows: July 1, 1915, 150 shares; October 1, 300 shares; January 1, 1916, 250 shares; April 1, 1916, 350 shares. Profits July 1, 1915, to January 1, 1916, $200; from January 1, 1916, to July 1, 1916, $450. Find the book value of the shares on this last date, the profits being apportioned every six months.

Suggestion: First find the book value of the shares on January 1, 1916, and then on July 1. Proceed as on page 304.

2. On January 1, 1916, there were in force shares with book value as follows: 460 shares, $130.60 each; 340 shares, $112.50 each; 200 shares, $89.40 each; 150 shares, $58.50; 530 shares, $32.80; 412 shares, $15.50. Shares were subscribed for as follows: January 1, 1916, 200; March 1, 250; May 1, 300. Profits from January 1, 1916, to July 1, 1916, $2300. Find the book values on the last named date.

3. In an association starting business April 1, 1918, shares were subscribed for as follows: April 1, 1918, 400; July 1, 350; October 1, 500; January 1, 1919, 250; April 1, 1919, 700. Withdrawn October 1, 1918, 20 shares of the April 1 series, and April 1, 1919, 50 shares of the July 1, 1918, series. Earnings April 1, 1918, to July 1, 1918, $50; July 1, 1918, to January 1, 1919, $350; January 1 to July 1, $850. Find book value of the shares of each series on this date.

Suggestion: The book value is computed for July 1, 1918, although the association has been in operation only three months, because January 1 and July 1 are customary dates for closing the books.

In computing the book value for January 1, 1919, consider only 380 shares of the April 1 series, since 20 shares were withdrawn during the period July 1, 1918, to January 1, 1919. Similarly in computing the book value for July 1, 1918, consider only 300 shares of the July 1, 1918, series, since 50 shares of this series were withdrawn during this period.

FOREIGN EXCHANGE

321. **Foreign Trade.** The foreign trade of the United States amounts to several billion dollars a year. Thus, in a recent year our imports amounted to $2,197,884,000, and our exports to $4,333,659,000.

322. **Kind of Money Used in Payment Among Nations.** Gold is the only money that is used freely between the various nations. Since practically immediate payment must be made for all our foreign sales and purchases, it is clear that, unless some means are found for setting off our imports against our exports, immense quantities of gold must be transported to and from our shores each year.

The bankers, however, have succeeded so well in cancelling mutual indebtedness among nations that in a recent year our total exports of gold were enough to pay for about 3.5% of our imports, and our total imports of gold were enough to pay for about 2.2% of our exports.

323. **The Bill of Exchange.** When a banker writes an order on another banker in this country, this order is called a *draft* (see page 127). When he issues such an order on a bank in a foreign country the order is called a *bill of exchange.* The only difference between a draft and a bill of exchange is that one is drawn on a foreign bank and the other on a bank in the United States.

324. **Par of Exchange on England.** When a bill of exchange on England is bought in the United States the bill calls for a certain number of pounds sterling, while it is paid for in dollars. Hence the question arises: How many dollars equal one pound sterling? By actual weight, the pure gold in the pound sterling is equal to the weight of pure gold in $4.8665 of American gold coin. This is par of exchange; that is, when exchange is at par a man in New York must pay $4.8665 for one English pound.

325. **Par of Exchange on France and Germany.** Exchange on Paris is usually given by stating how many francs and centimes can be bought for \$1.00. The rate of exchange on Berlin is given by stating how much 4 marks will cost.

Thus, a rate of 5.29 on Paris means that 5 francs 29 centimes may be bought for \$1.00, while a rate of .91¼ on Berlin means that 4 marks will cost \$.91¼.

Par of exchange on Paris is 5.1813 francs, and on Berlin \$.942.

326. **Par of Exchange on Some Other Nations.** Following are monetary units of some other nations and the par of exchange in dollars of these units.

Country	Name of Unit	Par of Exchange
Canada, British Honduras ...	Dollar	\$1.00
Belgium, Switzerland	Franc (Fr.)	0.1930
Italy	Lira (L.)	0.1930
Scandinavian Countries	Crown (Cr.)	0.2680
Austria-Hungary	Crown (Cr.)	0.203

327. **Rate of Exchange.** The actual rate of exchange charged varies from time to time, but usually within narrow limits.

WRITTEN EXERCISES

Find the cost of the following bills of exchange:

	Face of Bill	Rate of Exchange
1.	£560 (pounds sterling)	\$4.8555
2.	4800 marks	\$.894
3.	15360 francs	5.12 francs
4.	10370 crowns	\$.2695

5. During the great war one ship brought to New York 2,864,000 pounds sterling. What was the value of this gold if the pound pieces averaged \$4.650 in value? (The gold pieces were below par because they were worn.)

328. **Income Tax Laws.** On January 1, 1914, a law went into effect levying a tax on all personal incomes in the United States above a certain amount. This law was amended by an Act of September 8, 1916, and on October 3, 1917, a law was enacted imposing special war income taxes.

329. **Exemptions Under the Law of 1916.** Under the act of 1916, all net incomes above $3000 per annum are taxable in the case of persons not having a family, while all net incomes above $4000 are taxable in the case of married persons living with their families.

Thus, a single man having a net income of $10,000 has a taxable income of $7000, while a married man having a $10,000 income has a taxable income of $6000.

330. **Exemptions Under the Law of 1917.** Under the act of 1917 an additional tax is imposed on all net incomes above $1000 per annum in the case of single persons and on incomes above $2000 per annum in the case of married persons and heads of families. The amount of $200 is added to the $2000 of exempted income for each dependent child.

Thus, under the Act of 1917 an unmarried person having an income of $5000 pays tax on $4000, and a married person with no dependent children pays tax on $3000.

A married person with 3 dependent children and an income of $5000 pays tax on $5000 − $2600 = $2400.

331. **The War Tax an Additional Tax.** The act of 1917 does not repeal the act of 1916, but imposes an *additional tax*.

That is, to compute the tax on a certain income under these two acts, compute the tax under each act separately and add the results. Thus a single person having an income of $5000 pays tax on $2000 under the act of 1916, and an additional tax on $4000 under the act of 1917.

332. Normal Taxes under the Acts of 1916 and 1917. The act of 1916 imposes a normal tax of 2% on all incomes above the exempted amounts, and the act of 1917 imposes an additional normal tax of 2% on all incomes above the exempted amounts.

333. Surtaxes. The acts of 1916 and 1917 both provide for a graduated increase of the tax. The taxes above the normal 2% provided for in each act are called *surtaxes*.

The table on page 310 shows the surtaxes imposed by these acts.

Example 1. Find the income tax under the act of 1916 of a married man having an annual income of \$75,000.

Solution: 2% on \$71,000 = \$1420
1% on 20,000 = 200
2% on 20,000 = 400
3% on 15,000 = 450
Total tax..... \$2450

Example 2. Find the income tax under the act of 1916 of a single man having an annual income of \$37,000.

Solution: 2% on \$34,000 = \$680
1% on 17,000 = 170
Total tax..... \$850

Example 3. A man having a family of 4 dependent children has an annual income of \$17,500. Find his combined income tax under the acts of 1916 and 1917.

Solution: 2% on \$14,700 = \$294 (normal tax, law 1917)
2% on 13,500 = 270 (normal tax, law 1916)
1% on 2500 = 25
2% on 2500 = 50
3% on 2500 = 75
4% on 2500 = 100
5% on 2500 = 125
Total tax.. \$939

334. Table Showing Income Tax. The rates of taxation imposed by the two acts of 1916 and 1917, and also the combined rates, are shown in the following table.

INCOME TAX TABLE

	LAW 1916	LAW 1917	TOTAL SURTAX RATE	NET INCOME	TOTAL SURTAX
NORMAL TAX					
Single Persons					
On incomes above $1,000	......	2%			
On incomes above 3,000	2%				
Married Persons					
On incomes above $2,000	......	2%			
On incomes above 4,000	2%				
SURTAXES					
On net income between:					
$5,000 and $7,500	None	1%	1%	$5,000	$00
7,500 and 10,000	None	2%	2%	7,500	25
10,000 and 12,500	None	3%	3%	10,000	75
12,500 and 15,000	None	4%	4%	12,500	150
15,000 and 20,000	None	5%	5%	15,000	250
20,000 and 40,000	1%	7%	8%	20,000	500
40,000 and 60,000	2%	10%	12%	40,000	2,100
60,000 and 80,000	3%	14%	17%	60,000	4,500
80,000 and 100,000	4%	18%	22%	80,000	7,900
100,000 and 150,000	5%	22%	27%	100,000	12,300
150,000 and 200,000	6%	25%	31%	150,000	25,800
200,000 and 250,000	7%	30%	37%	200,000	41,300
250,000 and 300,000	8%	34%	42%	250,000	59,800
300,000 and 500,000	9%	37%	46%	300,000	80,800
500,000 and 750,000	10%	40%	50%	500,000	172,800
750,000 and 1,000,000	10%	45%	55%	750,000	297,800
1,000,000 and 1,500,000	11%	50%	61%	1,000,000	435,300
1,500,000 and 2,000,000	12%	50%	62%	1,500,000	740,300
Over 2,000,000	13%	50%	63%	2,000,000	1,050,300

The rate given under the head "Total Surtax Rate" is the sum of the surtax rates in the two laws enacted in 1916 and 1917. Hence, instead of computing the two surtaxes separately and adding, the total surtax may be found at once.

In the last column the total surtax is computed for net incomes such as $5,000, $7,500, etc. To use this to compute the surtax on a net income of $47,000 add 12% of $7,000 to $2,100.

PROBLEMS

1. What is the income tax, under the act of 1916, of a single person having an annual income of $38,000? Also find his combined tax under the acts of 1916 and 1917.

2. A married person having three dependent children has an annual income of $47,500. Find his income tax under the act of 1916. What is the average rate of tax on his whole income?

3. What is the income tax of a single person on a yearly income of $6000 under the act of 1916? What is the average rate of tax on his whole income?

4. What is the income tax of a single person on a yearly income of $79,000 under the act of 1916? What is the average rate of tax on his whole income?

5. What is the income tax of a single person on a yearly income of $500,000 under the act of 1916? What is the average rate on his whole income?

6. Find the combined income tax of a married person, having no dependent children, under the acts of 1916 and 1917, if his income is $6000. Find the average rate on his whole income. What would be his average rate if he had 4 children?

7. Find the combined income tax of an unmarried person under the acts of 1916 and 1917 if his annual income is $6000. Find the average rate on his whole income.

8. Find the combined income tax of an unmarried person on $79,000 under the acts of 1916 and 1917. Find average rate on the whole income.

9. Find the combined income tax of an unmarried person on $270,000 under the acts of 1916 and 1917. Find the average rate on the whole income.

THE METRIC SYSTEM

335. Use of Metric System. The metric system of measure has been adopted in most civilized countries except England and the United States. In these two countries its use is *permitted*, but *not made compulsory.* Manufacturers in England and the United States who seek foreign markets, use the metric system in describing their goods. It is used by scientists in all parts of the world.

336. The Meter. The fundamental unit of the metric system is the unit of length called the meter. The meter is very nearly equal to 39.37 inches, and this has been established by the Congress of the United States as its legal equivalent.

337. Subdivisions of the Meter. The meter is divided into tenths, hundredths, and thousandths.

One-tenth of a meter is a *decimeter*, one hundredth of a meter is a *centimeter*, and one thousandth of a meter is a *millimeter.*

338. Multiples of the Meter. Ten meters is one decameter, one hundred meters is one hectometer, and one thousand meters is one kilometer.

Practically the whole trouble in learning the tables of metric weights and measures consists in learning the Latin words: deci = ten, centi = one hundred, milli = thousand; and the Greek words deca = ten, hekto = hundred, and kilo = thousand. The Latin words are used to designate parts of the principal units, as decimeter (tenth of meter), centimeter (hundredth of meter). The Greek words are used to designate multiples of the principal units, as decameter (ten meters), hectometer (hundred meters).

The most important fact about the metric system is that each unit is ten times as great as the next lower unit. This makes the system exceptionally convenient for expression in our decimal system of numeration. Thus, 3 kilometers, 4 hectometers, 8 decameters, 6 meters. 2 decimeters, and 8 centimeters is written 3486.28 meters.

339. **The Metric and English Units of Length.** The lengths of the meter, decimeter, centimeter, and millimeter, respectively in inches are: 39.37, 3.937, .3937, and .03937. To find these we have only to divide 39.37 by 10, 100, and 1000, respectively.

Example 1. One English mile is how many kilometers? Find result to the nearest hundredth of a kilometer.

Solution: $\frac{5280 \times 12}{39.37} = \text{(kilometers)} = 1609.13 \text{ (meters)} = 1.61.$

Example 2. One kilometer is what part of an English mile? Express as a decimal to the nearest hundredth of a mile.

Solution: One kilometer $= \frac{39.37 \times 1000}{12}$ feet $= 3280.8$ feet.

$$\frac{3280.8}{5280} = .62 = \text{number of miles in one kilometer.}$$

Hence, *to find the number of kilometers, multiply the number of English miles by 1.61, and to find the number of English miles multiply the number of kilometers by .62.*

WRITTEN EXERCISES

The following records in aviation speeds were reported for one year. Fill in the blank columns:

Distance		Time		Speed	
Kilometers	Miles	Hours	Minutes	Kilometers per hour	Miles per hour
45.664	....	....	15	......	......
84.665	....	....	30	......	......
168.244	....	1	....	......	......
234.431	....	2	....	......	......
310.387	....	3	....	......	......
325.9	....	4	....	......	......
407.67	....	5	....	......	......

340. Square Measure. The units of area, or square measure, in the metric system, are naturally the squares of the units of length. Thus, the square meter, the square centimeter, and the square kilometer are units of area.

One hundred square meters are called an *are*. One hundred *ares*, or one *hectare*, is the unit used in measuring pieces of land where the acre would be used by us.

THE TABLE OF SQUARE MEASURE

1 square meter	= 1 centare
100 centares	= 1 are
100 ares	= 1 hectare
100 hectares	= 1 square kilometer

WRITTEN EXERCISES

1. One centare is how many square feet? Express in decimals to the nearest hundredth.

2. One hectare is how many acres? First find the number of square feet in one acre, and also in one hectare. (See page 319 for answer.)

3. In France, a farm is sold at 3500 francs per hectare. How many dollars per acre is this? (19¢ = 1 franc.)

4. In Germany a farm sold at 3000 marks per hectare. How many dollars per acre is this? (24¢ = 1 mark.)

5. How many square kilometers are there in one square mile? Express in decimals to the nearest hundredth.

6. How many square miles are there in one square kilometer?

7. The areas of France and Germany before the war were 207,054 and 208,830 square miles respectively. How many square kilometers were there in each of these?

8. How many square kilometers are there in your own state?

341. **Units of Cubic Measure.** The units of cubic measure in the metric system are the cubes of the units of length. Thus the cubic meter, the cubic decimeter, and the cubic millimeter are units of volume.

The cubic decimeter is the unit of dry and liquid measure. It is called the *liter*. Ten liters are one *dekaliter*. One hundred liters and one thousand liters are called the *hectoliter* and the *kiloliter* respectively.

A *milliliter* is one cubic centimeter. This is used a great deal in scientific measurements of liquids, and is denoted by ccm. The hectoliter is used for measuring substances such as we measure by the bushel or the barrel.

WRITTEN EXERCISES

1. Find the number of cubic inches in one liter. How does this compare with the number of cubic inches in a liquid quart? (The number of cubic inches in 4 quarts, or a gallon, is 231.)

2. One liquid quart is how many liters? Give decimal correct to the nearest thousandth.

3. One liter is how many liquid quarts? Give result in a decimal correct to the nearest thousandth.

4. One liter is how many dry quarts? (There are 32 dry quarts in one bushel, and one bushel contains 2150.42 cubic inches.)

5. One cubic centimeter (ccm.) is how many cubic inches?

6. One hectoliter is how many bushels?

7. One bushel is how many hectoliters?

8. One cubic meter is how many cubic yards?

9. An excavation is 18 meters long, 16 meters wide, and 1.6 meters deep. How many cubic meters is this? How many cubic yards is it?

342. **Weights.** The principal unit of weight is the *gram*, which is the weight of one cubic centimeter of pure water at a temperature of 39°.

The subdivisions of the gram into tenths, hundredths, and thousandths are called *decigram*, *centigram*, and *milligram*. Ten grams, one hundred grams, and one thousand grams are called *dekagram*, *hectogram*, and *kilogram*, respectively.

The gram is a very small unit of weight, being equal to 0.03527 ounces. For this reason the kilogram, which is equal to 2.2046 pounds, is used as the unit in most commercial transactions. Butter, meat, flour, sugar, etc., are bought and sold by the kilogram. The gram and milligram are used for scientific purposes.

One cubic decimeter = 1000 cubic centimeters. Hence, one cubic decimeter or one liter holds 1000 grams of water, or 1 kilogram. One cubic meter of water weighs 1000 kilograms.

WRITTEN EXERCISES

1. How many kilograms make one ton of 2000 pounds?

2. How many kilograms make one long ton of 2240 pounds?

3. A tank of water is 1.8 meters long, .9 meters wide, and .4 meters deep. How many kilograms of water does it hold?

4. A cylindrical water tank is 1.4 meters in diameter, and 1.8 meters high. How many liters of water does it hold?

5. A gasoline tank is .75 meters in diameter, and .85 meters high. How many liters of gasoline does it hold?

6. Reduce 1.750 cubic decimeters to cubic centimeters.

7. Reduce 3 kilometers, 14 decameters, 3 meters, 7 decimeters, 5 centimeters, and 3 millimeters to millimeters.

8. Reduce 3976480 millimeters to centimeters, to decimeters, to meters, to decameters, to hectometers, to kilometers.

The problems on this page should be compared with similar problems on denominate numbers where the American tables of weights and measures are used. The immense superiority of the metric system is apparent. Practically all the reduction can be made orally.

1. Reduce 3461.74 meters to kilometers and meters.
Solution: 3461.74 meters = 3 kilometers + 461.74 meters.

2. Reduce 15 kilometers and 385 meters to meters.

3. Reduce 14 kilograms 680 grams to grams.

4. Reduce 3 kilometers 143 meters to meters.

5. Reduce 7 hectoliters 36 liters to liters.

6. Reduce 18749 grams to kilograms.

7. Reduce 4562 meters to kilometers.

8. Reduce 24960 liters to hectoliters.

9. Divide 384 kilograms 480 grams by 7.

10. Multiply 1 kilometer 790 meters by 8
Suggestion: Reduce to meters.

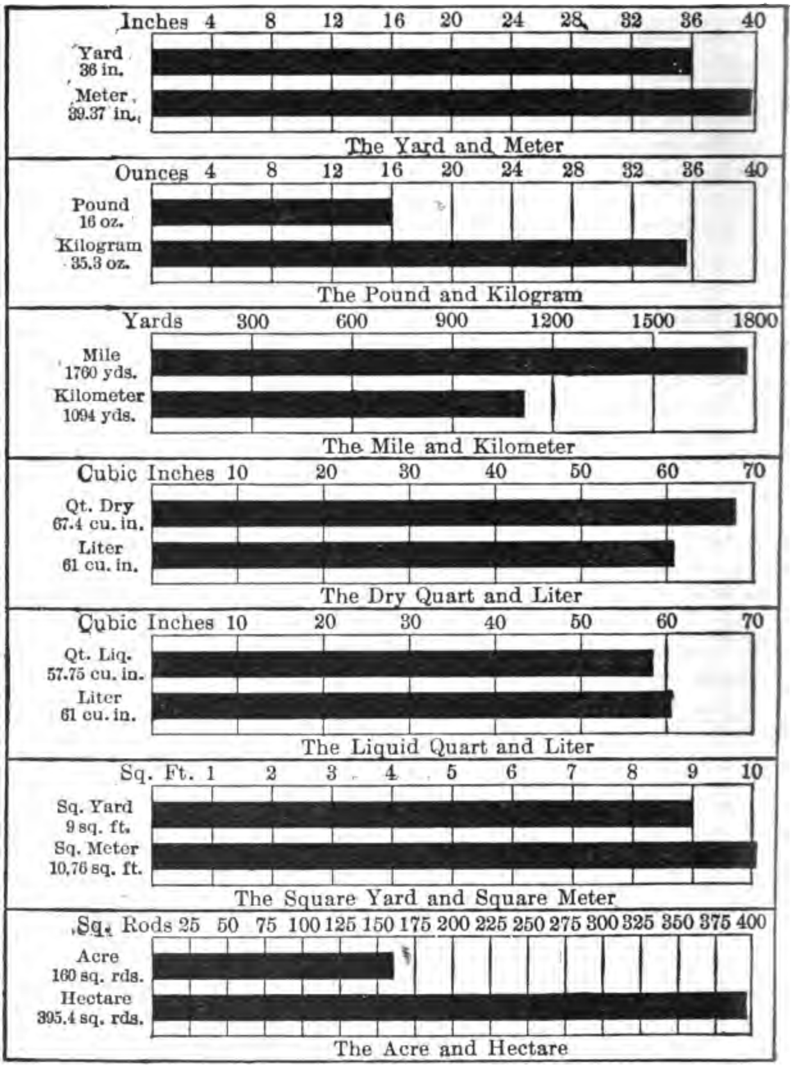
11. An automobile goes 340 kilometers 522 meters in 5 hours. What is the average speed per hour?

12. A rectangular field is 125 meters wide, and 360 meters long. How many hectares does it contain?

13. A piece of timber is 4.7 meters long, 32 centimeters wide, and 14 centimeters thick. Find its cubic contents in cubic meters. Also find it in cubic decimeters.

14. How many kilograms of water does a cylindrical tank hold which is 83 centimeters in diameter, and 1.42 meters high?

Relations between American and metric units of measure.



Relations between American and metric units.

MEASURES OF LENGTH

Metric Units		Equivalents in American Units
Kilometer (km.),	1,000 meters	0.62137 mile, or 3280 ft. 10 in.
Meter (m.)	1 meter	39.37 inches
Decimeter (dm.)	1/10 meter	3.937 inches
Centimeter (cm.)	1/100 meter	0.3937 inches
Millimeter (mm.)	1/1000 meter	0.03937 inches

MEASURES OF SURFACE

Metric Units		Equivalents in American Units
Hectare (ha.)	10,000 square meters	2.471 acres
Are (a.)	100 square meters	119.6 square yards
Centare (m^2.)	1 square meter	1,500 square inches

MEASURES OF CAPACITY

Metric Units			Equivalents in American Units	
Names	Number of liters	Cubic measure	Dry measure	Liquid or wine measure
Kiloliter	1,000	1 cu. meter	1.308 cu. yds.	284.17 gallons
Hectoliter	100	1/10 cu. meter	2 bu. 3.35 pks.	26.417 gallons
Dekaliter	10	10 cu. decimeters	9.02 quarts	2.6417 gallons
Liter	1	1 cu. decimeter	0.908 quarts	1.0567 quarts
Deciliter	1/10	.1 cu. decimeter	6.1022 cu. inches	0.845 gills
Centiliter	1/100	1 cu. centimeter	0.061 cu. inches	0.27 fluid drams
Milliliter	1/1000	10 cu. centimeters	1.6102 cu. inches	0.388 fluid ounces

Quantity of water whose weight at maximum density is equal to the unit opposite	Metric names	Metric Units Number of grams	Equivalents in American Units Avoirdupois weight
1 cu. meter	Miller or Tonneau (t.)	1,000,000	2204.6 pounds
1 hectoliter	Quintal (q.)	100,000	220.46 pounds
1 decaliter	Kilogram (k.)	1,000	2.2046 pounds
1 deciliter	Hectogram (hg.)	100	3.5274 ounces
1 cu. centimeter	Gram (g.)	1	15.432 grains
1 cu. millimeter	Milligram (mg.)	1/10000	.01543 grains

343. Literal Notation. Some parts of arithmetic may be simplified by using certain abbreviations which are really characteristic of another branch of mathematics called *algebra.*

We have already used the equations:

$$\textit{Length} \times \textit{width} = \textit{area}.$$
$$\textit{Length} \times \textit{width} \times \textit{depth} = \textit{volume}.$$

Instead of the words length, width, depth, area, and volume, we will now use merely the initial letters: l, w, d, a, v.

Then the above equations read:

$$l \times w = a.$$
$$l \times w \times d = v.$$

When we speak of the product of two letters, as l and w, we mean the product of the numbers which they represent.

344. Omission of Multiplication Sign. It is customary not to write the sign "$\times$" between two letters, whose product is indicated. Thus we write lw instead of $l \times w$, and lwd instead of $l \times w \times d$, similarly $3a$ means $3 \times a$.

When a product has one factor expressed by means of a letter, and the other by means of an Arabic numeral the numeral is placed first. That is, we write $3a$ and not $a3$. $3ab$ means $3 \times a \times b$.

Notice the difference between this notation and the Arabic and Roman notation. Thus, lw means l multiplied by w, while 56 means 50+6.

345. A Number Symbol is any expression used to represent a number.

In practice the expression "number symbol" is made to refer to special symbols, other than the ordinary words.

Thus, 47, VII, l, w, lw, are number symbols.

346. Squares and Cubes. The square of a number represented by n is written n^2. That is $n^2 = n \times n$ or nn. (See page 228.)

Similarly, the cube of a number represented by n is written n^3. That is, $n^3 = n \times n \times n$ or nnn.

347. Exponents. The small numbers 2 and 3 used thus to indicate square and cubes are called *exponents*.

ORAL EXERCISES

Find the values of the following:

1. 2^2	5. 4^2	9. $(\frac{1}{2})^2$	13. $(\frac{2}{3})^2$	17. $(\frac{3}{4})^2$
2. 2^3	6. 6^3	10. $(\frac{1}{2})^3$	14. $(\frac{2}{3})^3$	18. $(\frac{3}{4})^3$
3. 3^2	7. 5^2	11. $(\frac{1}{3})^2$	15. $(\frac{3}{5})^2$	19. $(\frac{4}{5})^2$
4. 3^3	8. 5^3	12. $(\frac{1}{3})^3$	16. $(\frac{3}{5})^3$	20. $(\frac{4}{5})^3$

348. General Products. In reading products we must be careful to notice that an exponent applies only to the letter or numeral next to which it is written. Thus, $3\times2^2=3\times4=12$, and not $(3\times2)\ (3\times2)=36$.

ORAL EXERCISES

Find the values of the following:

1. 2×3^2	5. $\frac{2}{3}\times3^2$	9. $2\times(\frac{2}{3})^2$	13. 3×4^2
2. 2×3^3	6. $\frac{2}{3}\times3^3$	10. $2\times(\frac{2}{3})^3$	14. 3×4^3
3. 2×2^2	7. $3\times(\frac{1}{2})^2$	11. $2\times(\frac{1}{4})^2$	15. $3\times(\frac{3}{4})^2$
4. 2×2^3	8. $3\times(\frac{1}{2})^3$	12. $4\times(\frac{1}{4})^3$	16. $3\times(\frac{3}{4})^3$

349. Sum of Products. When two products are connected by the sign + or the sign − the products are to be found before the addition or subtraction is performed.

Thus, $2\times3+3\times4=6+12=18$, and $5\times6-3\times7=30-21=9$.

ORAL EXERCISES

Find the results in the following:

1. $4\times6-2\times3$	3. $7\times2+5\times3$	5. $12\times6-3\times8$
2. $5\times9+3\times7$	4. $10\times9-5\times6$	6. $8\times9+3\times6$

350. The Equation. An *equation* consists of two number symbols connected by the sign =.

Thus, $lw=a$, $lwd=v$, are equations.

The sign, =, is called the sign of *equality*.

The two number expressions connected by the equality sign are called the *members* of the equation.

Thus, lw and a are the members of the equation $lw=a$.

351. Interpretation of Equations. Since the letters used in an equation are simply abbreviations, an equation may be translated into words. Thus, $lw=a$ means "length multiplied by width equals area."

Example. 1. If b=base, r=rate, and p=percentage, translate the equation $br=p$ into words.

Solution: Base multiplied by *rate* equals *percentage*.

Example 2. If p=principal, r=rate, t=time, and i=interest, interpret the equation $prt=i$.

Solution: Principal multiplied by *rate* and the product by the *time* equals *interest*.

352. Advantage of Using the Equations. Practically every rule of arithmetic may be stated in the form of an equation. One advantage of the equation form of the arithmetic rule is that the equation is much shorter, and hence easier to remember than the same rule written out fully in words. Other advantages will appear when you come to study algebra.

ORAL EXERCISES

Translate each of the following equations into words:

Equation	Meaning of letters
1. $lw=a$	(l=length of rectangle, w=width, a=area).
2. $lsd=v$	(l=length of solid, w=width, d=depth, v= volume).

	Equation	Meaning of letters
3.	$pn=c$	(p = price per unit, n = number of units, c = cost).
4.	$vt=s$	(v = velocity, t = time, s = space or distance).
5.	$\frac{a}{b}=\frac{ma}{mb}$	(a, b, m, any numbers).
6.	$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bc}$	(a, b, c, d, any numbers).
7.	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$	(a, b, c, d, any numbers).
8.	$br=p$	(b = base, r = rate, p = percentage).
9.	$prt=i$	(p = principal, r = rate, t = time, i = interest).
10.	$pr=c$	(p = price or cost, r = rate commission, c = commission).
11.	$\frac{bh}{2}=a$	(b = base, h = altitude, a = area, all of a triangle).
12.	$\frac{h}{2}(b+c)=a$	(a, b, and c, are bases of a trapezoid, h and altitude and area).
13.	$\sqrt{a^2+b^2}=c$	(a and b are the sides of a right triangle and c its hypothenuse.)
14.		(same as in Example 13).
15.	$\pi r^2=a$	(r and a are radius and area of circle, and $\pi=3\frac{1}{7}$ or 3.1416).
16.	$bh=v$	(b = area of base, h = altitude, v = volume of prism or cylinder).
17.	$\frac{bh}{3}=v$	(b = area of base, h = altitude, v = volume of pyramid or cone).
18.	$4\pi r^2=a$	(r = radius and a = area of surface of sphere.)
19.	$\frac{4}{3}\pi r^3=v$	(r = radius, and v = volume of sphere).

353. Use of Equation in Solving Problems. The equation is the most important single instrument ever devised for the solution of problems.

A large share of the subject of algebra consists of a study of the equation and its uses. We shall now study some of the ways in which an equation may be changed in the solution of a problem.

354. How an Equation May be Changed. The following are the ways in which an equation may be changed in the solution of a problem.

(a) *The same number may be added to both members.*
(b) *The same number may be subtracted from both members.*
(c) *Both members may be multiplied by the same number.*
(d) *Both members may be divided by the same number.*

Example 1. What is the value of x in the equation $x-4=12$?
Solution: If x less 4 is 12 then x must be $12+4=16$.

This may be found directly by adding 4 to both members of $x-4=12$ and noticing that $x-4+4=x$.

Example 2. What is the value of x in $x+7=20$.
Solution: Subtracting 7 from both members we have: $x+7-7=20-7$ or $x=13$.

Example 3. $\frac{x}{8}=2$, find the value of x.

Solution: Multiply both members by 8. Then $\frac{x}{\cancel{8}}\times\cancel{8}=2\times 8$ or $x=16$.

Example 4. $4x=20$, find the value of x.

Solution: Dividing both members by 4 we have: $\frac{4x}{4}=\frac{20}{4}$ or $x=5$.

These examples show how the changes indicated in (a), (b), (c), (d), may be made in an equation. We shall now show how these operations may be used in solving problems in arithmetic.

The principles used in these examples have already been used repeatedly in solving problems. Thus, in Example 4 the principle of product and factors is used.

Example 1. From the equation $lw=a$ (length×width equals area) derive rules for finding the length and also the width, when the other elements of the problem are given.

Solution: (a) Divide both members of $lw=a$ by l.

Then,

$$\frac{lw}{l}=\frac{a}{l} \text{ or } w=\frac{a}{l} \qquad (1)$$

(b) Divide both members of $lw=a$ by w.

$$\frac{lw}{w}=\frac{a}{w} \text{ or } l=\frac{a}{w} \qquad (2)$$

Example 2. From $lwd=v$ (length×width×depth equals volume) derive rules for finding the length, the width or the depth when the other elements of the problem are given.

(3) *Solution:* (a) Dividing both members of $lwd=v$ by wd, $\frac{lwd}{wd}=\frac{v}{wd}$ or $l=\frac{v}{wd}$ (3)

(b) Dividing by ld, $\frac{lwd}{ld}=\frac{v}{ld}$ or $w=\frac{v}{ld}$ (4)

(c) Dividing by lw, $\frac{lwd}{lw}=\frac{v}{lw}$ or $l=\frac{v}{lw}$ (5)

EXERCISES

1. Translate equations (1) and (2) into words.

2. Translate equations (3), (4), and (5) into words.

3. From $pn=c$ (price×numbers of articles = cost) derive equations given p and n. Translate each equation into words.

4. From $vt=s$ (see exercise 4, page 322), derive equations giving v and t. Translate each into words.

5. From $br=p$ (Example 8, page 323), derive equations giving b and r. Translate each into words.

6. From $prt=i$ (Example 9, page 323), derive equations giving p, r and i. Translate each into words.

Example. 1. From $\pi r^2=a$, derive an equation giving r in terms of a and π.

Solution: Dividing both members of $\pi r^2=a$ by π, $r^2=\frac{a}{\pi}$.

Taking square roots of both sides: $r=\sqrt{\frac{a}{\pi}}$.

(Taking a square root or a cube root of both members of an equation may be regarded as dividing both members by the same number.)

Example. 2. Find the radius of a circle whose area is 100 square inches.

Solution: $r=\sqrt{\frac{a}{\pi}}=\sqrt{\frac{100}{\pi}}=\sqrt{31.8309}=5.642.$

Before approximating the square root of $\dfrac{100}{3.1416}$ reduce to a four-place decimal.

Example 3. From $4\pi r^2=a$ (Example 21, page 323), derive an equation giving r in terms of a and π.

Suggestion: Divide by 4π and take square roots, obtaining $r=\sqrt{\frac{a}{4\pi}}$.

Example 4. Find the radius of a sphere the area of whose surface is 100 square inches.

Suggestion: Substitute 100 for a in $r=\sqrt{\frac{a}{4\pi}}$, obtaining $r=\sqrt{\frac{25}{\pi}}=\sqrt{7.9577}$.

Carry out each step fully.

EXERCISES

1. Find the radius in rods of a circle whose area is one acre (160 square rods).
2. What is the radius of a circle whose area is one square foot? Find radius in inches.
3. Find in inches the radius of a sphere whose surface area is 1 square foot.
4. Find the radius of a sphere whose surface is equal to that of a foot cube.

EXERCISES

1. From $pr=c$ (Example 10, page 323), derive equations giving p and r. Translate each into words.

2. From $\frac{bh}{2}=a$ (Example 11, page 323), derive equations giving b and h. Translate each into words.

Suggestion: First multiply each number by 2 giving $bh=2a$.

3. From $\pi rs=a$ (Example 20, page 323), derive equations giving r and s. Translate each into words.

4. Find the radius of a cone if its lateral area is 50 square inches and its slant height is 8 inches.

5. Find the slant height of a cone if its lateral area is 200 square inches and its radius 4 inches.

6. Find the base of a triangle if its area is 350 square inches and its altitude 7 inches.

7. Find the altitude of a triangle if its area is 64 square inches and its base 11 inches.

8. From $\frac{bh}{3}=v$ (Example 17, page 323), derive equation giving b and h. Translate each into words.

9. Find the altitude of a pyramid if its volume is 250 cubic inches and the area of its base is 40 square inches.

10. Find the area of the base of a cone, if its volume is 85 cubic inches and its altitude 5 inches.

11. Find the radius of the base of a cone, if its altitude is 10 inches and its volume 150 inches.

Suggestion: First find the area of its base and then find the radius of the base, as in Example 2, page 326.

12. A pyramid, with a square base has an altitude of 12 inches, and a volume of 1600 cubic inches. Find a side of its base.

355. Volume of a Cube. To find the volume of a cube, we multiply the length of an edge by itself twice.

Thus, the number of cubic feet in a cubic yard is $3 \times 3 \times 3$, and the number of cubic inches in a cubic foot is $12 \times 12 \times 12$.

356. The Cube of a Number. The product obtained by using a number three times as a factor is called the cube of that number.

Thus, $3^3 = 27$, $12^3 = 1728$, and $(\frac{1}{2})^3 = \frac{1}{8}$.

The problem of finding the length of the edge of a cubical box when its volume is given consists in finding one of the three equal factors of the number representing its volume.

357. The Cube Root of a Number. One of the three equal factors of a number is called the cube root of the number.

The cube root of a number is indicated by placing the symbol $\sqrt[3]{}$ over the number.

Thus, $\sqrt[3]{8} = 2$, $\sqrt[3]{1728} = 12$, and $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$.

358. Perfect Cubes. A number which is the cube of an integer or a fraction is called a perfect cube.

Thus, 8, 27, 64, and $\frac{1}{27}$ are all perfect cubes.

The cube root of a perfect cube can be found exactly.

Thus, $\sqrt[3]{1000} = 10$, and $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$.

359. Approximate Cube Roots. In the case of numbers such as 2, which are not perfect cubes, we can only find approximations to the cube root.

Thus, $1.2^3 = 1.728$, and $1.3^3 = 2.197$. Hence, $\sqrt[3]{2}$ lies between 1.2 and 1.3. (Compare approximations to $\sqrt{2}$, page 228.)

Again, $1.26^3 = 2.000376$ and $1.2599^3 = 1.999900-$. Hence $\sqrt[3]{2}$ lies between 1.2600 and 1.2599, and is therefore very close to 1.26.

In the case of small numbers, it is always possible to find, at sight, two consecutive integers between which the cube root lies.

Thus, the cube root of 50 lies between 3 and 4 because $3^3 = 27$ and $4^3 = 64$.

State two consecutive integers between which each of the following lies:

1. $\sqrt[3]{85}$	4. $\sqrt[3]{489}$	7. $\sqrt[3]{935}$
2. $\sqrt[3]{62}$	5. $\sqrt[3]{539}$	8. $\sqrt[3]{849}$
3. $\sqrt[3]{291}$	6. $\sqrt[3]{730}$	9. $\sqrt{732}$

360. The First Approximation of a Cube Root. The first approximation to cube root is made by the following:

Rule. (1) *Divide the number into groups of three digits each, beginning from the right, or at the decimal point if there is one.*

(2) *Find two consecutive integers between which lies the cube root of the first group to the left, and annex to these as many zeros as there are groups remaining.*

(3) *Estimate the root between these numbers.*

Example 1. Find a first approximation of $\sqrt[3]{9264891}$.

Solution: The cube root of the left group in 9,264,891 lies between 2 and 3. Hence the required root lies between 200 and 300. Moreover, 9 is much nearer $2^3=8$ than $3^3=27$. Hence we take 210 as the first approximation.

Find a first approximation to each of the following:

10. $\sqrt[3]{43780}$	13. $\sqrt[3]{47824916}$	16. $\sqrt[3]{392478}$
11. $\sqrt[3]{591920}$	14. $\sqrt[3]{293784}$	17. $\sqrt[3]{478293}$
12. $\sqrt[3]{19824}$	15. $\sqrt[3]{487392}$	18. $\sqrt[3]{924783}$

Example 2. Find $\sqrt[3]{73}$. Correct to two decimal places.

Solution: We take 4.3 as the first approximation, and divide 73 by the square of 4.3.

Thus, $73 \div 4.3^2 = 73 \div 18.49 = 3.948$. Hence, $4.3 \times 4.3 \times 3.948$ equals nearly 73. We now take one-third the sum of these factors, obtaining 4.18 as the second approximation. Dividing 73 by 4.18^2, we get 4.1780 as a quotient. Hence, the cube root of 73 lies between 4.18 and 4.178, and is nearer 4.18 than 4.17.

$$\boxed{\begin{array}{r} 4.3 \\ 4.3 \\ 3.948 \\ \hline 3\overline{)12.548} \\ \hline 4.18 \end{array}}$$

Example. Find $\sqrt[3]{2}$ correct to three decimal places.

Solution: $(1.2)^3 = 1.728$, and $(1.3)^3 = 2.197$; we take 1.26 as the first approximation.

$$2 \div (1.26)^2 = 2 \div 1.5876 = 1.2597.$$

One-third the sum of the factors 1.26, 1.26, 1.2597, is 1.2599. Hence the required root lies between 1.26 and 1.2599, and therefore 1.260 is the nearest approximation to three decimal places.

$$\boxed{\begin{array}{r} 1.26 \\ 1.26 \\ 1.2597 \\ \hline 3\overline{)3.7797} \\ \hline 1.2599 \end{array}}$$

361. Rule for Approximating Cube Root. To approximate the cube root of a number, use the following rule:

(1) *Estimate a first approximation, and divide the number by the square of this approximation.*

(2) *To the quotient thus obtained, add twice the first approximation, and divide the sum by 3, thus obtaining a second approximation.*

(3) *Continue this process as far as may be necessary to obtain the required closeness of approximation.*

The root always lies between the last quotient and the next preceding approximation.

The approximation is accurate to at least as many decimal places as the last quotient and the next preceding approximation are alike.

WRITTEN EXERCISES

Find each of the following accurate to two places of decimals:

1. $\sqrt[3]{3}$	4. $\sqrt[3]{465}$	7. $\sqrt[3]{52976}$
2. $\sqrt[3]{5}$	5. $\sqrt[3]{4926}$	8. $\sqrt[3]{194780}$
3. $\sqrt{25}$	6. $\sqrt[3]{19600}$	9. $\sqrt[3]{1246492}$

10. Find, to the nearest hundredth of an inch, the length of an edge of a cube whose volume is 280 cubic inches.

11. Find the edge of a cubical reservoir which will hold 100,000 gallons of water.

Suggestion: Reduce to cubic feet and approximate square root.

362. Square and Cube Root Tables. Engineers and others who frequently need to approximate square and cube roots, have tables giving the roots of all integers from 1 to 1000 or higher, and also of some decimal fractions. On the next two pages, parts of such a table are given.

Example 1. Find the square root of 3500.

Solution: From the table we find that the square root of 35 is 5.9161. Hence, the square root of 3500 is 59.161.

Example 2. Find the square root of 1790.

Solution: Since 1790 is much nearer 1800 than 1700, we find by means of the table that $\sqrt[2]{1800}=42.43$, and use this as a first approximation. Dividing 1790 by 42.43, we get 42.19 as a quotient. Hence the second approximation is 42.31. On dividing we find this to be the nearest approximation in two decimals.

Example 3. Find the cube root of 47,760.

Solution: By the table the cube root of 48,000 is 36.342, which we take as the first approximation to the cube root of 47,760. Squaring and dividing, we get 36.165 as a quotient. The average of the three factors 36.342, 36.342, and 36.165, is 36.282. This approximation is correct to two places of decimals. That is, the required root is 36.28.

From these examples we see that the table can be used to find the approximations to roots of numbers not given in the table.

WRITTEN EXERCISES

Using the table on the following pages, find the following:

1. $\sqrt{35}$	2. $\sqrt{79}$	3. $\sqrt{489}$
4. $\sqrt{7940}$	5. $\sqrt[3]{8310}$	6. $\sqrt[3]{586}$
7. $\sqrt[3]{1392}$	8. $\sqrt[3]{586}$	9. $\sqrt[3]{591}$
10. $\sqrt{9476}$	11. $\sqrt[3]{76548}$	12. $\sqrt[3]{5178}$
13. $\sqrt[3]{1490}$	14. $\sqrt[3]{9142}$	15. $\sqrt[3]{837}$

No.	Square Root	Cube Root	No.	Square Root	Cube Root	No.	Square Root	Cube Root
.1	.3162	.4642	1.85	1.360	1.228	5.1	2.258	1.721
.15	.3873	.5313	1.9	1.378	1.239	5.2	2.280	1.732
.2	.4472	.5848	1.95	1.396	1.249	5.3	2.302	1.744
.25	.500	.6300	2.	1.4142	1.2599	5.4	2.324	1.754
.3	.5477	.6694	2.1	1.449	1.281	5.5	2.345	1.765
.35	.5916	.7047	2.2	1.483	1.301	5.6	2.366	1.776
.4	.6325	.7368	2.3	1.517	1.320	5.7	2.387	1.786
.45	.6708	.7663	2.4	1.549	1.339	5.8	2.408	1.797
.5	.7071	.7937	2.5	1.581	1.357	5.9	2.429	1.807
.55	.7416	.8193	2.6	1.612	1.375	6.	2.4495	1.8171
.6	.7746	.8434	2.7	1.643	1.392	6.1	2.470	1.827
.65	.8062	.8662	2.8	1.673	1.409	6.2	2.490	1.837
.7	.8367	.8879	2.9	1.703	1.426	6.3	2.510	1.847
.75	.8660	.9086	3.	1.7321	1.4422	6.4	2.530	1.857
.8	.8944	.9283	3.1	1.761	1.458	6.5	2.550	1.866
.85	.9219	.9473	3.2	1.789	1.474	6.6	2.569	1.876
.9	.9487	.9655	3.3	1.817	1.489	6.7	2.588	1.885
.95	.9747	.9830	3.4	1.844	1.504	6.8	2.608	1.895
1.	1.	1.	3.5	1.871	1.518	6.9	2.627	1.904
1.05	1.025	1.016	3.6	1.897	1.533	7.	2.6458	1.9129
1.1	1.049	1.032	3.7	1.924	1.547	7.1	2.665	1.922
1.15	1.072	1.048	3.8	1.949	1.560	7.2	2.683	1.931
1.2	1.095	1.063	3.9	1.975	1.574	7.3	2.702	1.940
1.25	1.118	1.077	4.	2.	1.5874	7.4	2.720	1.949
1.3	1.140	1.091	4.1	2.025	1.601	7.5	2.739	1.957
1.35	1.162	1.105	4.2	2.049	1.613	7.5	6.757	1.966
1.4	1.183	1.119	4.3	2.074	1.626	7.7	2.775	1.975
1.45	1.204	1.132	4.4	2.098	1.639	7.8	2.793	1.983
1.5	1.225	1.145	4.5	2.121	1.651	7.9	2.811	1.992
1.55	1.245	1.157	4.6	2.145	1.663	8.	2.8284	2.
1.6	1.265	1.170	4.7	2.168	1.675	8.1	2.846	2.008
1.65	1.285	1.182	4.8	2.191	1.687	8.2	2.864	2.017
1.7	1.304	1.193	4.9	2.214	1.698	8.3	2.881	2.025
1.75	1.323	1.205	5.	2.2361	1.7100	8.4	2.898	2.033
1.8	1.342	1.216		2.				

No.	Square Root	Cube Root	No.	Square Root	Cube Root	No.	Square Root	Cube Root
8.5	2.915	2.041	30	5.4772	3.1072	65	8.0623	4.0207
.6	2.933	2.049	31	5.5678	3.1414	66	8.1240	4.0412
.7	2.950	2.057	32	5.6569	3.1748	67	8.1854	4.0615
.8	2.966	2.065	33	5.7446	3.2075	68	8.2462	4.0817
.9	2.983	2.072	34	5.8310	3.2396	69	8.3066	4.1016
9.	3.	2.0801	35	5.9161	3.2711	70	8.3666	4.1213
.1	3.017	2.088	36	6.	3.3019	71	8.4261	4.1408
.2	3.033	2.095	37	6.0828	3.3322	72	8.4853	4.1602
.3	3.050	2.103	38	6.1644	3.3620	73	8.5440	4.1793
.4	3.066	2.110	39	6.2450	3.3912	74	6.6023	4.1983
.5	3.082	2.118	40	6.3246	3.4200	75	8.6603	4.2172
.6	3.098	2.125	41	6.4031	3.4482	76	8.7178	4.2358
.7	3.114	2.133	42	6.4807	3.4760	77	8.7750	4.2543
.8	3.130	2.140	43	6.5574	3.5034	78	8.8318	4.2727
.9	3.146	2.147	44	6.6332	3.5303	79	8.8882	4.2908
10.	3.1623	2.1544	45	6.7082	3.5569	80	8.9443	4.3089
11.	3.3166	2.2240	46	6.7823	3.5830	81	9.	4.3267
12.	3.4641	2.2894	47	6.8557	3.6088	82	9.0554	4.3445
13.	3.6056	2.3513	48	6.9282	3.6342	83	9.1104	4.3621
14.	3.7417	2.4101	49	7.	3.6593	84	9.1652	4.3795
15.	3.8730	2.4662	50	7.0711	3.6840	85	9.2195	4.3968
16.	4.	2.5198	51	7.1414	3.7084	86	9.2736	4.4140
17.	4.1231	2.5713	52	7.2111	3.7325	87	9.3276	4.4310
18.	4.2426	2.6207	53	7.2801	3.7563	88	9.3808	4.4480
19.	4.3589	2.6684	54	7.3485	3.7798	89	9.4340	4.4647
20.	4.4721	2.7144	55	7.4162	3.8030	90	9.4868	4.4814
21.	4.5826	2.7589	56	7.4833	3.8259	91	9.5394	4.4979
22.	4.6904	2.8020	57	7.5498	3.8485	92	9.5917	4.5144
23.	4.7958	2.8439	58	7.6158	3.8709	93	9.6437	4.5307
24.	4.8990	2.8845	59	7.6811	3.8930	94	9.6954	4.5468
25.	5.	2.9240	60	7.7460	3.9149	95	9.7468	4.5629
26.	5.0990	2.9625	61	7.8102	3.9365	96	9.7980	4.5789
27.	5.1962	3.	62	7.8740	3.9579	97	9.8489	4.5947
28.	5.2915	3.0366	63	7.9373	3.9791	98	9.8995	4.6104
29.	5.3852	3.0723	64	8.	4.	99	9.9499	4.6261

TABLE OF FOOD VALUES

363. Amount of Food Needed by an Individual. The food value of a substance is measured in terms of a unit called a *calory*. The number of calories needed by a grown person depends on the amount of physical work he does, on his size, and also on his age, and varies from 2000 to over 5000 calories per day. The amounts needed by growing children are shown in the following table:

Age	Calories per day	Age	Calories per day
Under 1	300–900	10–13	girls, 1800–2400
1–2	900–1200		boys, 2200–3000
3–5	1200–1500	14–17	girls, 2200–2600
6–9	1400–2000		boys, 2500–4000

From 10% to 15% of the total food should be protein, growing children needing a larger proportion of protein food than the adult.

364. Tables of Food Values. The table below gives the number of calories per pound (except in the case of milk, where the quart is the unit) of each of the three principal food constituents, and also the total food value.

Kind of Food	Protein Calories	Fats Calories	Carbohydrates Calories	Total Calories
Asparagus	27	5	51	83
Bacon, smoked	172	2425	....	2597
Beans, baked	125	102	356	583
Beans, dried lima	328	61	1196	1585
Beans, string, fresh	38	12	125	175
Beef, porterhouse	346	731	....	1077
Beef, roast	428	1131	....	1559
Beef, round	345	522	....	867
Beef, shoulder	298	400	....	698
Beef, sirloin	300	657	....	957
Bread, white	169	49	956	1174
Butter	18	3470	....	3488
Cabbage	26	8	87	121
Cheese, American	523	1466	5	1994

TABLE OF FOOD VALUES

Kind of Food	Protein Calories	Fats Calories	Carbohydrates Calories	Total Calories
Chicken, broilers....	232	57	....	289
Chocolates..........	234	1988	550	2772
Cod, salt............	345	16	....	361
Cottoline...........	...	4082	....	4082
Crackers, oyster.....	205	429	1280	1914
Eggs (per lb.).......	216	279	....	595
Farina, Force, etc....	200	56	1284	1640
Fowl...............	249	502	....	751
Haddock...........	153	8	....	161
Halibut, steak.......	278	180	....	458
Ham, smoked.......	366	915	....	1281
Hickory nuts........	105	1041	78	1224
Lamb chops.........	334	1090	....	1424
Lamb, loin..........	390	984	....	1274
Lard...............	...	4082	....	4082
Milk (per quart).....	129	429	197	755
Mutton, leg.........	274	600	....	874
Oats, rolled.........	303	298	1202	1803
Oleomargarine.......	22	3388	....	3410
Oysters, canned.....	159	98	61	328
Peanuts, in shell.....	354	1187	335	1876
Peas, green.........	65	8	178	251
Pork chops	243	988	....	1231
Potatoes............	33	4	267	304
Rice...............	145	12	1434	1591
Salmon, canned......	354	306	....	660
Sugar..............	...	....	1814	1814
Turkey.............	292	751	....	1043
Wheat flour.........	203	41	1359	1603
White fish..........	415	265	....	680

Breakfast foods such as corn flakes, farina, force, grape nuts, shredded wheat, are of substantially the same food value. Compare rolled oats.

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